Final state $\psi_f(x_B, z_B, t_B)$

$$= \int \underbrace{d\Gamma}_{dx_A\, dz_A} e^{iS(\Gamma)/\hbar} \times \underbrace{\psi_i(x_A, z_A, t=0)}$$

initial state

$t_B = T$

$$= [\delta(x+d) + \delta(x-d)] \times \underbrace{\chi(z)}_{\text{enveloppe}} e^{-imv_0 z/\hbar}$$

initial velocity

Action $S(\Gamma) = \int_0^T L(t)\, dt$

Goal : compute $|\psi_f|^2$

phase factor $e^{-i\vec{p}\cdot\vec{r}/\hbar}$

$$L(t) = \frac{1}{2} m \left(v_x(t)^2 + v_z(t)^2 \right) - mg\, z(t)$$

[link with " university " course]

$$U(t, t_0) = e^{i \int_{t_0}^{t} H(t')\, dt'}$$

$$v_x(t) = v_{x_0} = \frac{x_B - x_A}{T}$$

$$v_z(t) = v_{z_0} - gt$$

$$z(t) = z_A + v_{z_0} t - \frac{1}{2} g t^2$$

Calculations ....

$$\hookrightarrow \quad S_{\cdot}(T) = \frac{1}{2T} m \Big[(x_B - x_A)^2 + (z_B - z_A)^2 \Big]$$

$$- \frac{mg}{2} (z_A + z_B) T \quad - \frac{1}{24} m g^2 T^3 .$$

$$\psi_f(B) = \int dx_A \, dz_A \; e^{iS(x_A, z_A)/\hbar} \; \psi_i(x_A, z_A)$$

$$S(x_A, z_A) = S(x_A) + S(z_A)$$

$$e^{iS} = e^{iS(x_A)} \; e^{iS(z_A)} \rightarrow$$ the integral separates

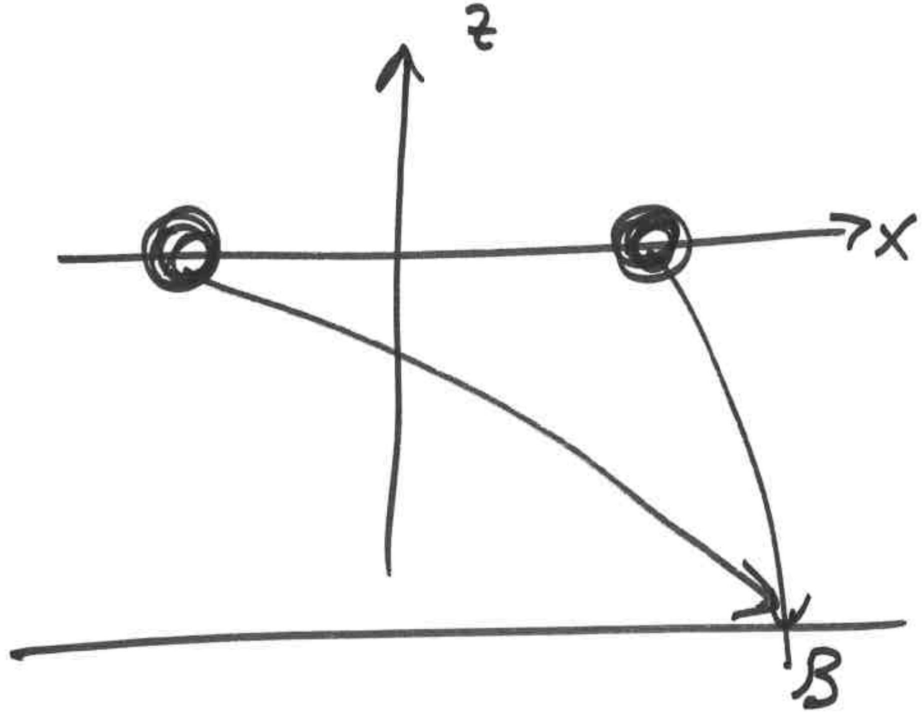
(5

$$\psi_f(B) = \int dx_A \, e^{iS(x_A)/\hbar} \left(\delta(x_A + d) + \delta(x_A - d) \right)$$

$$\times \int dz_A \, e^{iS(z_A)/\hbar} \qquad \chi(z_A) \, e^{-imv_0 z_A/\hbar}$$

Storey, Tannoudji, 1994

"Fermat principle"



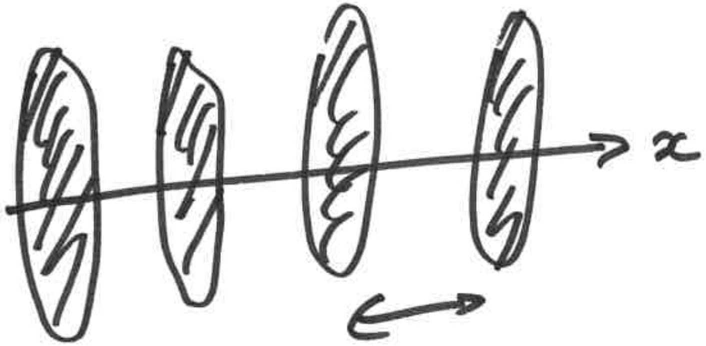
$$\text{integral } 1 = e^{iS(d)/\hbar} + e^{iS(-d)/\hbar}$$

$$\text{integral } 1 = e^{i \frac{m}{2T\hbar} (x_B - d)^2} + e^{i \frac{m}{2T\hbar} (x_B + d)^2}$$ (6

$$S(x_A) = \frac{m}{2T} (x_B - x_A)^2$$

$$\text{integral } 1 = e^{i \frac{m}{2T\hbar} (x_B^2 + d^2)} \times 2\cos\left(\frac{m x_B d}{2T\hbar}\right)$$

Measure : $|\psi_f(B)|^2 \propto \cos^2\left(\frac{m x_B d}{2T\hbar}\right)$

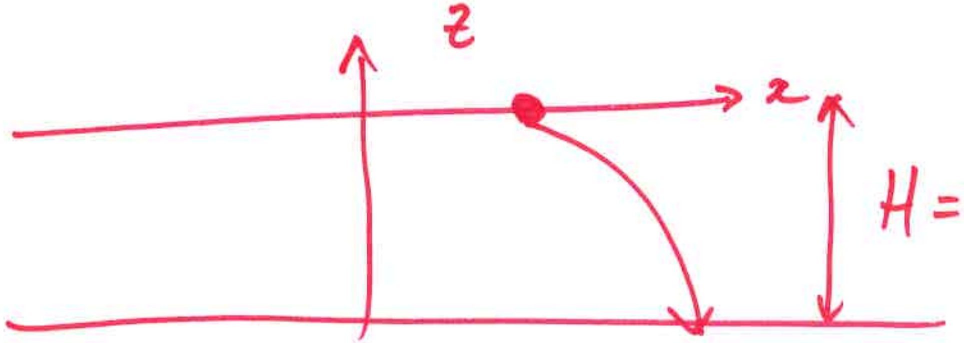


fringe spacing Δx_B

$$\frac{md}{2\pi\hbar}\,\Delta x_B = 2\pi \;\longrightarrow\; \boxed{\Delta x_B = \frac{hT}{md}}$$

The value of T is obtained from the 2^{nd} integral

$$\left(\int dz_A \ldots\right)$$



$$H = \frac{1}{2} g T^2 + v_0 T$$

Not pure point-like source

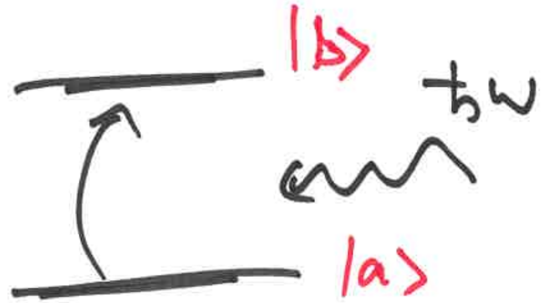
$$\psi_i(x_A) = \alpha\, e^{-\frac{(x_A-d)^2}{2\sigma_x^2}} + \beta\, e^{-\frac{(x_A+d)^2}{2\sigma_x^2}}$$

$$\delta(x_A+d) + \delta(x_A-d)$$

Raman transition

[Reminder of interaction of atom with E.M. field]

$$H = -\vec{\mu} \cdot \vec{B}(t)$$

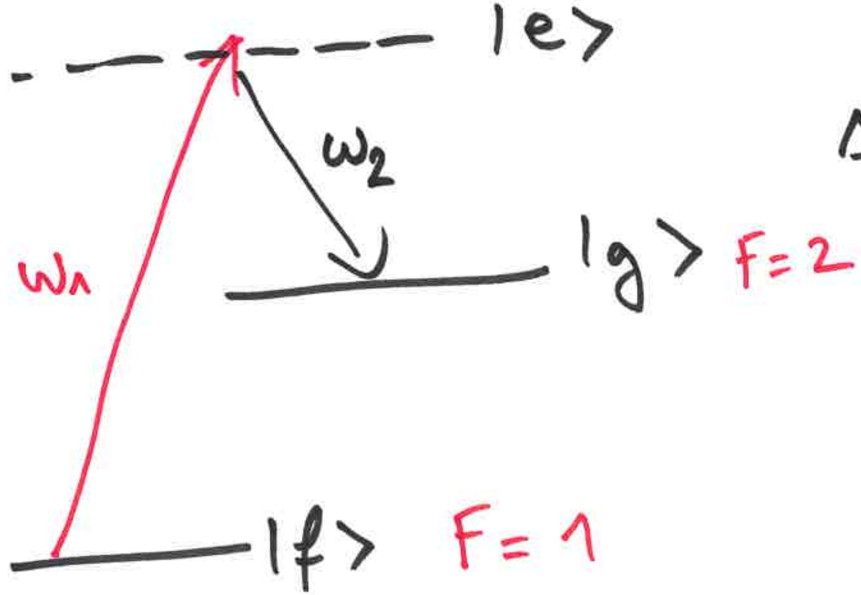


$$H = \begin{pmatrix} x & \frac{\mu}{2} e^{i\varphi} \\ \frac{\mu}{2} e^{-i\varphi} & x \end{pmatrix}$$

Ω = Rabi frequency

$$= \frac{\mu B_0}{\hbar}$$

$$\vec{B}(t) = \vec{B}_0 \cos(\omega t + \varphi)$$



$$\Delta\varphi = (\omega_1 - \omega_2)t + \varphi_1 - \varphi_2$$

^{87}Rb

$$|f\rangle \longrightarrow |g\rangle e^{+i\Delta\varphi}$$

$$|g\rangle \longrightarrow |f\rangle e^{-i\Delta\varphi}$$

$$\omega_1 - \omega_2 = G + \frac{\hbar k_{eff}^2}{2M} + \vec{v} \cdot \vec{k}_{eff}$$

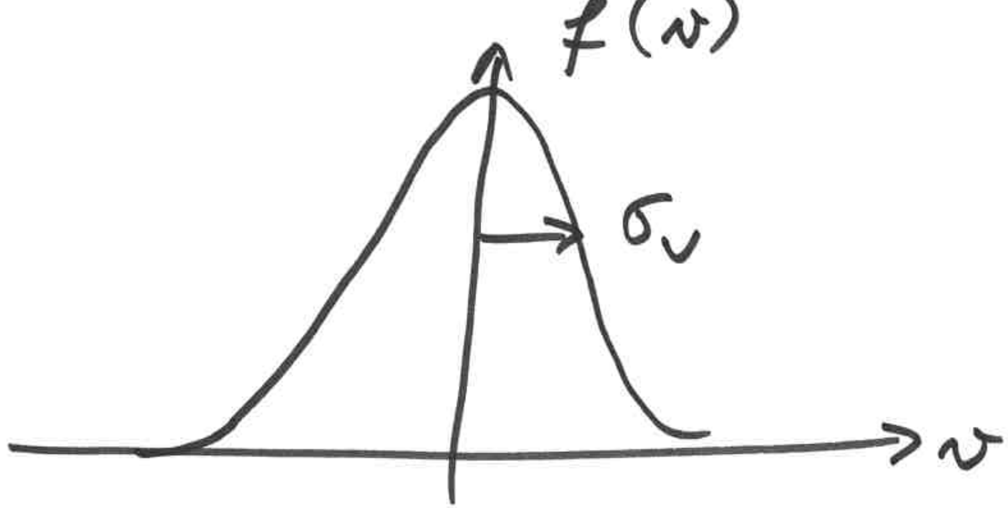
6.8 GHz

(Rb^{87})

$$k_{eff} = 2 \times \frac{2\pi}{\lambda}$$

$\lambda = 780$ nm

cloud of atoms



Boltzmann distribution

$$\frac{1}{2} m \sigma_v^2 = \frac{1}{2} k_B T$$

$$\sigma_v = \sqrt{\frac{k_B T}{m}} \sim 1000 \ m.s^{-1} \ @ \ 300\,K$$

To what should we compare σ_v ?

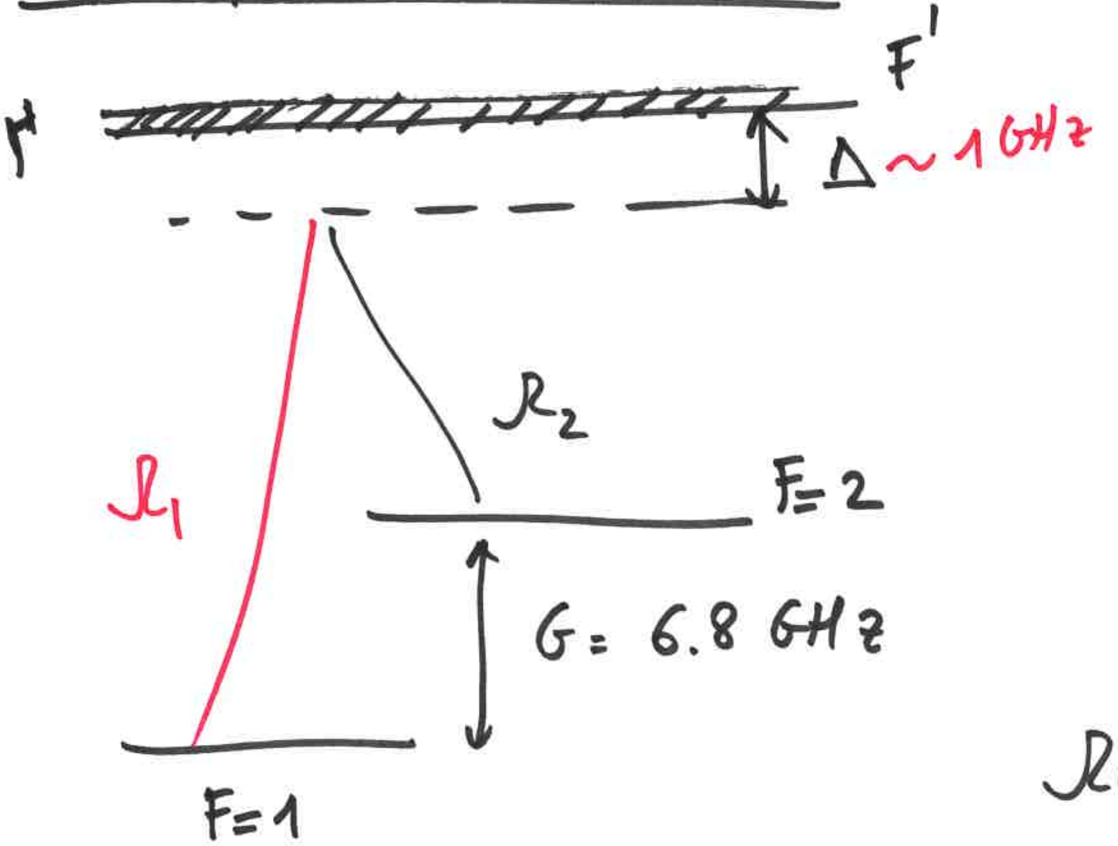
↳ Rabi frequency of the two-photon transition

$$\Omega = \frac{\vec{d} \cdot \vec{E}}{\hbar} \qquad d \sim e \times a_0$$

a_0 ↳ 0.5 Å

two photon transition

$\Delta \gg \Gamma' \sim 10^7 \, \mathrm{rad \cdot s^{-1}}$



$$\frac{\Delta}{\Gamma'} \sim 100 - 1000$$

$$\Omega_1 = \frac{d E_1}{\hbar}$$

$$\Omega_2 = \frac{d E_2}{\hbar}$$

$$\Omega_{eff} = \frac{\Omega_1 \Omega_2}{2\Delta}$$

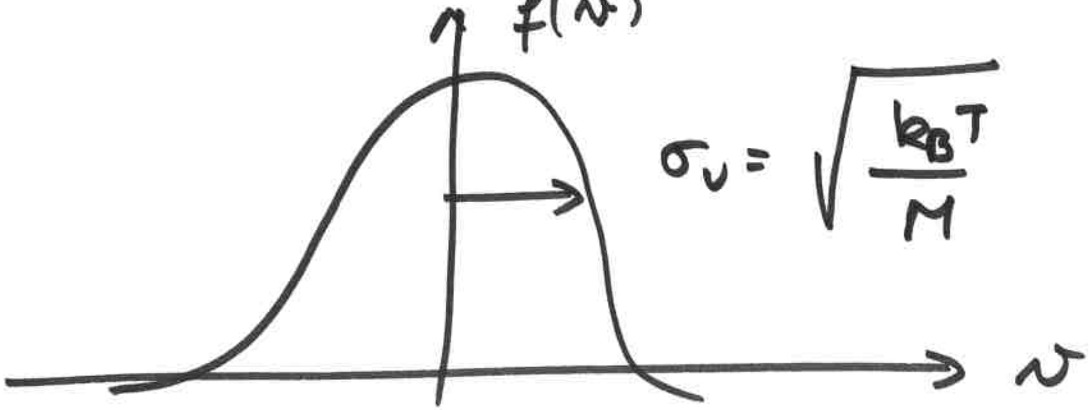
typical strength

$$\boxed{\frac{\Omega_{eff}}{2\pi} \sim 10 \text{ kHz}}$$

laser power

100 mW

waist ~ 10 mm



$$\sigma_v = \sqrt{\frac{k_B T}{M}}$$

$$\text{Doppler shift} = \frac{\vec{k}_{eff} \cdot \vec{v}}{2\pi} = \frac{4\pi/\lambda \; v}{2\pi} = \frac{2v}{\lambda}$$

For the Raman transition to be efficient, I should ensure that

$$\boxed{\frac{k_{eff}}{2\pi} > \frac{2\sigma_v}{\lambda} = \frac{2}{\lambda}\sqrt{\frac{k_B T}{M}}}$$

limited by laser power

$$\frac{2\sigma_v}{\lambda} = \frac{2}{780\,10^{-9}}\left(\frac{1.4\,10^{-23} \times 10^{-6}\,K}{87 \times 1.6\,10^{-27}}\right)^{1/2}$$

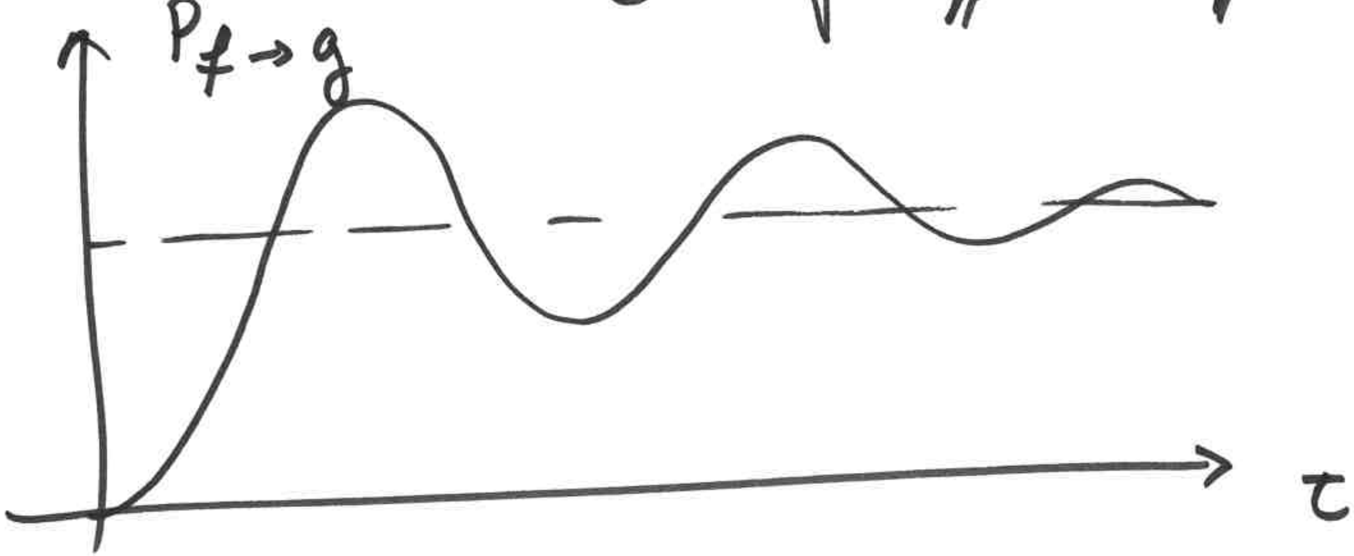
$$= 10^{6}\left(\frac{10^{-29}}{10^{-25}}\right)^{1/2} = 10^{6}\,10^{-2} = 10^{4}\ Hz$$

$$= \underline{10\ \ kHz}$$

1 μK

Rabi oscillation

$$\Omega = \sqrt{\Omega_{eff}^2 + (k_{eff} v)^2}$$




f(v)
v
leff