# Weak values in the study of quantum correlations

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July/20/2021

# Objective

- Determine the utility of weak values to study quantum correlations in general states.

# Structure

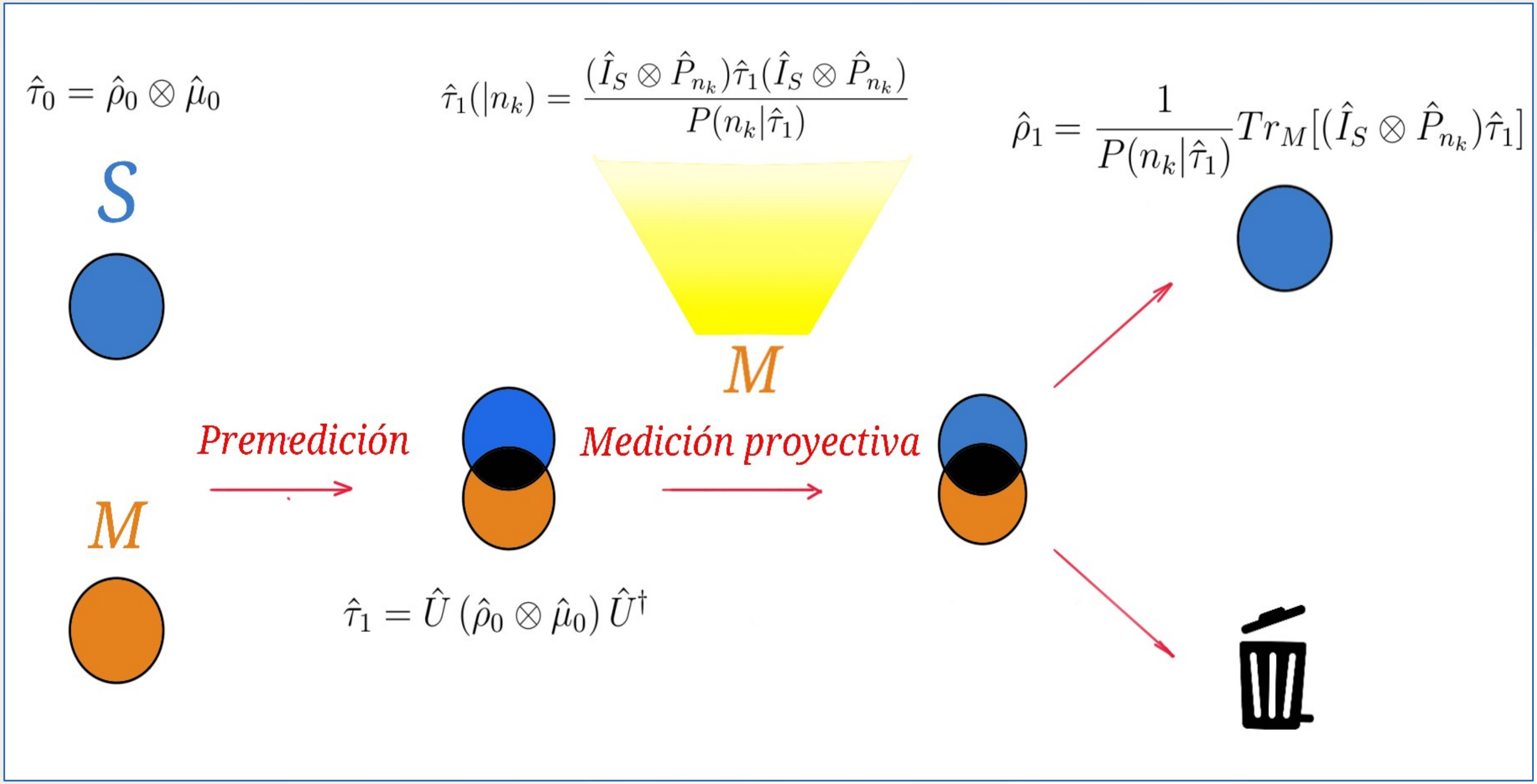
1. Weak measurements and weak values

2. Quantum entanglement criterion

3. Further results  and Conclusions

# 1. Weak measurements and weak values

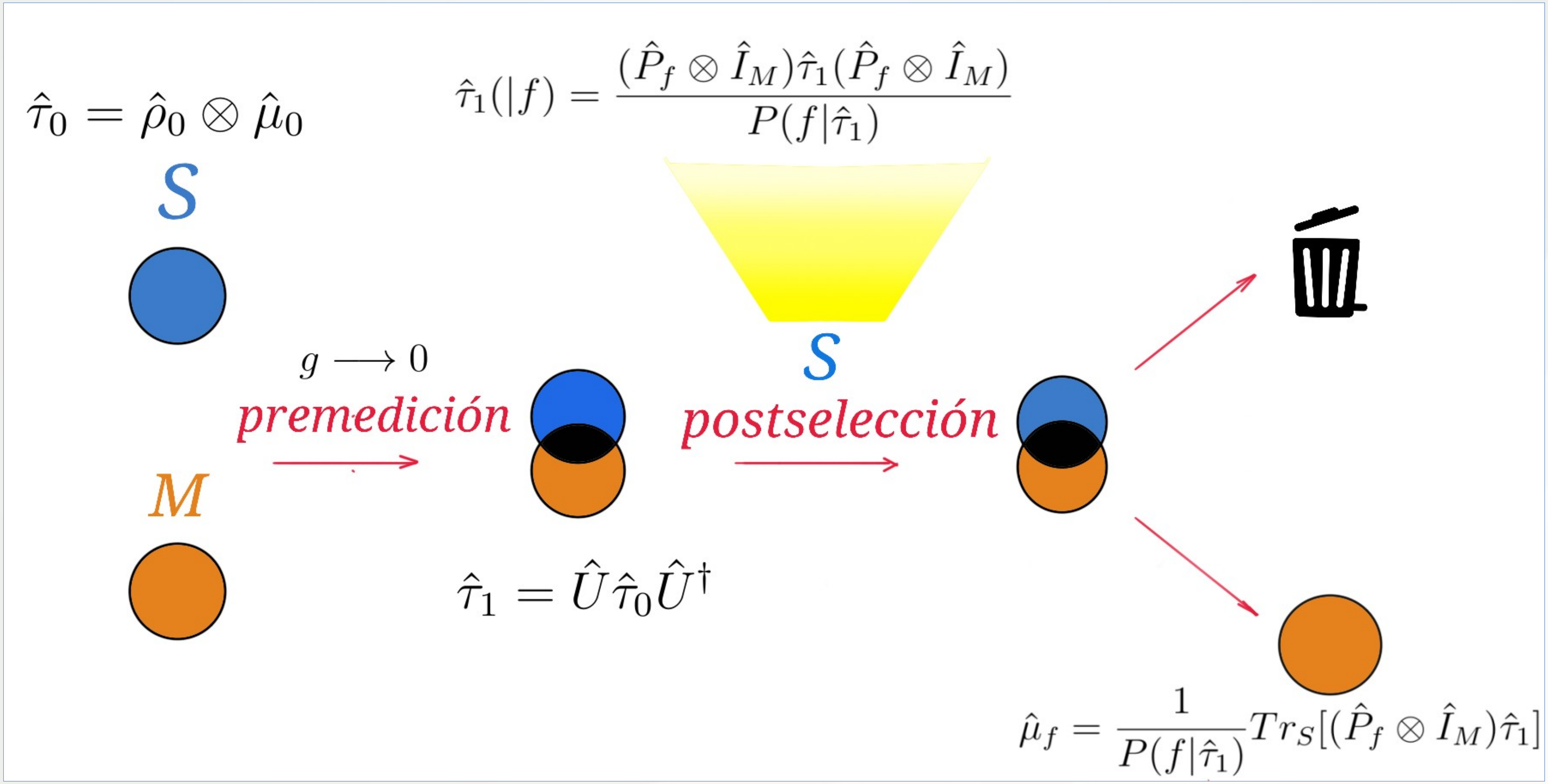
- Ancilla scheme




( Quanta, **2**, 18 (2013))

# 1. Weak measurements and weak values

- Weak measurements with postselection




$$\hat{U} = exp(\tfrac{-i}{\hbar} g\hat{S} \otimes \hat{N}), \quad g = \int_{t-\epsilon}^{t+\epsilon} k(t')dt'.$$

( Quanta, **2**, 18 (2013))

# 1. Weak measurements and weak values

- The state of the apparatus after the postselection is

$$\hat{\mu}_f = \hat{\mu}_0 + \frac{2g}{\hbar} Im(\langle \hat{S} \rangle_w \hat{N} \hat{\mu}_0).$$

With the definition of the weak value for pure states

$$\langle \hat{S} \rangle_w = \frac{\langle f | \hat{S} | s \rangle}{\langle f | s \rangle},$$

(Annals of physics, **376**,448 (2017))

# 2. Quantum entaglement criterion

- Entanglement in pure states of bipartite systems (A|B)

$$|\psi\rangle = |\psi\rangle_A \otimes |\psi\rangle_B.$$

In the configuration space

$$\psi(\mathbf{x}_A, \mathbf{x}_B, t) = \psi_A(\mathbf{x}_A, t)\psi_B(\mathbf{x}_B, t),$$

Separability conditions for amplitude and phase

$$\rho(\mathbf{x}_A, \mathbf{x}_B, t) = \rho_A(\mathbf{x}_A, t)\rho_B(\mathbf{x}_B, t),$$

$$S(\mathbf{x}_A, \mathbf{x}_B, t) = S_A(\mathbf{x}_A, t) + S_B(\mathbf{x}_B, t).$$

(Physics Letters A, **383**, 838 (2019))

# 2. Quantum entaglement criterion

- Consider a 2-particles system in the pure state

$$\psi(\mathbf{x}_1, \mathbf{x}_2, t) = \sqrt{\rho(\mathbf{x}_1, \mathbf{x}_2, t)}e^{iS(\mathbf{x}_1, \mathbf{x}_2, t)}.$$

If you calculate

$$\langle \hat{\mathbf{p}}_i \cdot \hat{\mathbf{p}}_j \rangle_w = \frac{\langle \mathbf{x}_1 \mathbf{x}_2 | \mathbf{p}_i \cdot \hat{\mathbf{p}}_j | \psi \rangle}{\langle \mathbf{x}_1 \mathbf{x}_2 | \psi \rangle}$$

$(i \neq j \in \{1, 2\})$

$$= \frac{\hat{\mathbf{p}}_i \cdot \hat{\mathbf{p}}_j \psi}{\psi}$$

$$\mathbf{u}_j = \frac{\hbar}{2m_j} \frac{\nabla_j \rho}{\rho} \qquad \mathbf{v}_j = \frac{\hbar}{m_j} \nabla_j S$$

$$= \langle \hat{\mathbf{p}}_i \rangle_w \cdot \langle \hat{\mathbf{p}}_j \rangle_w - \hbar m_j (\nabla_i \cdot \mathbf{u}_j + i \nabla_i \cdot \mathbf{v}_j),$$

Physics Letters A, **383**, 838 (2019)

# 2. Quantum entaglement criterion

If the state is separable, the last term vanishes:

$$\nabla_i \cdot \mathbf{u}_j = 0 \qquad \nabla_i \cdot \mathbf{v}_j = 0$$

Sep. in the amplitude    Sep. in the phase

The criterion can be given in simple terms if one defines the quantity:

In analogy to
$C_{\hat{A}\hat{B}} = \langle \hat{A}\hat{B} \rangle - \langle \hat{A} \rangle \langle \hat{B} \rangle$

Weak correlation

$$C^w_{\hat{A}\hat{B}} = \langle \hat{A}\hat{B} \rangle_w - \langle \hat{A} \rangle_w \langle \hat{B} \rangle_w.$$

$$C^w_{\hat{p}_i\hat{p}_j} = -\hbar m_j (\nabla_i \cdot \mathbf{u}_j + i \nabla_i \cdot \mathbf{v}_j)$$

# 3. Further Results and Conclusions

- One can generalize the entanglement criterion to n-particles system in a pure state. What you obtain then is a bipartite entanglement criterion
- You can generalize the weak correlation to mixed states in two non-equivalent ways. The generalizations allow you to stablish quantum discord criteria
- One of this generalizations has to do wo with the skew information defined by Wigner and Yanase
- Weak values are a useful tool for the study of quantum correlations in general systems

Thank you!