Relativistic effects in atom interferometry

Albert Roura

German Aerospace Center (DLR)
Institute of Quantum Technologies, Ulm

PART I (Theoretical Framework)

- Propagation phase & wave-packet evolution:
  - two-level atom as a quantum clock
  - matter-wave propagation in curved spacetime

- Full interferometer and separation phase

  Description in different reference frames

- Retardation & relativistic effects on laser pulses

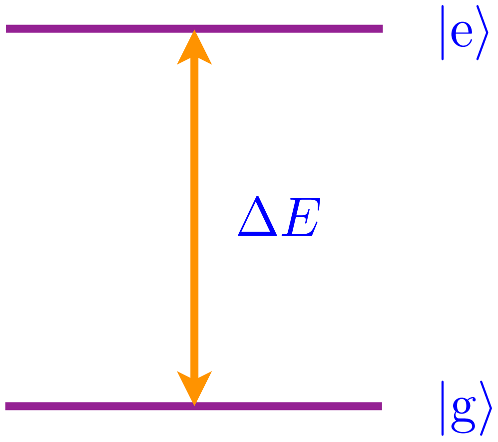
PART II (Experiments)

- Quantum phase from time dilation in atom interferometry and gravitational Aharonov-Bohm effect
- Quantum-clock interferometry
- Atom interferometers as freely falling clocks for time dilation measurements.
- GW detection & search of ultralight dark matter

Propagation phase & wave-packet evolution

Two-level atom as a quantum clock

- Proper time encoded in the relative phase between the two internal states (clock states).



- *Initialization* pulse:

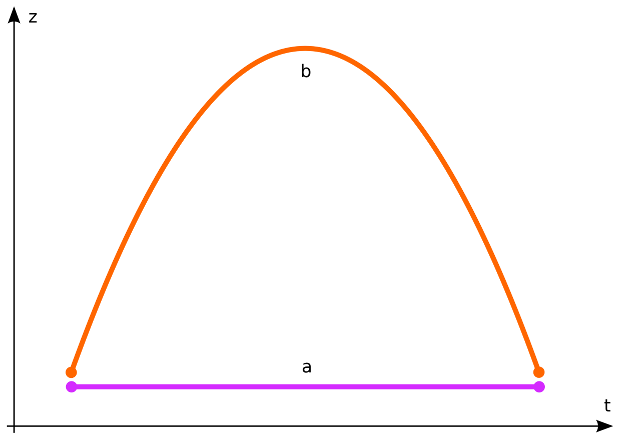
$$|\mathrm{g}\rangle \to \big|\Phi(0)\big\rangle = \frac{1}{\sqrt{2}}\Big(|\mathrm{g}\rangle + i\, e^{i\varphi}|\mathrm{e}\rangle\Big)$$

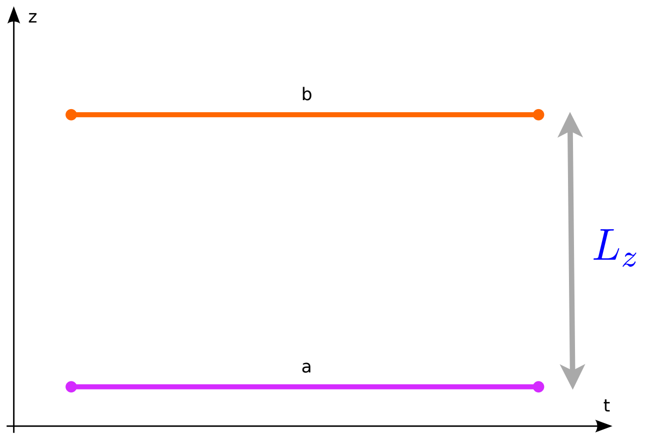
- Evolution:

$$\big|\Phi(\tau)\big\rangle \propto \frac{1}{\sqrt{2}}\Big(|\mathrm{g}\rangle + i\, e^{i\varphi} e^{-i\Delta E\,\tau/\hbar}|\mathrm{e}\rangle\Big)$$

Two-level atom as a quantum clock

- Comparison of independent clocks (after read-out pulse):





for optical *atomic clocks*

$\Delta E \sim 1\,\mathrm{eV} \qquad L_z \sim 1\,\mathrm{cm}$

$$\Delta\tau_b - \Delta\tau_a \approx \left(g\,L_z/c^2\right)\Delta t$$

Two-level atom as a quantum clock

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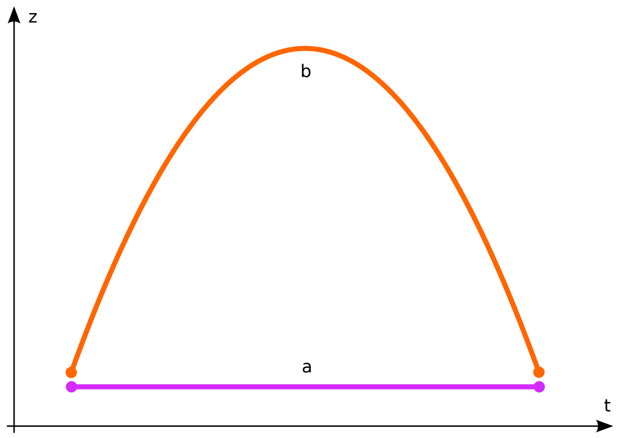

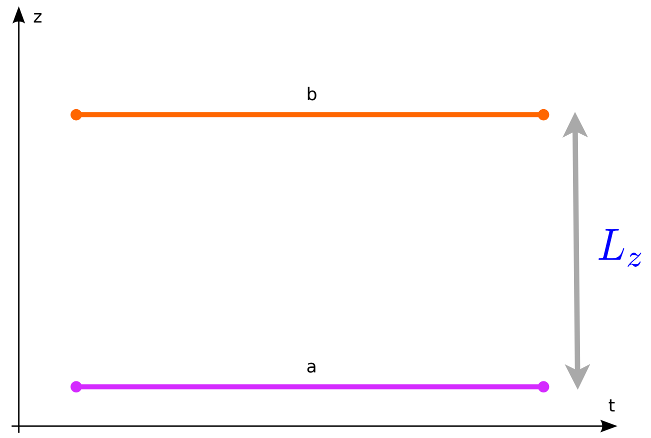

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Two-level atom as a quantum clock

- Theoretical description of the clock:
  - two-level atom (internal state):

$$\hat{H} = \hat{H}_1 \otimes |\mathrm{g}\rangle\langle\mathrm{g}| + \hat{H}_2 \otimes |\mathrm{e}\rangle\langle\mathrm{e}|$$

$$m_1 = m_\mathrm{g}$$

$$m_2 = m_\mathrm{g} + \Delta m$$

$$\Delta m = \Delta E / c^2$$

  - classical action for COM motion:

$$S_n\big[x^\mu(\lambda)\big] = -m_n c^2 \int d\tau = -m_n c \int d\lambda \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} \qquad (\, n = 1, 2 \,)$$

free fall

Two-level atom as a quantum clock

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  - classical action for COM motion:

$$S_n\big[x^\mu(\lambda)\big] = -m_n c^2 \int d\tau - \int d\tau \, V_n(x^\mu) \qquad (\, n = 1, 2 \,)$$

including external forces

Two-level atom as a quantum clock

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$$m_1 = m_\mathrm{g}$$

$$m_2 = m_\mathrm{g} + \Delta m$$

$$\Delta m = \Delta E / c^2$$

  - classical action for COM motion:

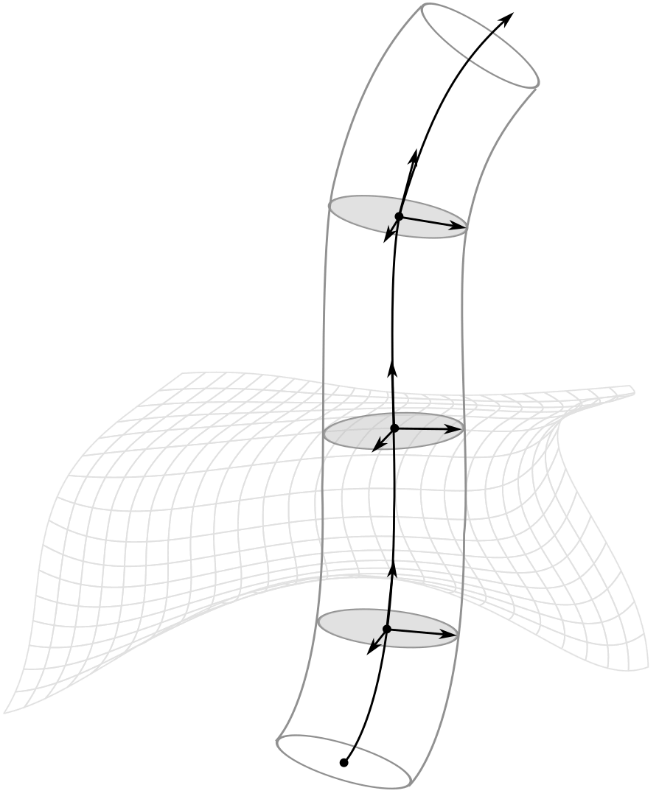
$$S_n\left[x^\mu(\lambda)\right] = -m_n c^2 \int d\tau \approx \int_{t_0}^{t} dt' \left(-m_n\, c^2 + \frac{1}{2} m_n \dot{\mathbf{x}}^2 - m_n\, U(t', \mathbf{x})\right)$$

free fall

Matter-wave propagation in curved spacetime

- Wave-packet evolution in terms of
  - *central trajectory* (satisfies *classical* e.o.m.) $X^\mu(\lambda)$
  - *centered* wave packet $\left|\psi_{\rm c}^{(n)}(\tau_{\rm c})\right\rangle$

$\Delta p/m \ll c$ $\Delta x \ll \ell$ ← curvature radius

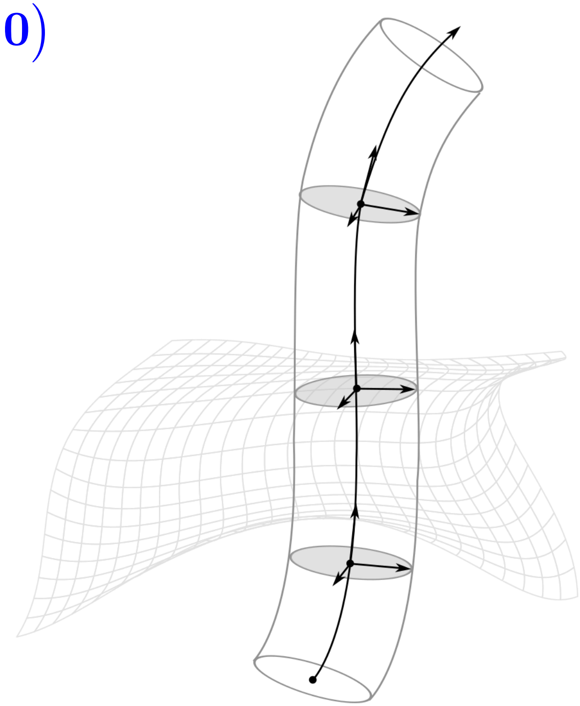


Nadja Augst & A.R.

- Metric in *Fermi-Walker* coordinates: $X^\mu(\tau_c) = \big(c\,\tau_c\,, \mathbf{0}\big)$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{00}\, c^2 d\tau_c^2 \,+\, 2\, g_{0i}\, c\, d\tau_c\, dx^i \,+\, g_{ij}\, dx^i dx^j$$

$$g_{00} = -\big(1 + \delta_{ij}\, a^i(\tau_c)\, x^j / c^2\,\big)^2 - R_{0i0j}(\tau_c, \mathbf{0})\, x^i x^j + O\big(|\mathbf{x}|^3\big)$$
$$g_{0i} = -\frac{2}{3} R_{0jik}(\tau_c, \mathbf{0})\, x^j x^k \,+\, O\big(|\mathbf{x}|^3\big)$$
$$g_{ij} = \delta_{ij} - \frac{1}{3} R_{ikjl}(\tau_c, \mathbf{0})\, x^k x^l \,+\, O\big(|\mathbf{x}|^3\big)$$



Nadja Augst & A.R.

- Expanding the action for the *centered wave packet:*

$$S_n\big[\mathbf{x}(t)\big] \,\approx\, \int d\tau_c \left[-m_n c^2 - V_n(\tau_c, \mathbf{0}) + \frac{m_n}{2}\, \mathbf{v}^2 - \frac{1}{2}\, \mathbf{x}^{\mathrm{T}} \Big(\mathcal{V}^{(n)}(\tau_c) - m_n \Gamma(\tau_c) \Big)\, \mathbf{x} \,-\, V^{(n)}_{\text{anh.}}(\tau_c, \mathbf{x}) \right]$$

$\Delta p / m \ll c$ $\qquad$ $\Delta x \ll \ell$

curvature radius

Comoving frame:

$$X^\mu(\tau_c) = (c\,\tau_c\,, \mathbf{0})$$

$$\hat{H}_n = m_n c^2 + V_n(\tau_c, \mathbf{0}) + \hat{H}_c^{(n)}$$

- Wave-packet evolution: $\big|\psi^{(n)}(\tau_c)\big\rangle = e^{i\mathcal{S}_n/\hbar}\,\big|\psi_c^{(n)}(\tau_c)\big\rangle$
  - *propagation phase*

$$\mathcal{S}_n = -\int_{\tau_0}^{\tau_c} d\tau_c'\,\big(m_n c^2 + V_n(\tau_c', \mathbf{0})\big)$$

  - *centered wave packet*

$$i\hbar\,\frac{d}{d\tau_c}\big|\psi_c^{(n)}(\tau_c)\big\rangle = \hat{H}_c^{(n)}\,\big|\psi_c^{(n)}(\tau_c)\big\rangle$$

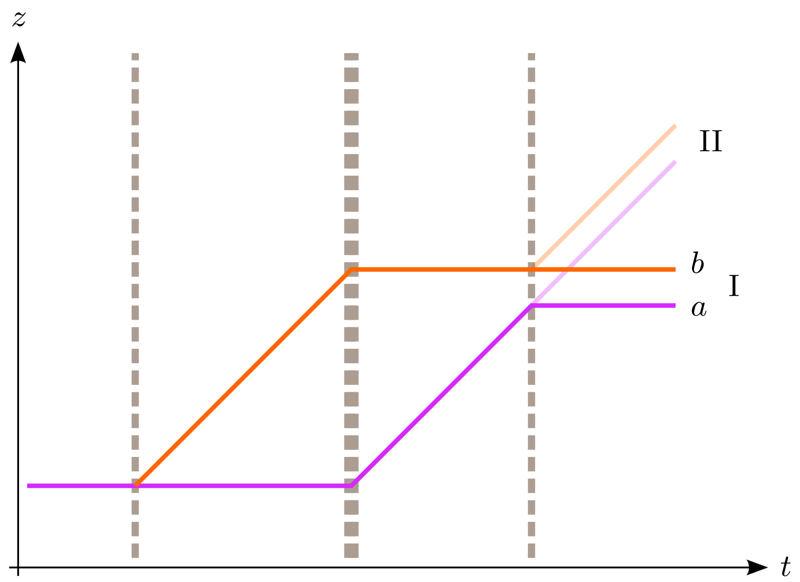
$$\hat{H}_c^{(n)} = \frac{1}{2m_n}\,\hat{\mathbf{p}}^2 + \frac{1}{2}\,\hat{\mathbf{x}}^{\mathrm{T}}\Big(\mathcal{V}^{(n)}(\tau_c) - m_n\Gamma(\tau_c)\Big)\,\hat{\mathbf{x}}$$

gravity-gradient tensor

$$\mathcal{V}_{ij}^{(n)}(\tau_c) = \partial_i\partial_j V_n(\tau_c, \mathbf{x})\big|_{\mathbf{x}=\mathbf{0}}$$

Full interferometer and separation phase

Full interferometer (including laser kicks)



propagation + *laser* phases

$$|\psi_\mathrm{I}\rangle = \frac{1}{2}\left(e^{i\phi_a} + e^{i\phi_b}\right)|\psi_\mathrm{c}\rangle$$

- Detection *probability* at the *exit port(s)*: $$\langle\psi_\mathrm{I}|\psi_\mathrm{I}\rangle = \frac{1}{2}\left(1 + \cos\delta\phi\right)$$

- *Phase shift*: $$\delta\phi = \phi_b - \phi_a \, + \, \delta\phi_\mathrm{sep}$$

Laser phases

DLR

- Phase acquired by an atomic wave packets *diffracted* by a laser pulse driving a **two-photon** process (*Bragg* or *Raman*), possibly described in a freely falling frame:

$$e^{-i\varphi}\, e^{-i\Delta\omega\, t}\, e^{i\mathbf{k}_{\mathrm{eff}}\cdot\mathbf{x}} = e^{-i\varphi}\, e^{-i\Delta\omega\, t_j}\, e^{i\mathbf{k}_{\mathrm{eff}}\cdot\mathbf{X}_j}\; e^{-i\Delta\omega\,(t-t_j)}\, e^{i\mathbf{k}_{\mathrm{eff}}\cdot(\mathbf{x}-\mathbf{X}_j)}$$

where $\Delta\omega = \omega_1 - \omega_2$, $\mathbf{k}_{\mathrm{eff}} = \mathbf{k}_1 - \mathbf{k}_2$ and the index j labels the pulse.

- For a **single-photon** transition, the phase $e^{-i\varphi}\, e^{-i\omega(t_j - \hat{n}\cdot\mathbf{X}_j/c)}$ at the center of the pulse envelope remains constant as the laser pulse propagates (co-propagation of *laser wave fronts* and *pulse envelope*).

  A different central position $\mathbf{X}_j$ means that the wave packet gets diffracted at a slightly different time t_j.

- The spatially dependent phase factor $e^{i\,\mathbf{k}_{\mathrm{eff}}\cdot\mathbf{x}}$ associated with the momentum kick matches the different hypersurfaces of *simultaneity* in the frames comoving with the central trajectory *before* and *after* the kick:

$$e^{-i(m_1 c^2/\hbar)\, t_{\mathrm{FW}}}\, e^{i\mathbf{k}_{\mathrm{eff}}\cdot\mathbf{x}} \leftrightarrow e^{-i(m_2 c^2/\hbar)\, (t_{\mathrm{FW}} - \mathbf{v}_{\mathrm{rec}}\cdot\mathbf{x}/c^2)} = e^{-i(m_2 c^2/\hbar)\, t'_{\mathrm{FW}}}$$

Separation phase

- Standard derivation of phase-shift contribution $\delta\phi_{\text{sep}}$ arising in case of a *relative displacement* of the *interfering wave packets* at each exit port: $\delta\mathbf{X}$ $\delta\mathbf{P}$

$$|\psi_{\mathrm{I}}(\mathbf{x},t)| = \frac{1}{2}\left|e^{i\phi_a}\psi_a(\mathbf{x},t) + e^{i\phi_b}\psi_b(\mathbf{x},t)\right| = \frac{1}{2}\left|e^{i\phi_a}e^{i\mathbf{P}_a\cdot(\mathbf{x}-\mathbf{X}_a)/\hbar}\psi_{\mathrm{c}}(\mathbf{x}-\mathbf{X}_a,t) + e^{i\phi_b}e^{i\mathbf{P}_b\cdot(\mathbf{x}-\mathbf{X}_b)/\hbar}\psi_{\mathrm{c}}(\mathbf{x}-\mathbf{X}_b,t)\right|$$
$$= \frac{1}{2}\left|e^{i\phi_a}e^{i\bar{\mathbf{P}}\cdot\delta\mathbf{X}/2\hbar}e^{-i\delta\mathbf{P}\cdot(\mathbf{x}-\bar{\mathbf{X}})/2\hbar}\psi_{\mathrm{c}}(\mathbf{x}-\bar{\mathbf{X}}+\delta\mathbf{X}/2,t) + e^{i\phi_b}e^{-i\bar{\mathbf{P}}\cdot\delta\mathbf{X}/2\hbar}e^{i\delta\mathbf{P}\cdot(\mathbf{x}-\bar{\mathbf{X}})/2\hbar}\psi_{\mathrm{c}}(\mathbf{x}-\bar{\mathbf{X}}-\delta\mathbf{X}/2,t)\right|,$$

  resulting in $\langle\psi_{\mathrm{I}}|\psi_{\mathrm{I}}\rangle = \frac{1}{2}\left(1 + C\cos\delta\phi'\right)$ with $C = \left|\left\langle\psi_{\mathrm{c}}(t)\middle|\hat{\mathscr{D}}(\delta\mathbf{X},\delta\mathbf{P})\middle|\psi_{\mathrm{c}}(t)\right\rangle\right| \leq 1$ and $\delta\phi' = \phi_b - \phi_a + \delta\phi_{\text{sep}}$,

  where $\boxed{\delta\phi_{\text{sep}} = -\,\bar{\mathbf{P}}\cdot\delta\mathbf{X}/\hbar}$ (*separation phase*)

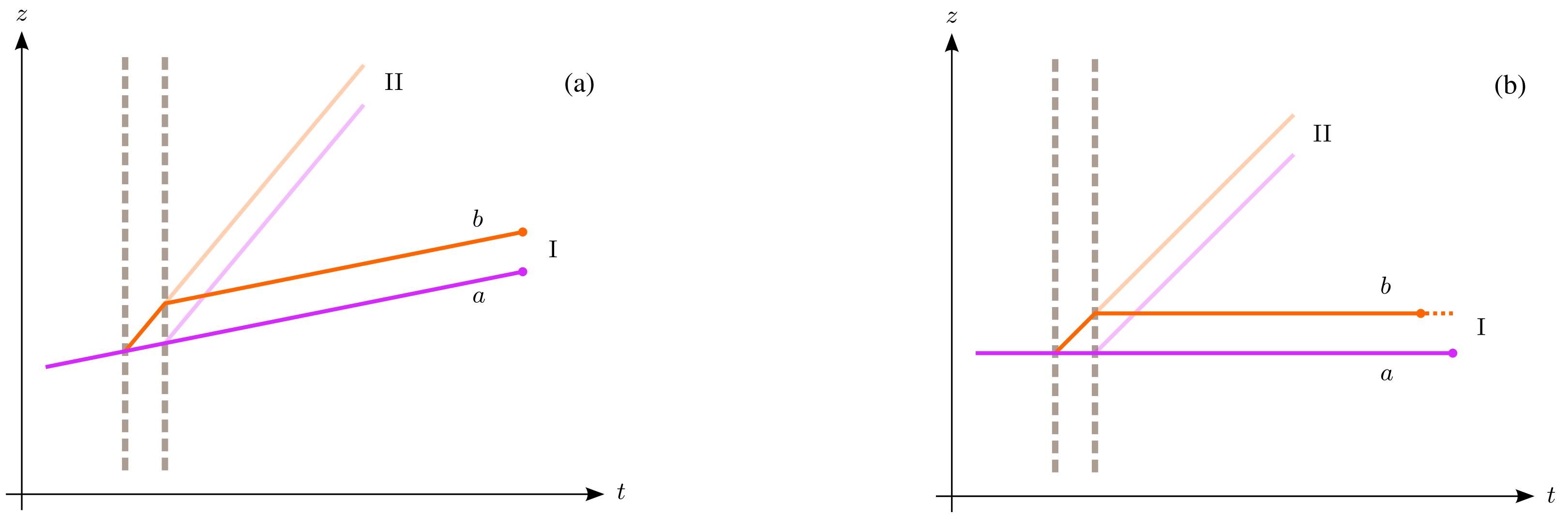
  (additional phase contribution $\arg\left(\left\langle\psi_{\mathrm{c}}(t)\middle|\hat{\mathscr{D}}(\delta\mathbf{X},\delta\mathbf{P})\middle|\psi_{\mathrm{c}}(t)\right\rangle\right)$ vanishes for $\psi_{\mathrm{c}}(-\mathbf{x}) = \pm\,\psi_{\mathrm{c}}(\mathbf{x})$)

- No relativistic aspects involved in the above considerations.

  But there is an interesting, intuitive interpretation in terms of the *relativity of simultaneity* (next slide).

Appendix C.3 in [A. Roura, *Phys. Rev. X* **10**, 021014 (2020)]

Separation phase

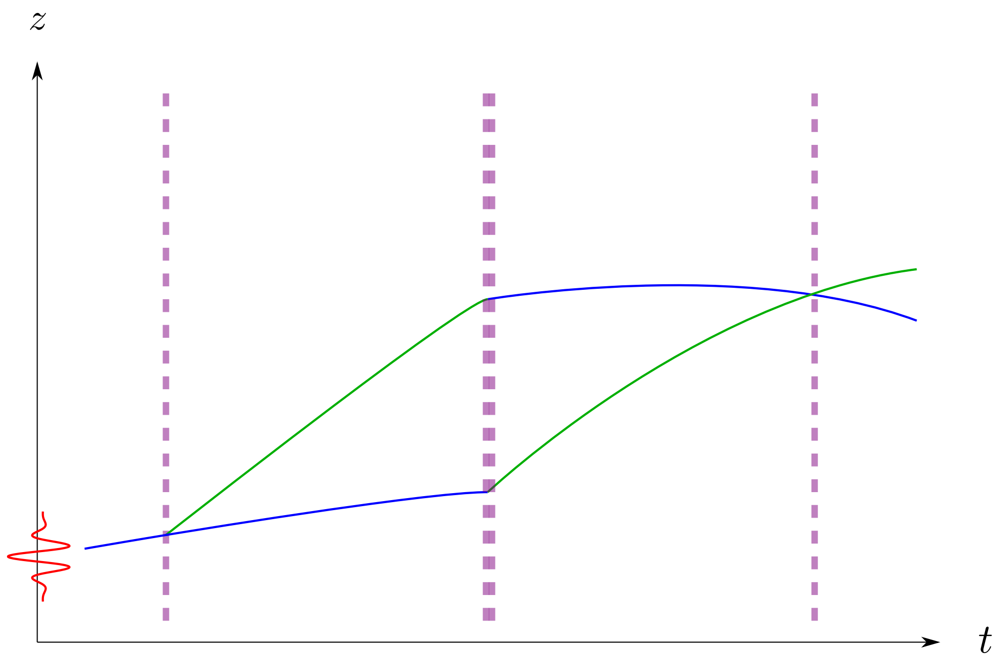


- Relativity of simultaneity for spatially separated spacetime events: $\Delta t_{\mathrm{FW}} = -(v_z/c^2)\Delta z$

- Relativistic interpretation of the separation phase:

  When combined with the propagation phase → proper-time difference between the two arms in the frame where the exit port is at rest.

Arguing in a freely falling frame

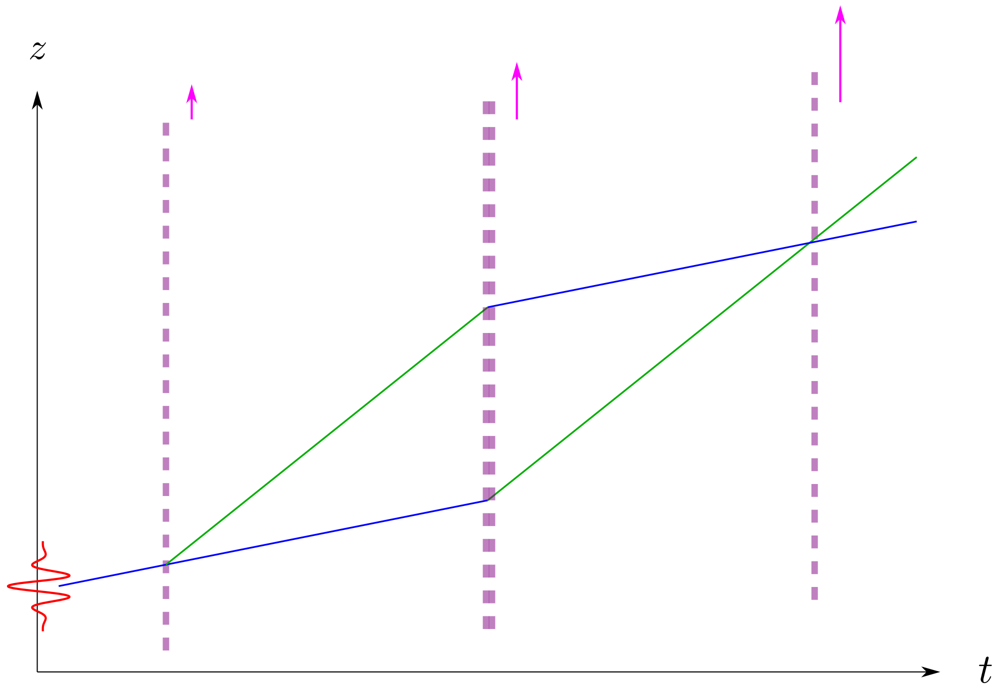


- *Uniform* gravitational field ⟶ *same proper time* along the two arms of a Mach-Zehnder interferometer.

  Even for asymmetric Ramsey-Bordé interferometer ⟶ proper-time difference independent of g.

- In the *lab frame*: cancelation of *special relativistic* and *gravitational* time dilation (*kinetic* and *potential* terms in the classical action).

Arguing in a freely falling frame



- In the *freely falling frame*: central trajectories are straight lines, proper times manifestly independent of g.
- *Timing* shift of *pulse envelope* due to retardation effects dependent on g ⟶ resulting proper-time difference suppressed by v_{rec}/c compared to naively expected gravitational time dilation.
- Detailed discussion of *relativistic* and *retardation* effects for *laser pulses* in the next section.

Main reference:

PHYSICAL REVIEW X **10,** 021014 (2020)

Gravitational Redshift in Quantum-Clock Interferometry

Albert Roura

Institute of Quantum Technologies, German Aerospace Center (DLR), Söflinger Straße 100, 89077 Ulm, Germany and Institut für Quantenphysik, Universität Ulm, Albert-Einstein-Allee 11, 89081 Ulm, Germany

https://doi.org/10.1103/PhysRevX.10.021014

- **Relativistic** description of atom interferometry in **curved spacetime**.
- Wave-packet propagation including **external forces** and even **guiding potentials**.
- **Relativistic** interpretation of the **separation phase** in open interferometers.

Other relevant references:

PHYSICAL REVIEW D **78,** 042003 (2008)

General relativistic effects in atom interferometry

Savas Dimopoulos,[1] Peter W. Graham,[2,1] Jason M. Hogan,[1] and Mark A. Kasevich[1]
[1]*Department of Physics, Stanford University, Stanford, California 94305, USA*
[2]*SLAC, Stanford University, Menlo Park, California 94025, USA*
(Received 27 March 2008; published 18 August 2008)

http://dx.doi.org/10.1103/PhysRevD.78.042003

Also earlier work by *Christian Bordé*.

Exact Semiclassical Phase Shifts for Relativistic Atom Interferometers in Flat Spacetime

Hunter Swan* and Jason M. Hogan†
Department of Physics, Stanford University, Stanford, California 94305, USA
(Dated: July 14, 2026)

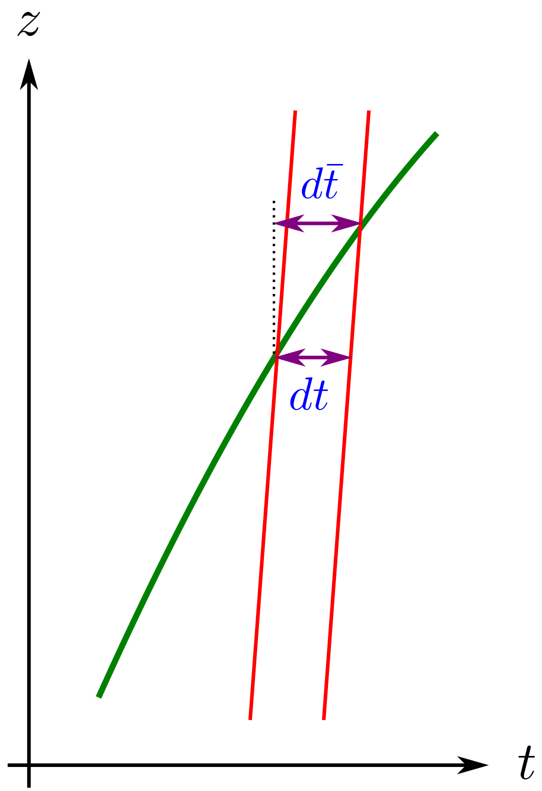
https://arxiv.org/abs/2607.10042

Retardation & relativistic effects on laser pulses

Propagation of laser wave fronts

- Proper time along a freely falling world line (geodesic) and elapsed between two light rays.
  - **Retardation** effect due to the finite speed of light:

$$d\bar{t} = dt + (\hat{\mathbf{n}} \cdot \bar{\mathbf{v}}/c)\, d\bar{t} + O\big(1/c^3\big)$$



Propagation of laser wave fronts

- Proper time along a freely falling world line (geodesic) and elapsed between two light rays.

  - **Retardation** effect due to the finite speed of light: (stationary spacetime)

$$d\bar{t} = dt + (\hat{\mathbf{n}} \cdot \bar{\mathbf{v}}/c)\, d\bar{t} + O\big(1/c^3\big) \longrightarrow \frac{d\bar{t}}{dt} = \frac{1}{1 - \hat{\mathbf{n}} \cdot \bar{\mathbf{v}}/c}$$

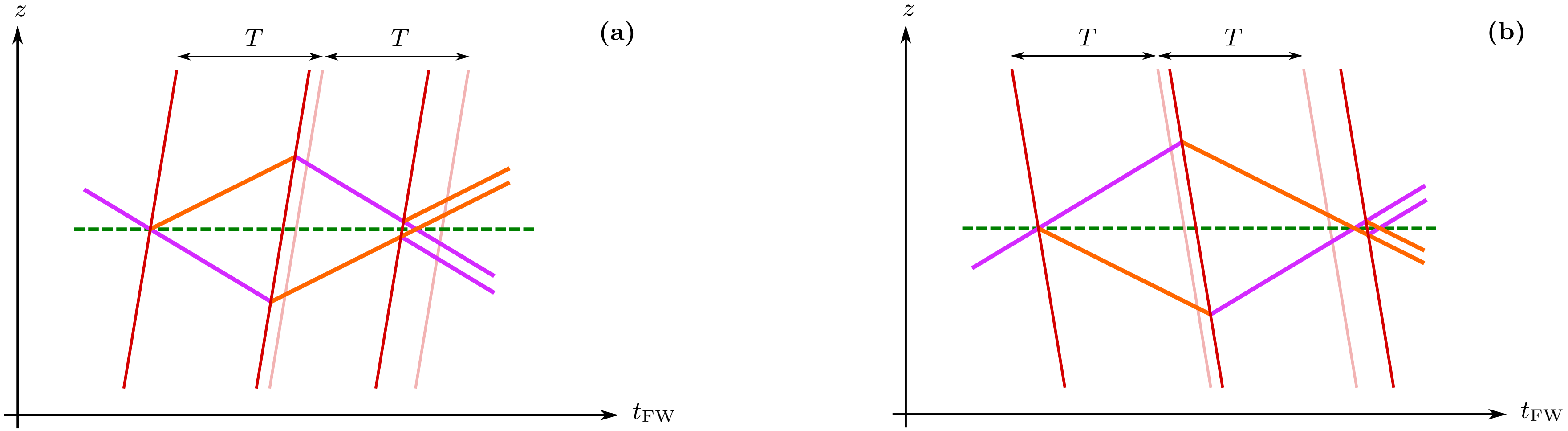
  - Relativistic **time dilation**:

$$\frac{d\bar{\tau}}{d\bar{t}} = 1 - \frac{1}{2\,c^2}\left(\frac{d\bar{\mathbf{X}}}{d\bar{t}}\right)^2 + \frac{1}{c^2}\,U\big(\bar{t}, \bar{\mathbf{X}}\big) + O\big(1/c^4\big)$$

special relativistic

gravitational redshift

Atom interferometer based on single-photon transitions



- **Freely falling frame** comoving with the *mid-point* world line (Fermi-Walker frame):
  - light rays (laser wave fronts) have fixed slope,
  - shifts due to Doppler effect (*opposite sign* in reversed interferometer) and time dilation (*same sign*).

- It is sufficient to calculate the proper times along the *mid-point* world line rather than the actual *arm trajectories* (negligible higher-order corrections to total phase shift).

- Proper time as a function of the phase φ, invariant characterizing each laser wave front:

$$\frac{d\bar{\tau}}{d\varphi} = \frac{d\bar{\tau}}{d\bar{t}} \frac{d\bar{t}}{dt} \left(\frac{dt}{d\varphi} \right) = \frac{d\bar{\tau}}{d\bar{t}} \left(\frac{1}{1 - \hat{\mathbf{n}} \cdot \bar{\mathbf{v}}/c} \right) \left(\frac{dt}{d\varphi} \right)$$

- The Doppler factor can be (partially) compensated through a suitable frequency chirp:

$$\left(\frac{dt}{d\varphi} \right)_{\text{chirp}} = \left(1 - \hat{\mathbf{n}} \cdot \bar{\mathbf{v}}'/c \right) \left(\frac{dt}{d\varphi} \right)_0 \qquad (dt/d\varphi)_0 = 1/\omega_0$$

$$\bar{\mathbf{v}}'(\bar{t}) = \bar{\mathbf{v}}'_0 + \mathbf{g}' \, (\bar{t} - \bar{t}_0)$$

- Phase-shift calculation:

$$\delta\phi = -\frac{\Delta E}{\hbar}\left[\int_0^{\omega_0 T}\left(\frac{d\bar{\tau}}{d\varphi}\right)d\varphi - \int_{\omega_0 T}^{2\omega_0 T}\left(\frac{d\bar{\tau}}{d\varphi}\right)d\varphi\right]$$

- For an approximately uniform gravitational field, $\dot{\bar{\mathbf{X}}}(\bar{t}) = \bar{\mathbf{v}}_0 + \mathbf{g}\,(\bar{t} - \bar{t}_0)$ and

$$\delta\phi = \frac{\Delta E}{\hbar}\left[\frac{(\hat{\mathbf{n}}\cdot\Delta\mathbf{g})}{c}\,T^2 - 2\,\frac{\bar{\mathbf{v}}_0\cdot\mathbf{g}}{c^2}\,T^2 - 2\,\frac{\mathbf{g}^2}{c^2}\,T^3\right] + \delta\phi_{\mathrm{corr}}$$

- Partial compensation of the Doppler factor characterized by $\Delta\mathbf{g} = \mathbf{g} - \mathbf{g}'$ and $\Delta\bar{\mathbf{v}}_0 = \bar{\mathbf{v}}_0 - \bar{\mathbf{v}}'_0$.

- Corrections of order $1/c^2$ and proportional to $\Delta\mathbf{g}$ or $\Delta\bar{\mathbf{v}}_0$, due to only partial compensation of Doppler factor:

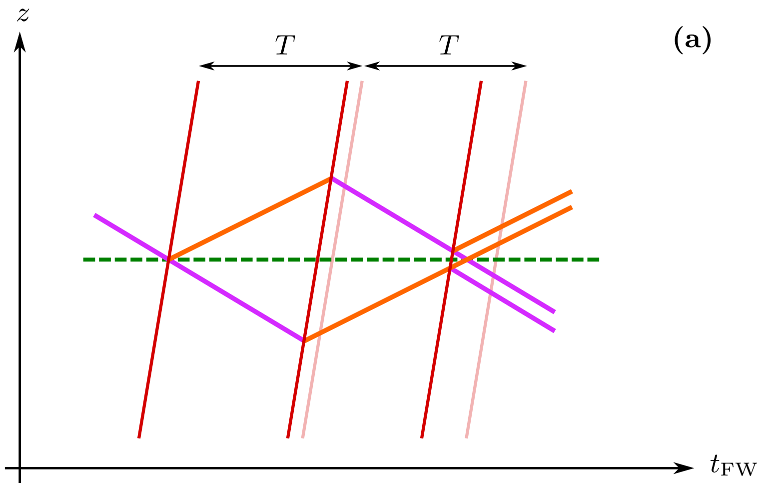
$$\delta\phi_{\text{corr}} = \frac{\Delta E}{\hbar} \left[2\, \frac{(\hat{\mathbf{n}} \cdot \Delta\bar{\mathbf{v}}_0)\, (\hat{\mathbf{n}} \cdot \mathbf{g})}{c^2}\, T^2 + \frac{(\hat{\mathbf{n}} \cdot \bar{\mathbf{v}}_0)\, (\hat{\mathbf{n}} \cdot \Delta\mathbf{g})}{c^2}\, T^2 + 3\, \frac{(\hat{\mathbf{n}} \cdot \mathbf{g})\, (\hat{\mathbf{n}} \cdot \Delta\mathbf{g})}{c^2}\, T^3 \right]$$

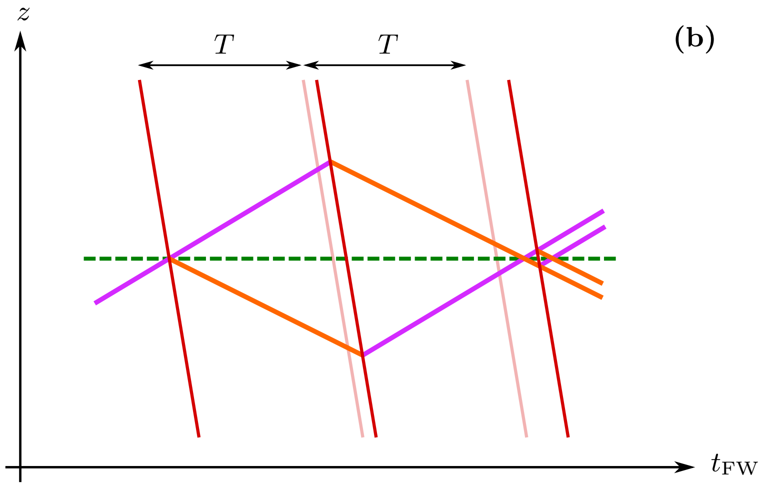
- The above results can be straightforwardly generalized to a time-dependent $\Delta\mathbf{g}(\bar{t})$.

  This can naturally account for *laser phase noise* and *vibrations* of retro-reflection *mirror*.

- In contrast to terms linear in $\hat{\mathbf{n}}$, the *time-dilation terms* and the contributions in $\delta\phi_{\text{corr}}$, which are *quadratic* in $\hat{\mathbf{n}}$, do not change sign for interferometers with opposite $\hat{\mathbf{n}}$ (**reversed interferometers**).

Reversed interferometers:





- **Freely falling frame** comoving with the *mid-point* world line (Fermi-Walker frame):
  - light rays (laser wave fronts) have fixed slope,
  - shifts due to <u>Doppler effect</u> (*opposite sign* in reversed interferometer) and <u>time dilation</u> (*same sign*).

- Result when ***subtracting*** the phase shift for the pair of ***reversed*** interferometers:

$$\left(\delta\phi_{\hat{\mathbf{n}}} - \delta\phi_{-\hat{\mathbf{n}}}\right)/2 = \frac{\Delta E}{\hbar}\,\frac{(\hat{\mathbf{n}}\cdot\Delta\mathbf{g})}{c}\,T^2$$

It corresponds to a **gravimeter** (accelerometer).

- Result when ***adding*** up the phase shift for the pair of ***reversed*** interferometers:

$$\left(\delta\phi_{\hat{\mathbf{n}}} + \delta\phi_{-\hat{\mathbf{n}}}\right)/2 = -2\,\frac{\Delta E}{\hbar}\left[\frac{\bar{\mathbf{v}}_0\cdot\mathbf{g}}{c^2}\,T^2 + \frac{\mathbf{g}^2}{c^2}\,T^3\right] + \delta\phi_{\mathrm{corr}}$$

It can be exploited in *time-dilation* measurements with atom interferometers acting as **freely falling clocks** (further discussed in tomorrow's lecture).

Pulse timing

- So far we have assumed that changes in the *timing* of the **pulse envelope** and the **laser wave fronts** coincide. This may not be the case due to *timing imperfections* or to a *lacking compensation* of *retardation* effects for the pulse envelope.

- A time shift ΔT of the *pulse envelope* with respect to the *reference wave front* for the π pulse leads to the following phase-shift contribution:

$$\delta\phi_{\text{timing}} = -2\left(\frac{\Delta E}{\hbar} - \omega_0\right)\Delta T = -\,2\,\delta_\pi\,\Delta T$$

Including *uncompensated retardation* effects, the detuning becomes

$$\delta_\pi = \frac{\Delta E}{\hbar}\left[1 + \frac{(\hat{\mathbf{n}}\cdot\Delta\bar{\mathbf{v}}_0)}{c} + \frac{(\hat{\mathbf{n}}\cdot\Delta\mathbf{g})}{c}\,T\right] - \omega_0$$

- Shifts in the timing of *all* laser pulses in a Mach-Zehnder interferometer can be accounted for by a time shift ΔT for the π pulse, and a change of the total interferometer time $2\,T'$.

  From the *retardation* effects of the *pulse envelope* (with no compensation) we have:

$$T' = T \left(1 + \frac{(\hat{\mathbf{n}} \cdot \mathbf{v}_\pi)}{c} \right) \qquad \Delta T = -\frac{(\hat{\mathbf{n}} \cdot \mathbf{g})}{2c}\, T^2$$

$$\mathbf{v}_\pi = \mathbf{v}_0 + \mathbf{g}\, T$$

- This leads to the following result for $\left(\delta\phi_{\hat{\mathbf{n}}} - \delta\phi_{-\hat{\mathbf{n}}}\right)/2$ at leading order:

$$\left(\delta\phi_{\hat{\mathbf{n}}} - \delta\phi_{-\hat{\mathbf{n}}}\right)/2 \;=\; \frac{\Delta E}{\hbar}\, \frac{(\hat{\mathbf{n}} \cdot \Delta\mathbf{g})}{c}\, T^2 - 2\, \delta_\pi\, \Delta T$$

  whereas the leading correction from T' in $\dfrac{\Delta E}{\hbar}\, \dfrac{(\hat{\mathbf{n}} \cdot \Delta\mathbf{g})}{c}\, (T')^2$ contributes to $\left(\delta\phi_{\hat{\mathbf{n}}} + \delta\phi_{-\hat{\mathbf{n}}}\right)/2$.

Atom interferometers based on <u>two-photon processes</u>

- Two *counter-propagating* laser *frequency components* with opposite linear Doppler shifts:

$$\bar{\omega}_j = \omega_j \left(\frac{1 - \hat{\mathbf{n}}_j \cdot \bar{\mathbf{v}}/c}{1 - \hat{\mathbf{n}}_j \cdot \bar{\mathbf{v}}'/c} \right) \quad \text{leading to} \quad \delta\phi = \mathbf{k}_{\text{eff}} \cdot \Delta\mathbf{g}\, T^2 \qquad \mathbf{k}_{\text{eff}} = \mathbf{k}_1 - \mathbf{k}_2$$

(additional terms of order $1/c^2$ and quadratic in $\hat{\mathbf{n}}$ are proportional to $\omega_1 - \omega_2$)

- Frequency component ω_2 always on + ***pulsed envelope*** for component ω_1 (control of pulse timing)

- For an *antisymmetric* frequency chirp $\alpha_1 = -\alpha_2 = \mathbf{k}_{\text{eff}} \cdot \mathbf{g}'/2$, the result of ***retardation*** effects is analogous to that for single-photon transitions:

$$\delta\phi \;=\; \mathbf{k}_{\text{eff}} \cdot \Delta\mathbf{g}\,(T')^2 - \,2\,\delta_\pi\,\Delta T \;=\; \Big(\mathbf{k}_{\text{eff}} \cdot \mathbf{g} - (\alpha_1 - \alpha_2)\Big)\,(T')^2 - \,2\,\delta_\pi\,\Delta T$$

with the same T' and ΔT of a previous slide.

- In case of a non-vanishing *symmetric* contribution $\bar{\alpha} = \alpha_1 + \alpha_2$, there is an additional phase-shift term due to *retardation* effects of the *frequency chirp* on the laser phases:

$$\delta\phi_{\bar{\alpha}} \;=\; \big(\alpha_1 \hat{\mathbf{n}}_1 - \alpha_2 \hat{\mathbf{n}}_2\big) \cdot (\mathbf{v}_\pi / c)\, T^2$$

Connected to a change of $\mathbf{k}_{\text{eff}}$ for the different pulses induced by the symmetric frequency chirp.

Main reference:

Quantum Sci. Technol. **10** (2025) 025004 https://doi.org/10.1088/2058-9565/ad9e2e

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PAPER

Atom interferometer as a freely falling clock for time-dilation measurements

Albert Roura

German Aerospace Center (DLR), Institute of Quantum Technologies, Wilhelm-Runge-Straße 10, 89081 Ulm, Germany

E-mail: albert.roura@dlr.de

Keywords: atom interferometry, relativistic time dilation, gravitational and relativistic measurements, long-baseline facilities, single-photon clock transition

https://doi.org/10.1088/2058-9565/ad9e2e

Main reference:

- Main focus on atom interferometers based on ***single-photon*** transitions.
- Useful framework for a detailed theoretical description of such interferometers.
- Simple, intuitive description of ***time dilation*** and ***retardation*** effects for the *laser pulses* working in the freely falling frame.
- Discussion of retardation effects for the ***timing*** of the ***pulse envelope*** (with or without compensation).
- Including also ***external*** (non-gravitational) ***forces*** and possible ***violations*** of the ***equivalence principle***.

Reference on retardation effects for two-photon processes:

PHYSICAL REVIEW A **96**, 023604 (2017)

Time delay and the effect of the finite speed of light in atom gravimeters

Yu-Jie Tan, Cheng-Gang Shao,* and Zhong-Kun Hu†

MOE Key Laboratory of Fundamental Physical Quantities Measurement, Hubei Key Laboratory of Gravitation and Quantum Physics, School of Physics, Huazhong University of Science and Technology, Wuhan 430074, People's Republic of China

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- The simple derivation relying on the freely falling frame that I outlined above agrees with the results for two-photon processes obtained in this reference with a much longer calculation.

DLR

Thank you for your attention.

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