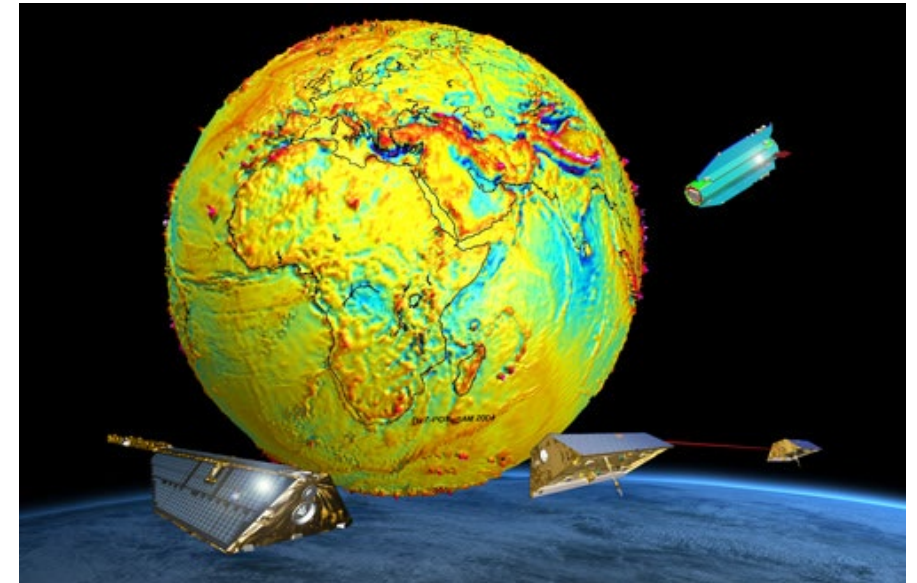


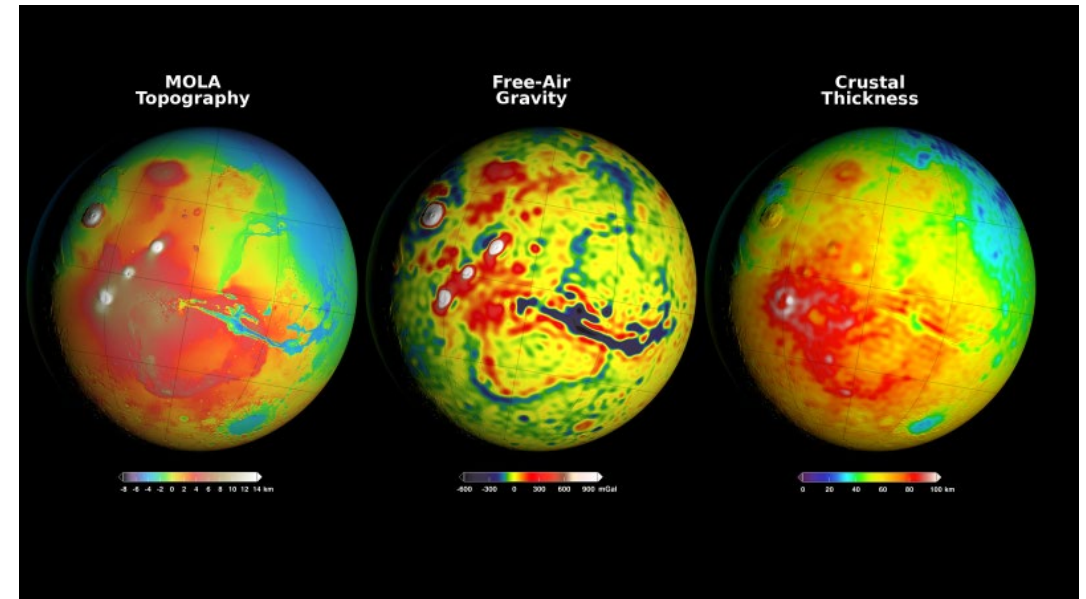
Earth Gravity and quantum

.. Why it is of interest and society benefit ..

Rene Forsberg, DTU Space, Denmark, rf@space.dtu.dk
with input from Tim Jensen (DTU) and Yannick Bidet and colleagues (ONERA)



Gravity fundamental in astrophysics and geophysics ...



NASA JPL

At Sagittarius A* (Milkyway black hole event horizon) $g = c^4 / 4GM \sim 3.5 \times 10^6 \text{ m/s}^2$

At surface of sun $\sim 273 \text{ m/s}^2$

At surface of earth "g" $\sim 9.76\text{-}9.83 \text{ m/s}^2$, at Mars: 3.72 m/s^2

Smallest gravity changes on earth measurable (pre-quantum): static $\sim 10^{-9} g$, $\sim$ orbit $10^{-12} g$

Knowledge of the Earth's Gravity Field and its time changes has important societal benefits

.. low-hanging fruits for quantum atom interferometry



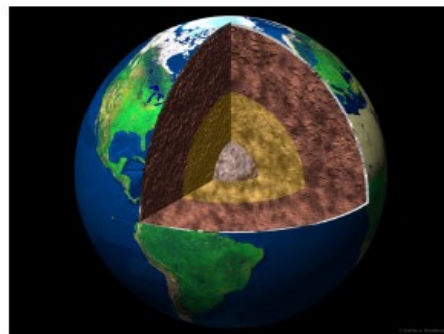
Glaciology



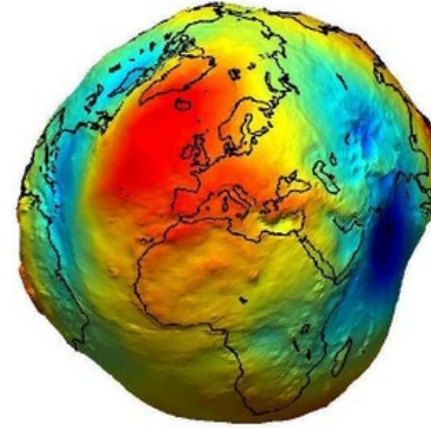
Oceanography



Hydrology

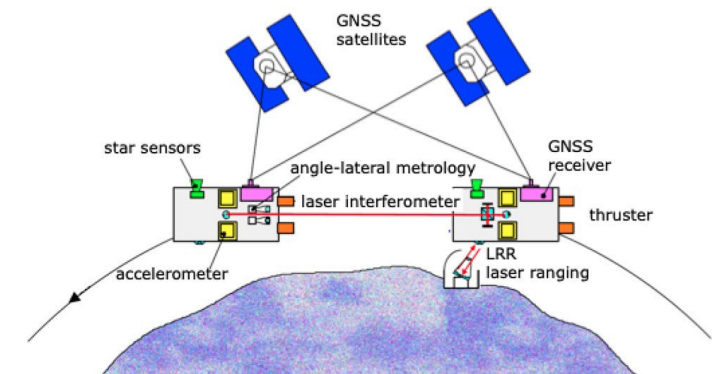
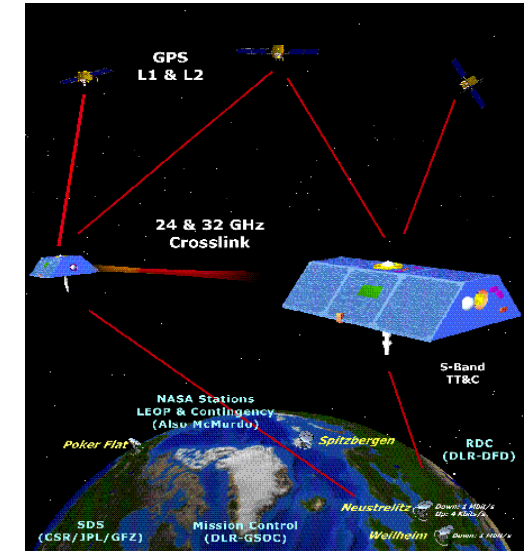
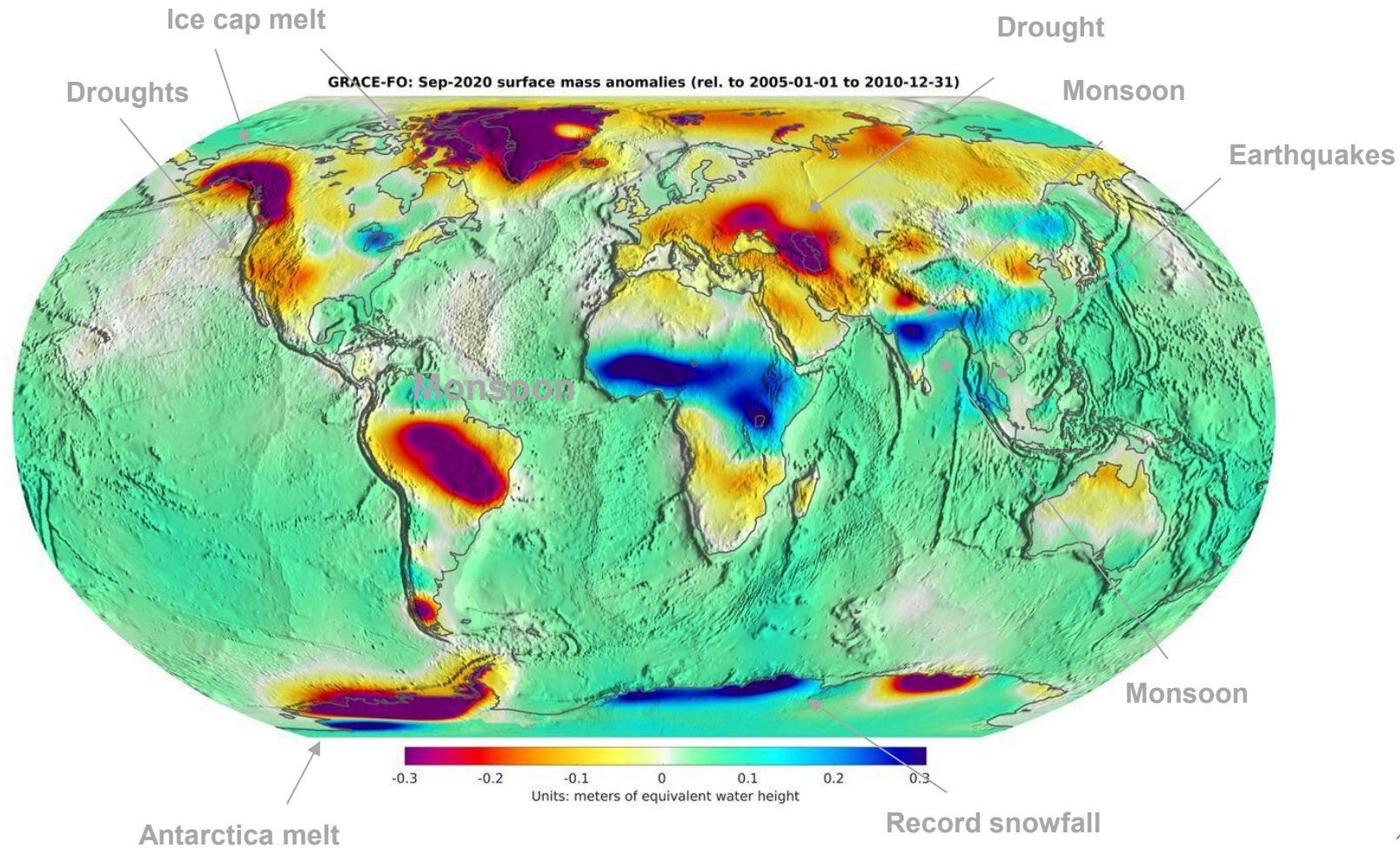


Internal Geophysics



- Melt of ice sheets and ice caps
- Global sea level rise
- Ocean currents (by combination with satellite radar altimetry)
- Droughts, flooding and precipitation / snowfall
- Geoid – national height systems
- Geophysics, mining and exploration
- Precise Inertial navigation
-

Global gravity field changes from space: GRACE / GRACE-FO (NASA/DLR 2002-17, 2018-)

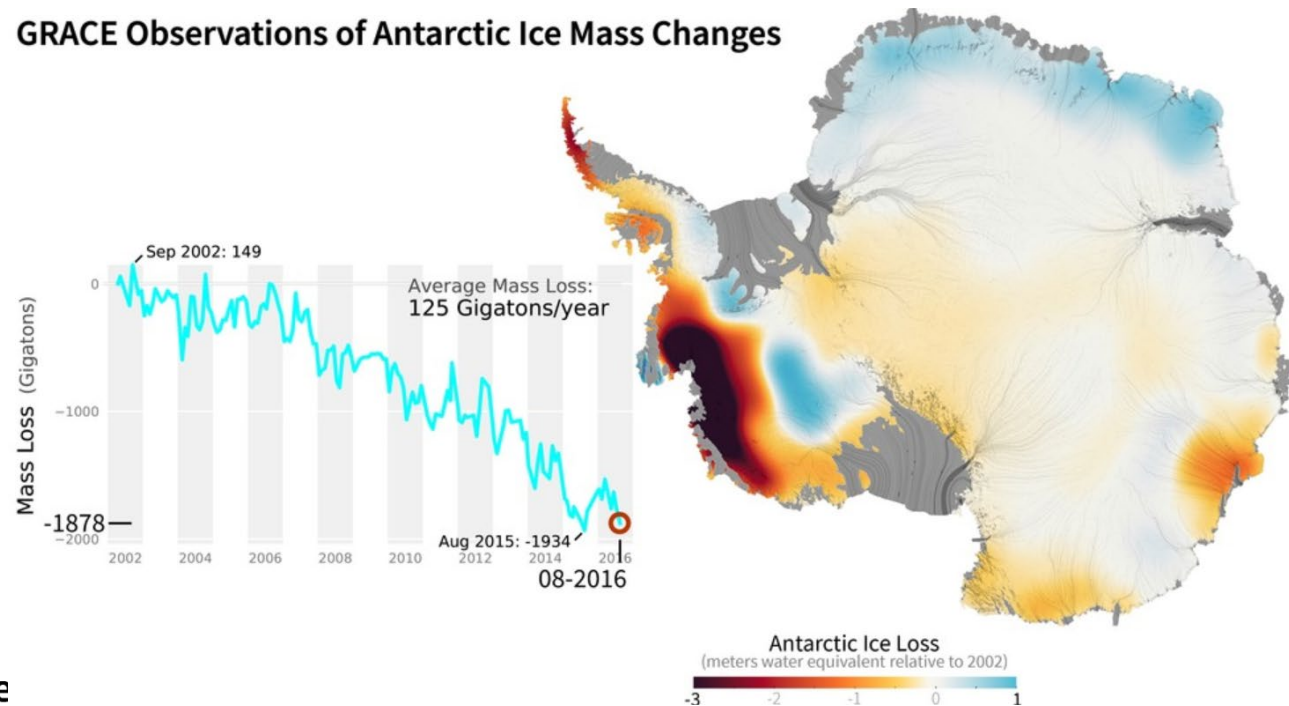


ESA NGGM/MAGIC - 2032

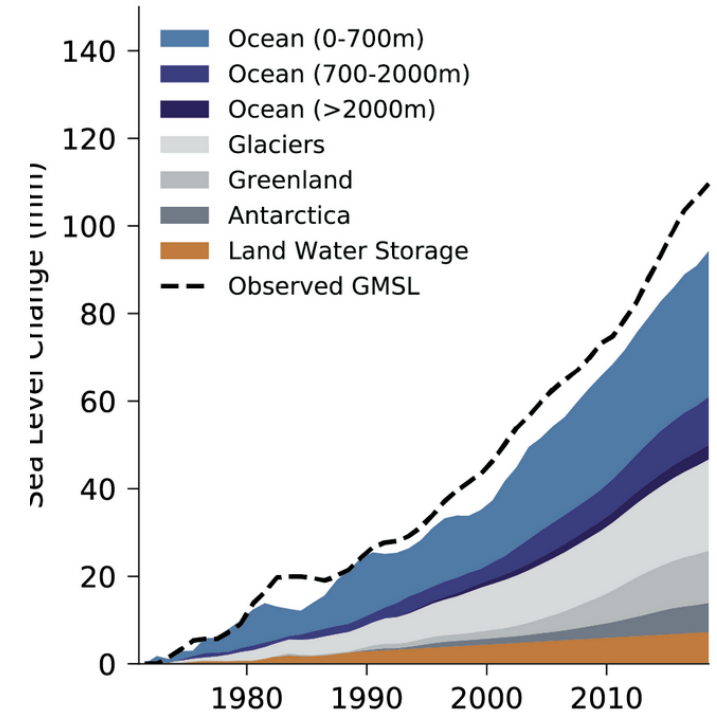
Gravity field and climate ...

- Measuring sea level changes and ice sheets .. *Geoid integral part of processing and theory (John Wahr, U Colorado, 1998 paper)*
- Geoids – ocean currents, global sea level rise inundation, coastal protection ..
- GIA (Glacial Isostatic Adjustment) – important corrections for full utilization of satellite gravity missions
- Sub-terrain hydrology and aquifers (quantum goal ..)

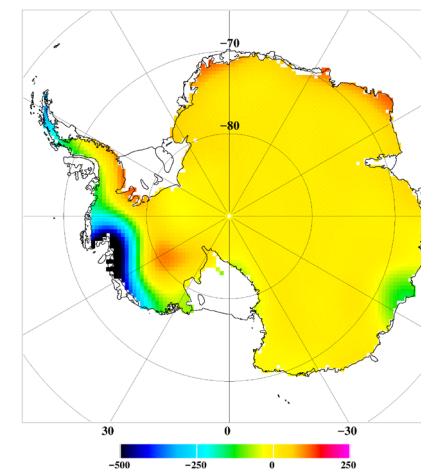
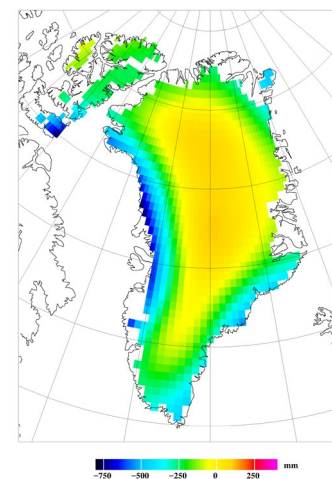
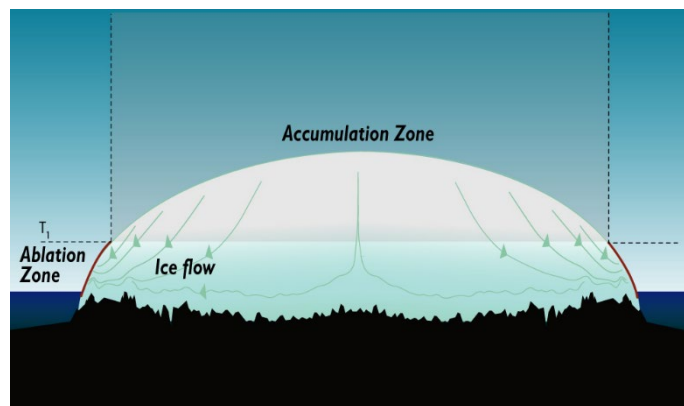
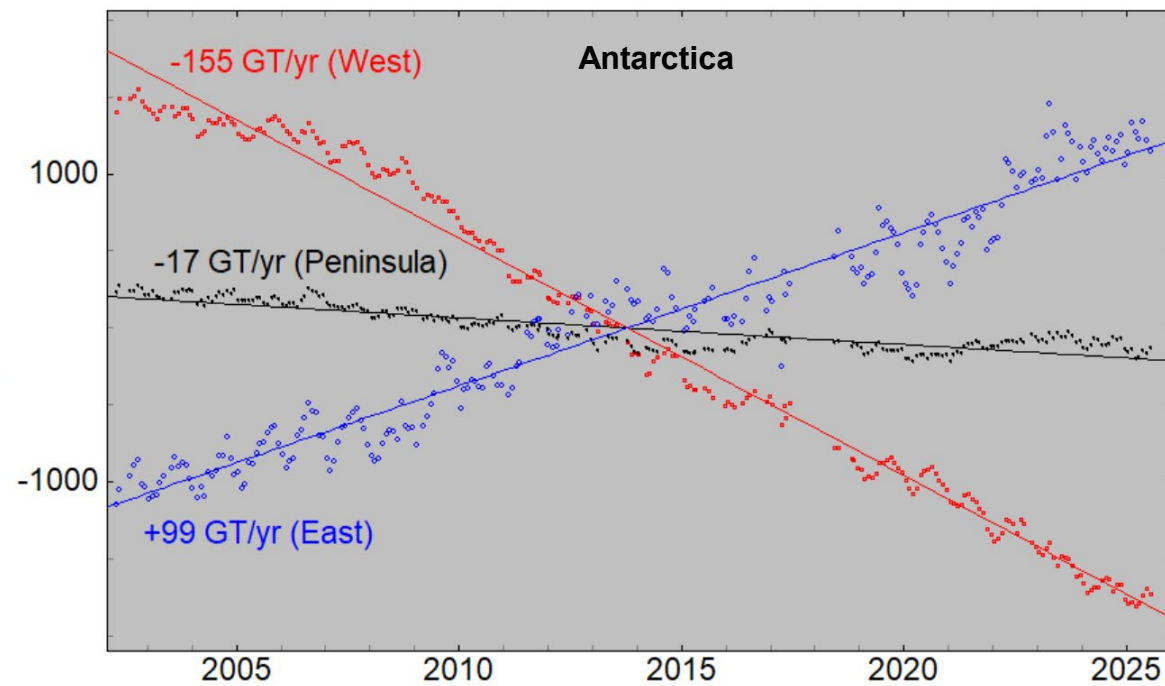
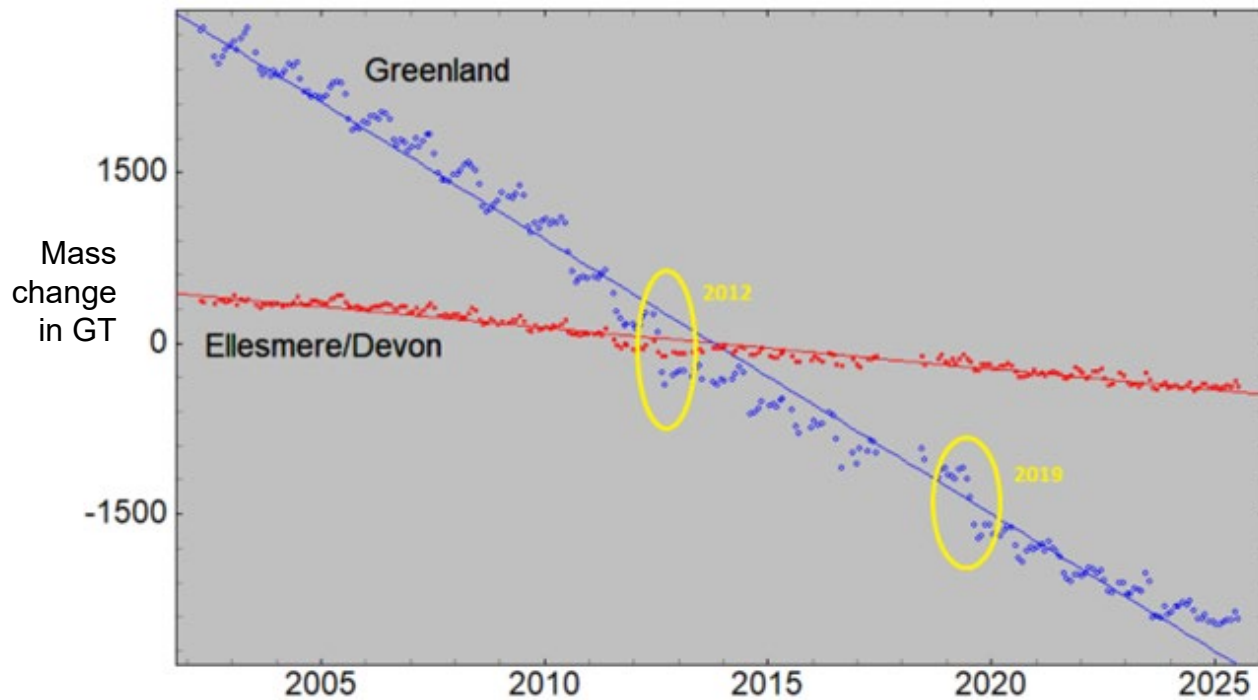
GRACE Observations of Antarctic Ice Mass Changes

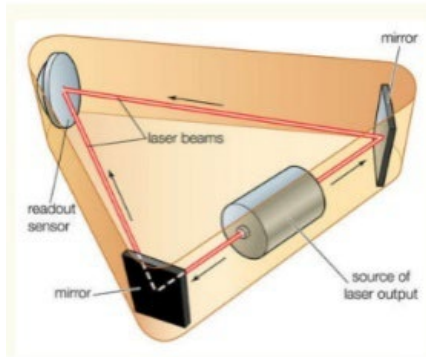
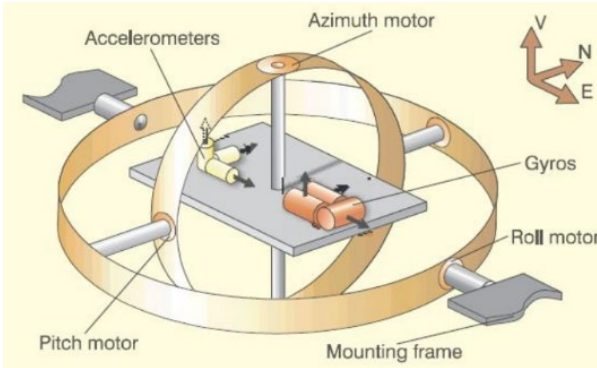


(b) Global Sea-Level Budget



Melting icesheets and global sea level rise – GRACE / GRACE-FO





Gravity field and inertial navigation

Inertial navigation essential in GNSS-denied environments

IMU's drift – due to gyro and accelerometer errors, *and unknown gravity field*

Commercial IMU's ~ 0.5 nm/hr (e.g. Honeywell F-16 RLG IMU)

High-end military systems ~ < 0.2 nm/hr (submarines, ICBMs)

Gravity field maps used for correcting high-end IMU's
(upload of grid models or 3D global models – NGA EGM2008)

*Gravity field is 3-dimensional (g_N , g_E , g_D) ..
Horizontal gravity = deflections of vertical*

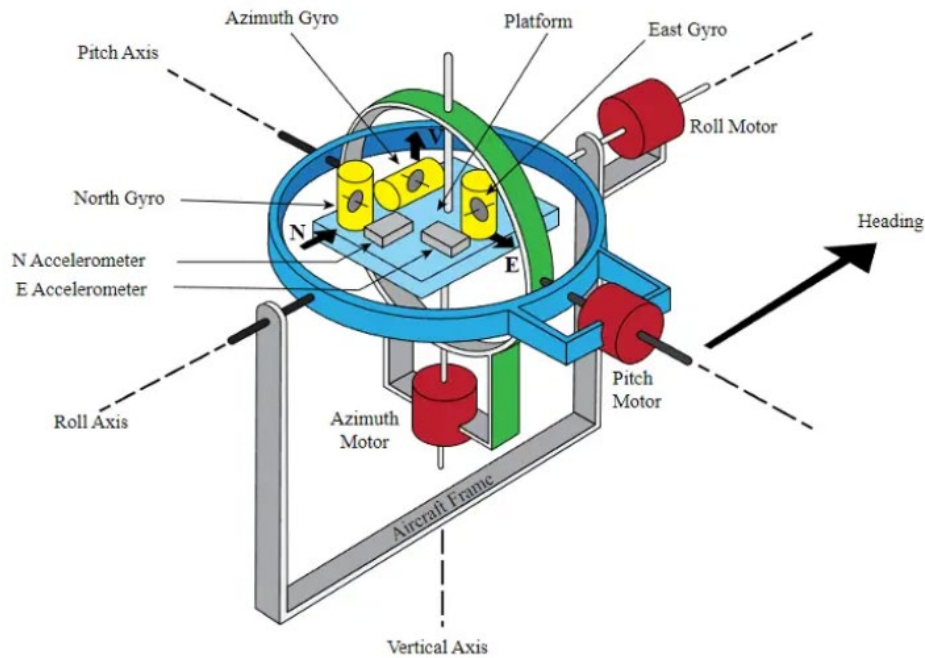
"Vector gravimetry"

Strapdown IMU and stabilised gravimeter,
Sabah Air Beech Kingair, DTU Malaysia 2014

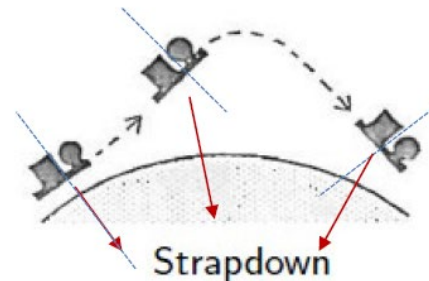
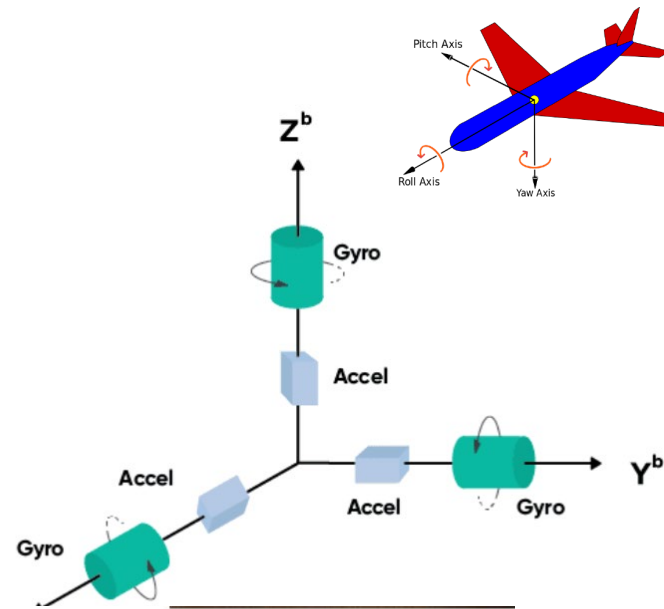


Inertial navigation – highly relevant due to GNSS jamming ...

Gyro-stabilised local-level platform



Strapdown IMU



Initial position and speed

$$(x_0, y_0) \text{ and } (v_{x,0}, v_{y,0})$$

Gravity acceleration along the profile:
 $g(x, y)$

Integration for velocity:

$$v_x(t) = v_{x,0}$$

$$v_y(t) = v_{y,0} + \int g(x(t), y(t)) dt$$

Integration for position:

$$x(t) = x_0 + \int v_x(t) dt = v_{x,0} t$$

$$y(t) = y_0 + \int v_y(t) dt$$

$$= y_0 + v_{y,0} t + \iint g(x(t), y(t)) dt$$



Figure 6.1: The iMAR iNAT-RQH-0001 IMU

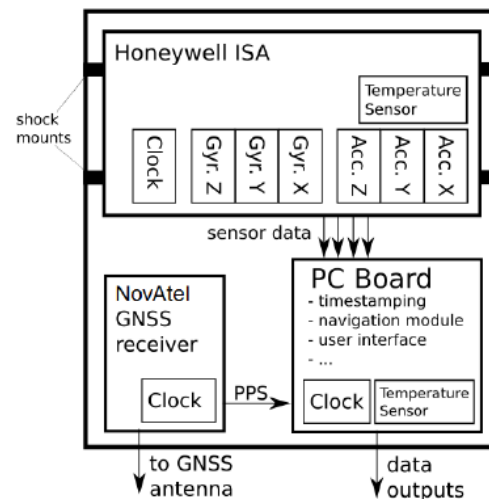


Figure 6.2: iMAR iNAT-RQH-0001 system design. Modified from [1, Fig. 6.2]

SPECIFICATIONS

Data output rate	≤ 300 Hz (1800 Hz internal)
Data latency	< 3 ms
Operating temperatures	-40°C to 71°C
Weight	≈ 8 kg
Size	$\approx 187 \times 128 \times 296$ mm (310 mm with cables)
Position accuracy	< 0.2 nmi/hr (0.4 km/hr)

PERFORMANCE CHARACTERISTICS	Gyroscopes	Accelerometers
Range	$\pm 800^{\circ}/\text{s}$	± 20 g
Drift (unaided) / offset	$< 0.003^{\circ}/\text{hr}$	$< 25 \mu\text{g}$
Bias instability (const. temp.)	$< 0.002^{\circ}/\text{hr}$	$< 10 \mu\text{g}$
Random walk	$< 0.0011^{\circ}/\sqrt{\text{hr}}$	$< 8 \mu\text{g}/\sqrt{\text{Hz}}$
Resolution	$< 0.001^{\circ}/\text{s}$	$< 5 \mu\text{g}$
Scale error	< 5 ppm	< 100 ppm
Linearity error	< 5 ppm	$< 20 \mu\text{g}/\text{g}^2$

IMU – gravimetry (iMAR / Honeywell)

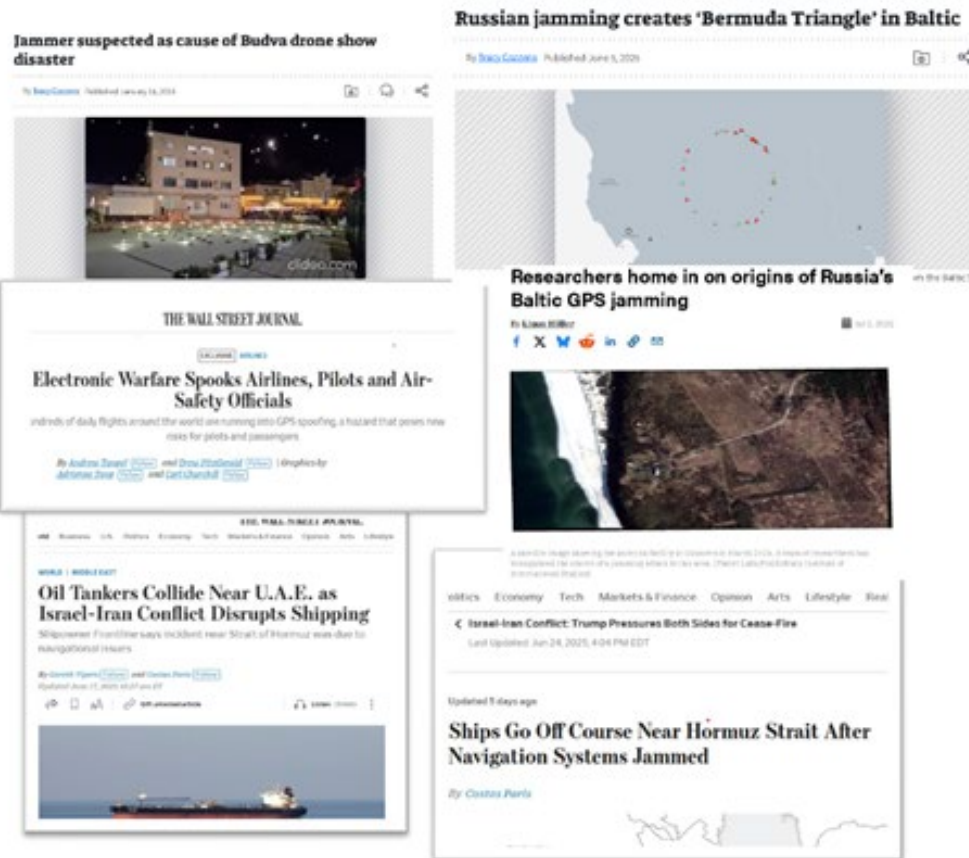
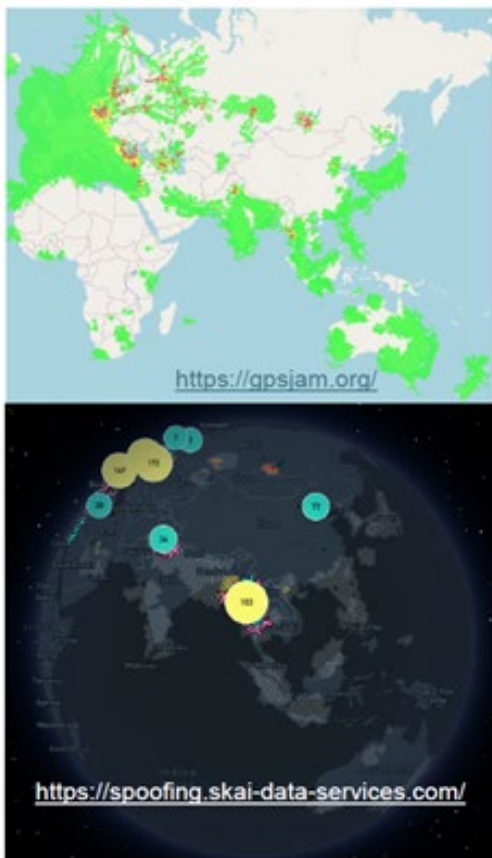
IMU's drift with time ...

Precise GPS updates of position and velocity can allow estimation of gravity vector

IMU+GPS => airborne gravimeter



Active GPS Jamming and Spoofing



Available SATCOM signals for PNTaaS Operation

Band	Freq	SoOP	Orbit
L	1 – 2 GHz	GNSS, Iridium Inmarsat	MEO GEO
S	2 – 4 GHz	GlobalStar TDRS COSMIC-2	MEO GEO LEO
C	4 – 8 GHz	Xona Intelsat, Telesat, SES, etc.	LEO GEO
X	8 – 12 GHz	WGS, Skynet, XTAR	GEO
Ku	10.7–12.7 GHz 12 – 18 GHz	OneWeb, SpaceX DBS, Viasat, SES	LEO GEO
Ka	17.8-18.6 GHz 19.7-202.2 GHz	Telesat, Kuiper, O3B mPower ViaSat, Telesat	LEO MEO GEO

Satbeams publishes footprints and frequencies

Earth gravity – geodesy and geophysics

$$g = 9.81\ 547\ 461\ \text{m/s}^2$$

Earth shape

Tide-forces

Geology

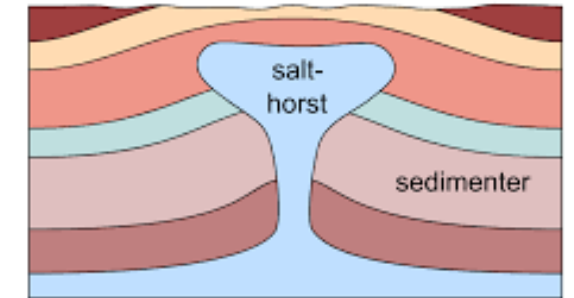
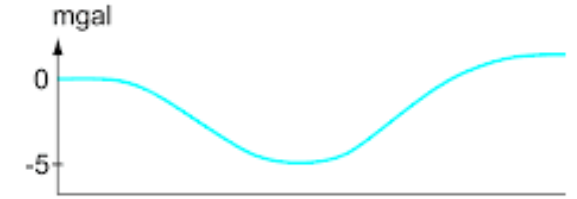
Air pressure

Hydrology

Mass changes

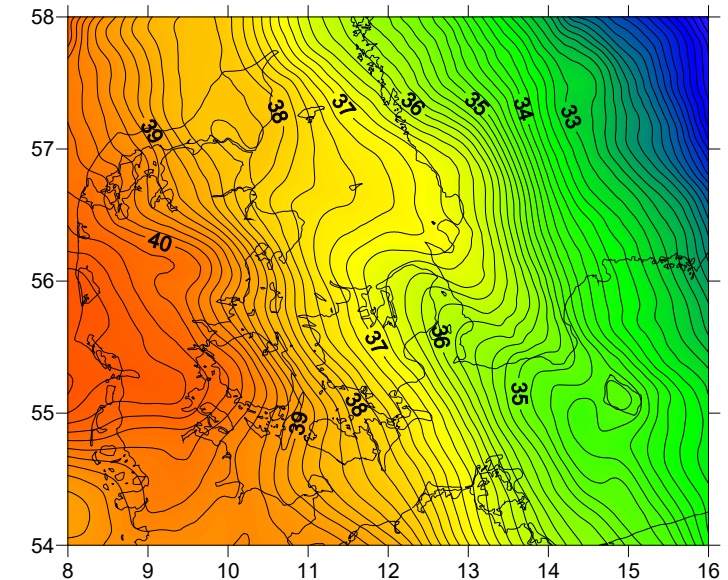
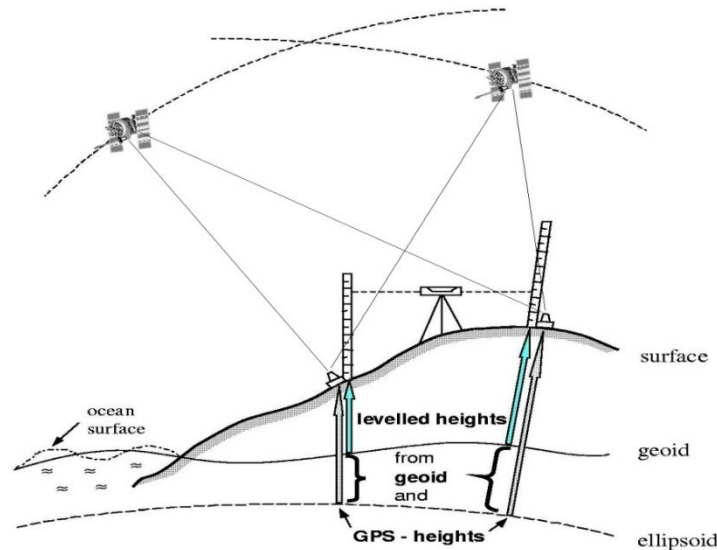
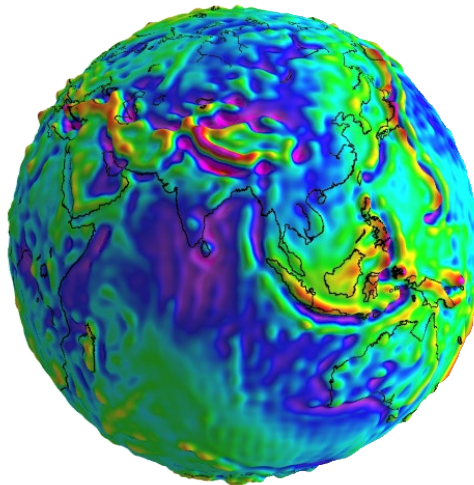
Primary applications:

- GPS and geodesy / geoid and height systems
- Oil, gas and mineral exploration
- Satellite orbits and navigation (inertial navigation / missiles / submarines)



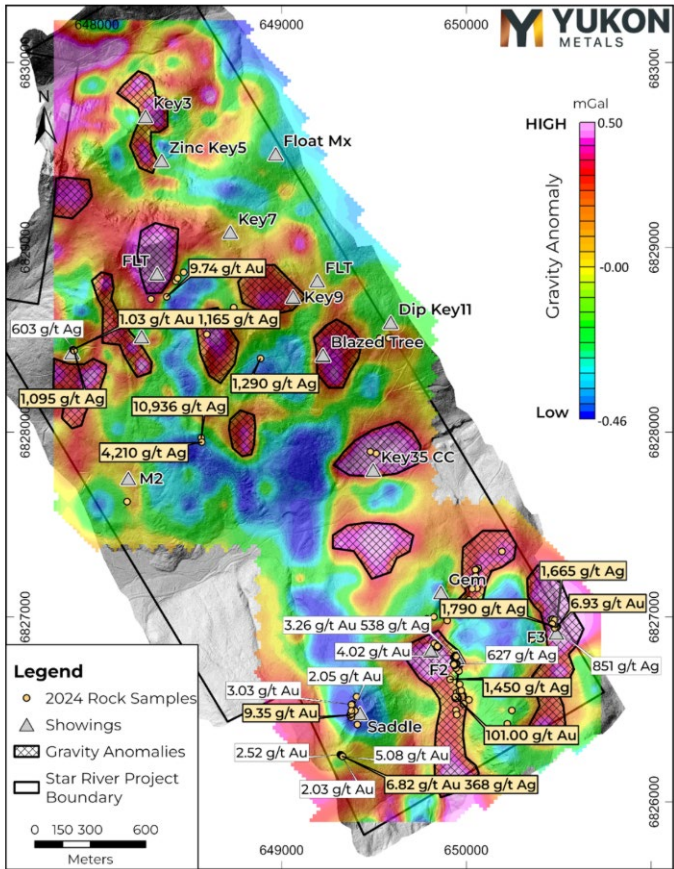
Salt domes – oil and gas exploration

Conventional unit: mGal – $10^{-5}\ \text{m/s}^2$

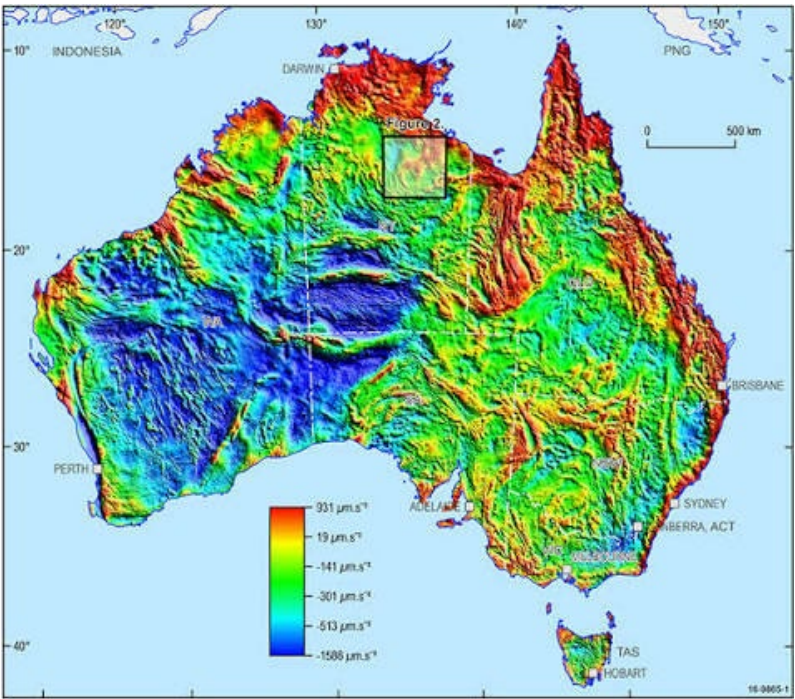


Geoid in Danmark (meter)

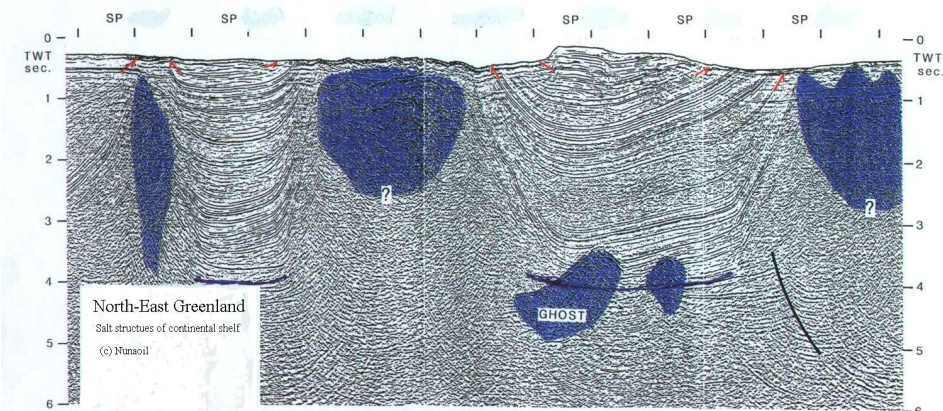
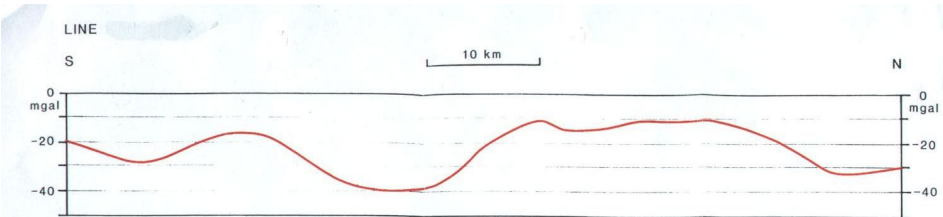
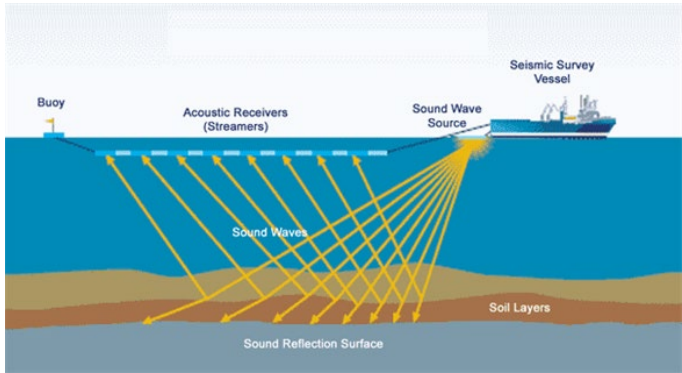
Geophysical exploration



Gravity always used together with seismic, magnetic, EM data ..

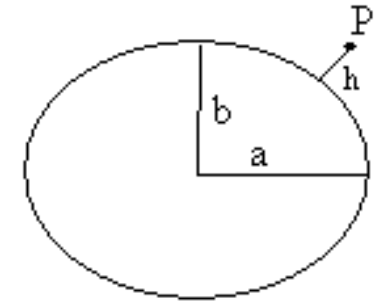


Australia Geoscience

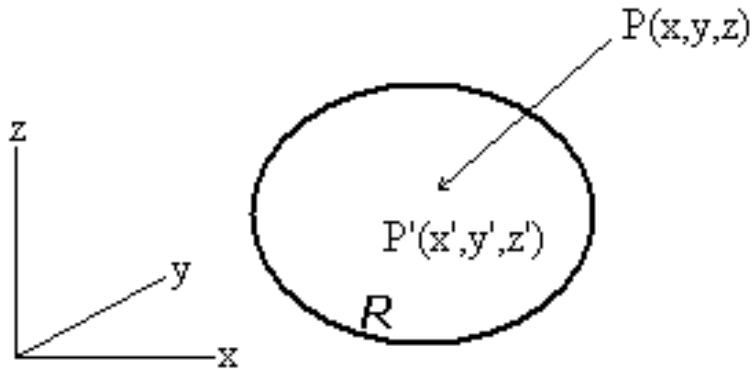


KANUMAS consortium

Geodesy: Some basic gravity concepts and textbooks ...



Newtonian potential and gravity field



$$V(x, y, z) = G \int_R \frac{\rho}{r} dR = \frac{GM}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}$$

$$\bar{g} = \nabla V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} \right)$$

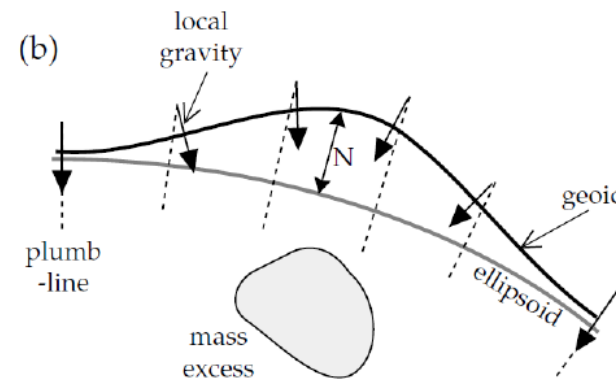
G: Gravitational constant; ρ : density

$W = V + \Phi_{\text{rotation}}$ physical potential on a rotating earth

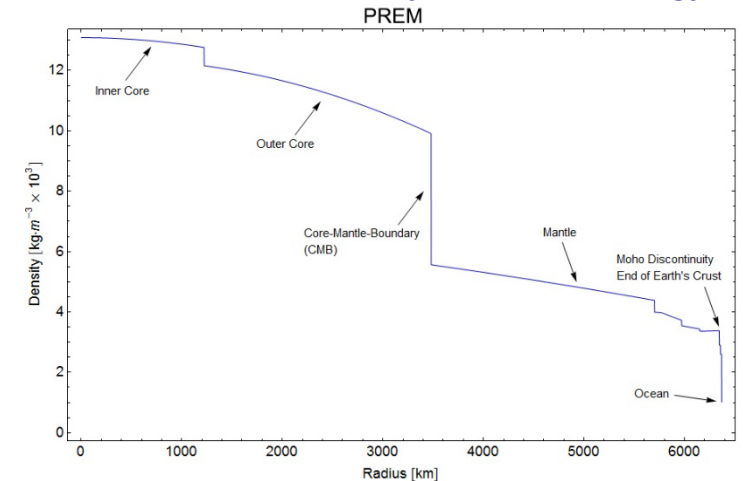
Reference ellipsoid defined by earth mass M, semimajor axis a, semi-minor axis b, rotation ω
GPS: WGS84
a = 6368137 km
b = 6356752 .. km
f = (a-b)/a ~ 1/298

Geoid height is deviation of equipotential from a selected reference ellipsoid (e.g., WGS84 = GRS80 ellipsoid)

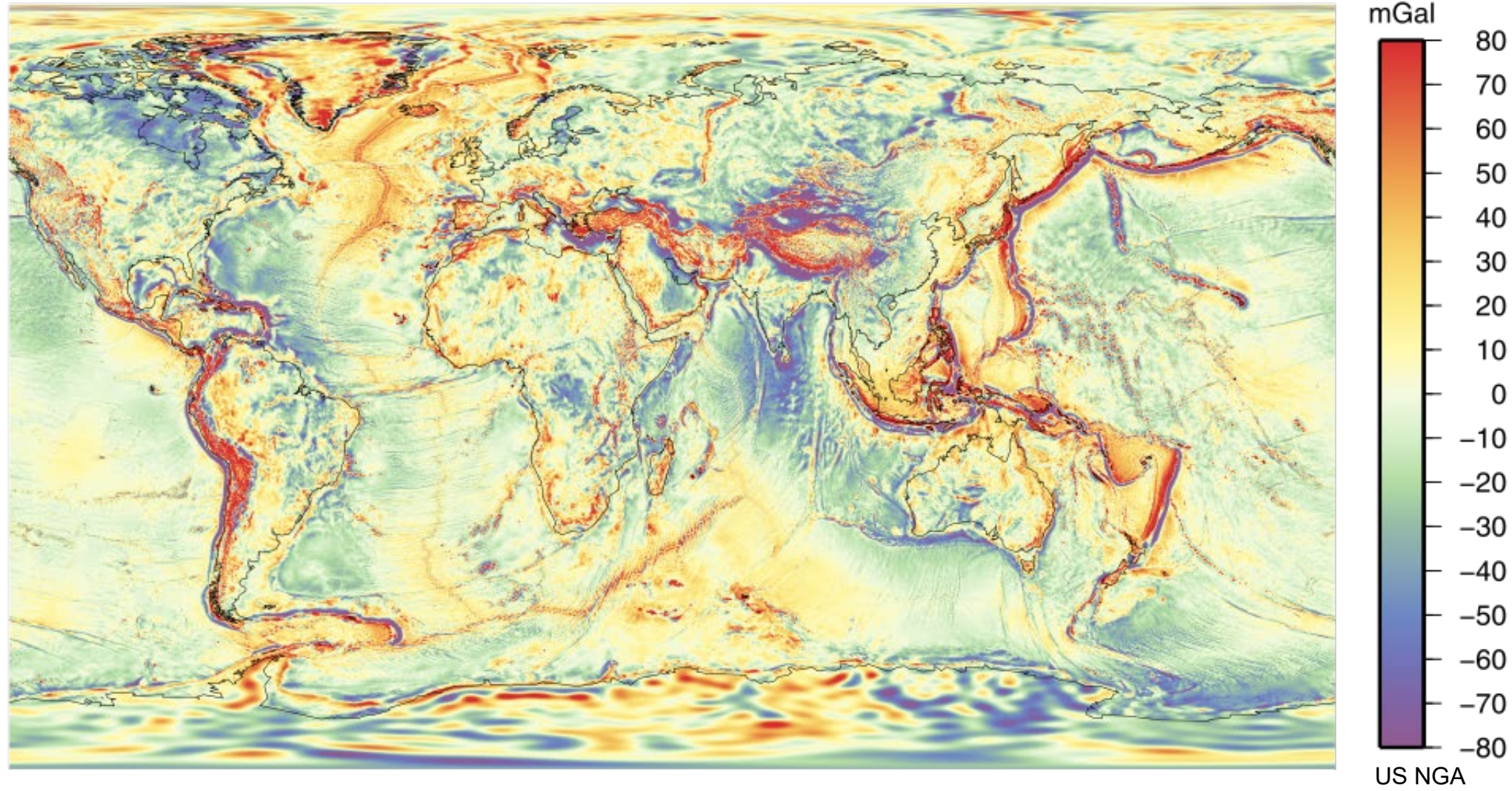
Gravity anomaly is deviation of gravity from normal gravity as expected on ellipsoid from theory .. Gravity anomalies measured in mgal (= 10^{-5} m/s^2)



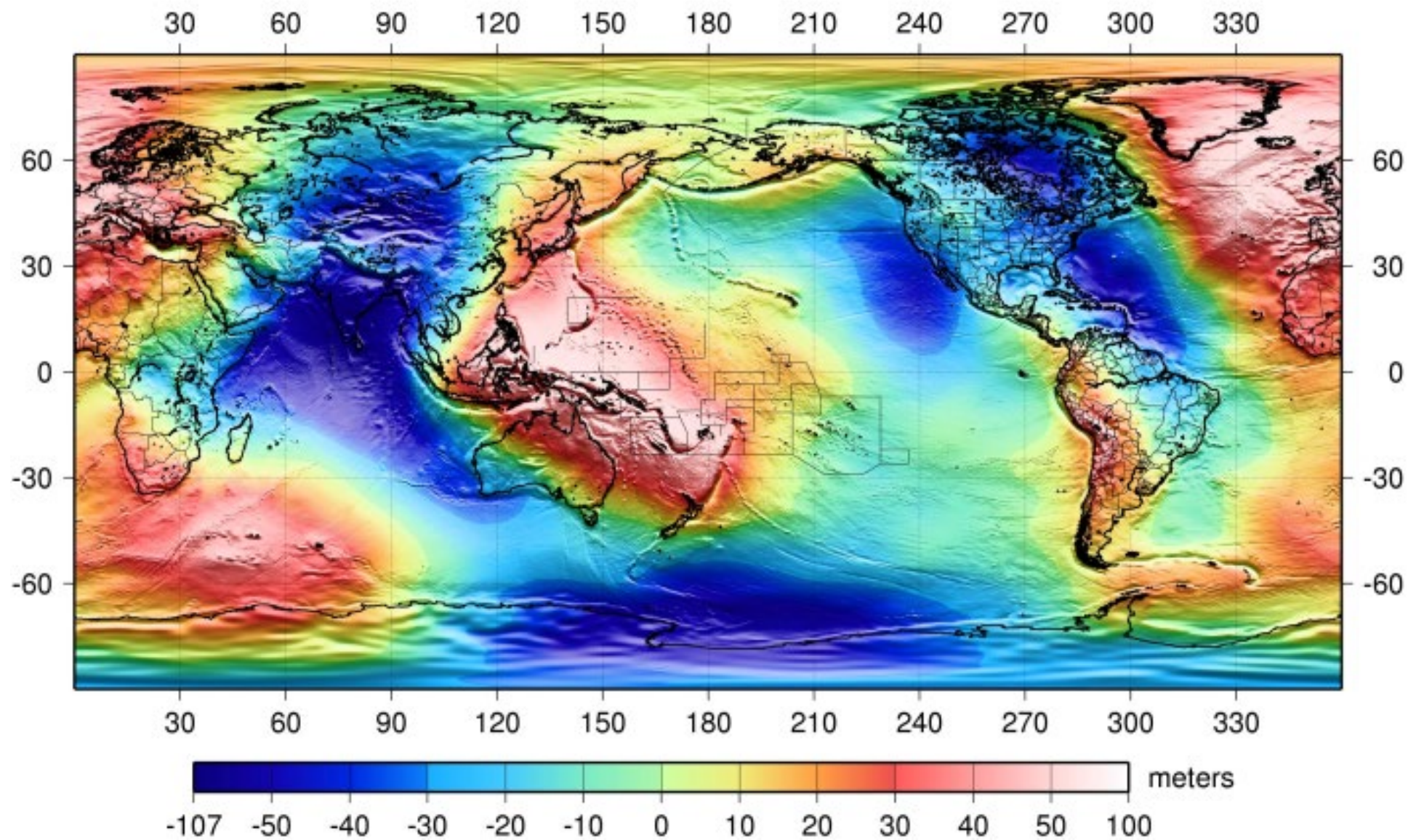
Real earth density from seismology



Free-air gravity anomaly from the Earth Gravitational Model 2008 (EGM2008)



EGM08 5 MINUTE GEOID UNDULATION



Basic geodetic coordinate systems

Earth-fixed – latitude φ , longitude λ , height h (over the ellipsoide)

Co-latitude: $\theta = 90^\circ - \varphi$. In consistent notation: φ also used for longitude at times ...

Cartesian: X, Y, Z

Local N, E, Up system ... *ellipsoid vertical n is different from local vertical g*

Direction in relation to ellipsoid normal: deflections of the vertical (ξ, η)

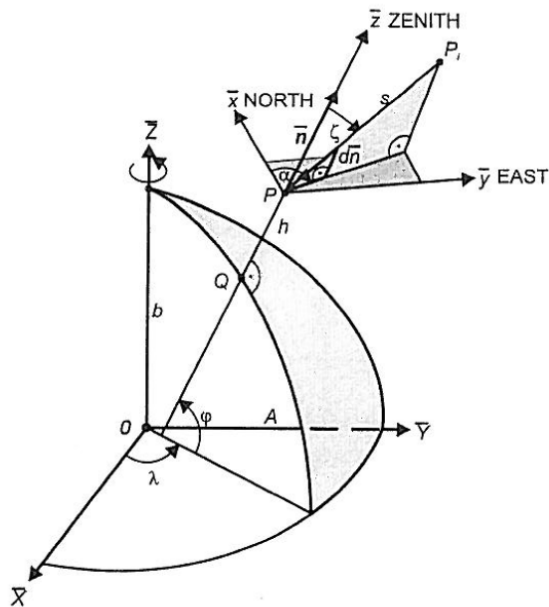


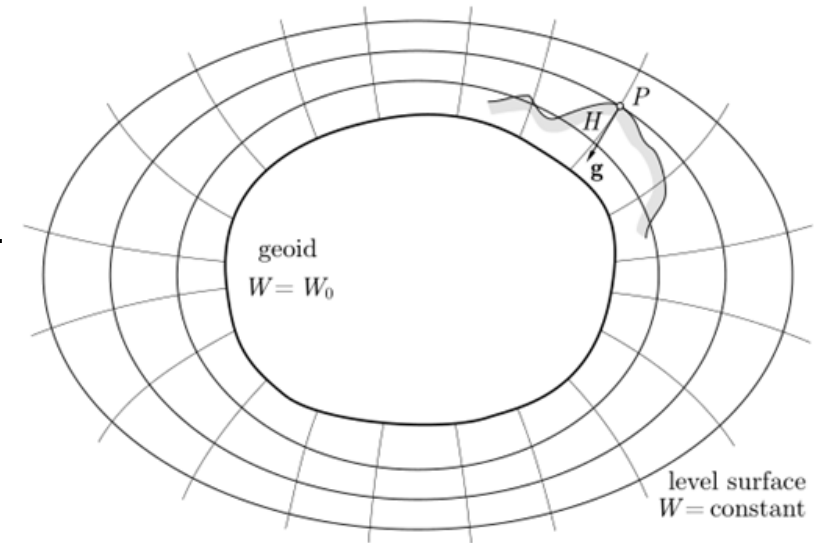
Fig. 4.6. Global and local ellipsoidal system

Ellipsoid parameters:

Flattening, eccentricity, radius of curvature (prime vertical curvature):

$$f = \frac{a-b}{a} \quad e = \frac{\sqrt{a^2 - b^2}}{a}$$

$$N = \frac{a}{(1 - e^2 \sin^2 \varphi)^{1/2}}$$



Ellipsoid chosen so it has a "best fit" to the geoid = the "undisturbed" sea surface

$$\bar{\mathbf{r}} = \begin{pmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \end{pmatrix} = \begin{pmatrix} (N+h)\cos\varphi\cos\lambda \\ (N+h)\cos\varphi\sin\lambda \\ ((1-e^2)N+h)\sin\varphi \end{pmatrix}$$

The Laplace equation and harmonic functions

Second derivatives: V_{xx} , V_{xy} , V_{xz} for a point mass easily derived by differentiation ..
 Second order gravity field *tensor* (symmetrical 3 x 3 matrix)

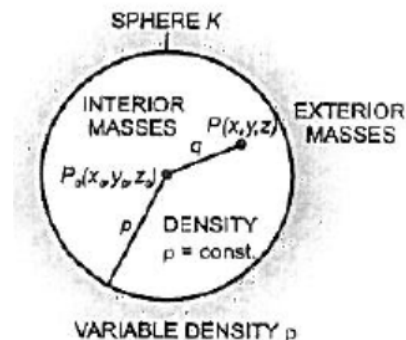
Globally:

$$\frac{\partial^2 V}{\partial X^2} = V_{xx} = -G \iiint_{\text{earth}} \frac{dm}{l^3} + 3G \iiint_{\text{earth}} \frac{(X - X')^2}{l^5} dm$$

Outside mass potential field will be harmonic:

$$\nabla^2 V = \Delta V = V_{xx} + V_{yy} + V_{zz} = 0$$

Inside mass a shrinking small sphere used to derive Poisson equation:



$$\Delta V = -4\pi G\rho$$

Used in borehole profiling to find densities

Harmonic functions have many "nice" properties: Mass can be derived from flux through an external surface



ESA GOCE 2009-13:

Global Ocean Circulation Explorer
 Direct measurement of tensor elements
 of gravity field in low orbit (~240 km)
 V_{xx} , V_{yy} , V_{zz} , V_{xy} , V_{xz} , V_{yz}
 (in local satellite system)

Enhed: Eotvos, 1 E = 10^{-9} s^{-2}

Relationships - geoid, gravity anomaly and deflections of the vertical ..

All relations based on harmonic functions => classic integral formulas ("filtrering") for the "anomalous" potential $T = W - U \dots$

i.e. ... if you know one compent of the gravity field anywhere on earth, you can compute all other quantities ... Nash and Jordan (1978 good overview for aerospace engineers ..)

PROCEEDINGS OF THE IEEE, VOL. 66, NO. 5, MAY 1978

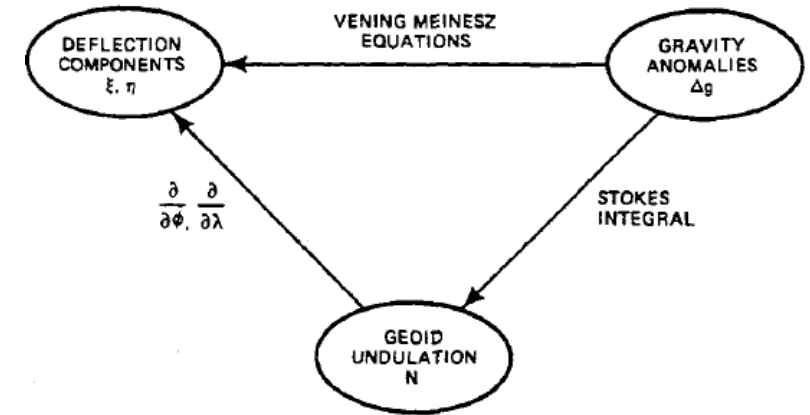


Fig. 2. Relationships among the gravimetric uncertainties.

NASH AND JORDAN: STATISTICAL GEODESY

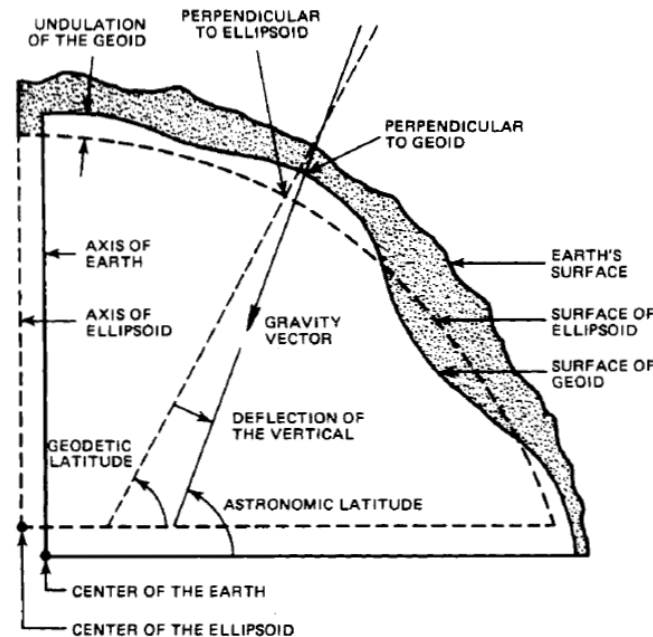


Fig. 1. Geodetic/gravimetric terminology.

$$N = \frac{R}{4\pi G} \int_S \Delta g(\psi, \alpha) F_0(\psi) dS$$

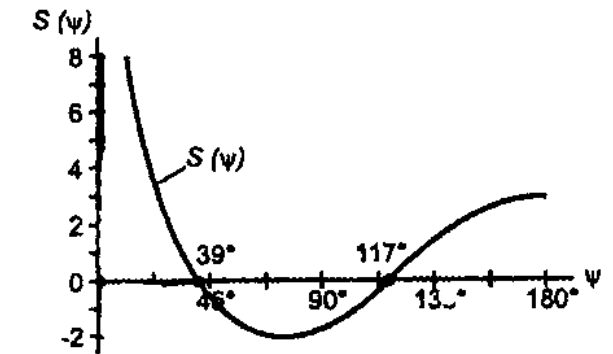
$$\begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = \frac{1}{4\pi G R^2} \int_S \frac{dF_0}{d\psi} \begin{Bmatrix} \cos \alpha \\ \sin \alpha \end{Bmatrix} \Delta g(\psi, \alpha) dS$$

$$\xi = -\frac{1}{R} \frac{\partial N}{\partial \phi}$$

$$\eta = -\frac{1}{R \cos \phi} \frac{\partial N}{\partial \lambda}$$

where

R mean radius of the geoid
 S sphere of radius R
 G average value of gravity over S
 $F_0(\psi)$ Stokes function.⁴

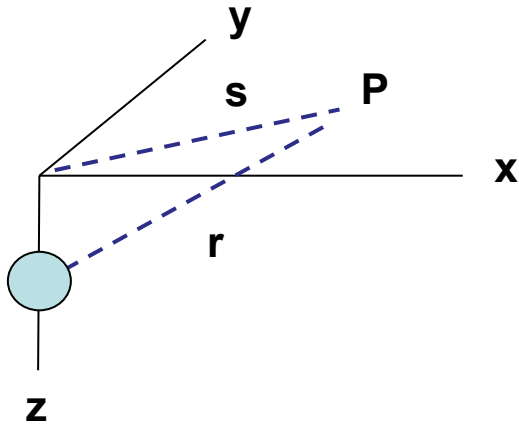


$$F_0(\psi) = \frac{1}{\sin(\frac{\psi}{2})} - 6 \sin \frac{\psi}{2} + 1 - 5 \cos \psi - 3 \cos \psi \log(\sin \frac{\psi}{2} + \sin^2 \frac{\psi}{2})$$

Getting the feel of gravity field quantities ... the point mass on a flat earth

Gravitational effect of a sphere of uniform density is the same as a point mass

Assume an (x, y, z) – system, here with z down for geophysics, and point mass at (0,0,z)



Potential

$$V = \frac{GM}{r}, \quad r = \sqrt{x^2 + y^2 + z^2}$$

Geoid

$$N = \frac{V}{\gamma} \quad (\text{Bruns formula})$$

Gravity anomaly

$$\delta g = GM \frac{z}{r^3}$$

Gravity gradients
tensor elements
(x,y,z) = (x₁, x₂, x₃)

$$V_{ij} = \frac{r^2 - 3x_i^2}{r^5} \quad (\text{for } i = j)$$

$$V_{ij} = \frac{3x_i x_j}{r^5} \quad (\text{for } i \neq j)$$

Conventional Units:

g.p.u. = 10 m²/s²

meter

mGal = 10⁻⁵ m/s²

Eotvos unit = 10⁻⁹/s²

G = newtonian
gravitational constant:

$6.6743 \times 10^{-11} \text{ m}^3 (\text{kg}^{-1} \text{ s}^{-2})$

*In conventional units for
gravity, density and m:*

$G = 0.00667$
 $\text{mGal} \cdot \text{m} / (\text{g}/\text{cm}^3)$

Solution to the Laplace equation – spherical coordinates: Spherical harmonics

$$\Delta V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \left(\frac{\partial^2 V}{\partial \lambda^2} \right)$$

Making solution to Laplace equation by separation of variables, and using constant k

$$V(r, \theta, \lambda) = V_1(r) V_2(\theta) V_3(\lambda)$$

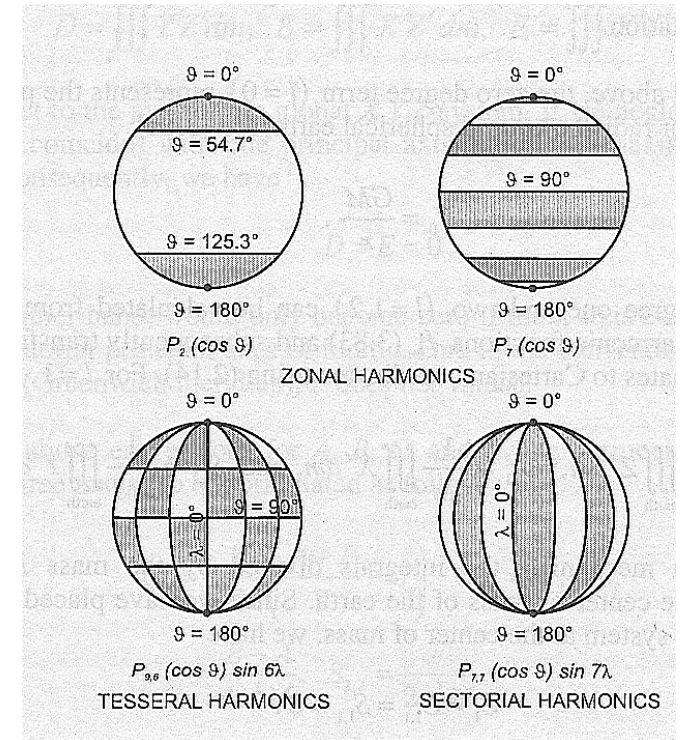
$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) = \mathbf{k}, \quad \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \left(\frac{\partial^2 V}{\partial \lambda^2} \right) = -\mathbf{k}$$

results in spherical harmonics and two radial solutions (for "inner" and "outer" fields)

$$r^n Y_{nm}(\theta, \lambda) \text{ and } r^{-(n+1)} Y_{nm}(\theta, \lambda)$$

for $n = 0, 1, \dots$ and $m = -n$ to n

Wellknown stuff in quantum mechanics also



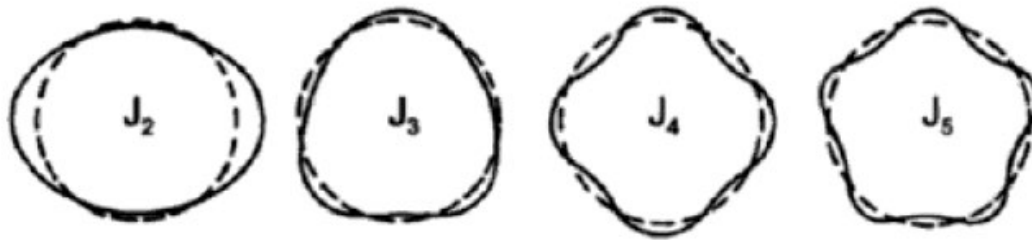


Figure 2.2: Illustration of the oscillation of the Legendre functions in the latitudinal direction, also known as the zonal harmonics. Notice that the Legendre polynomials only exist on the interval $\theta \in [0; \pi]$, meaning that these figures should be viewed as a cross-section of the Legendre polynomial, rotated around the north-south axis. The figure is from [Messerschmid and Bertrand \(1999\)](#)

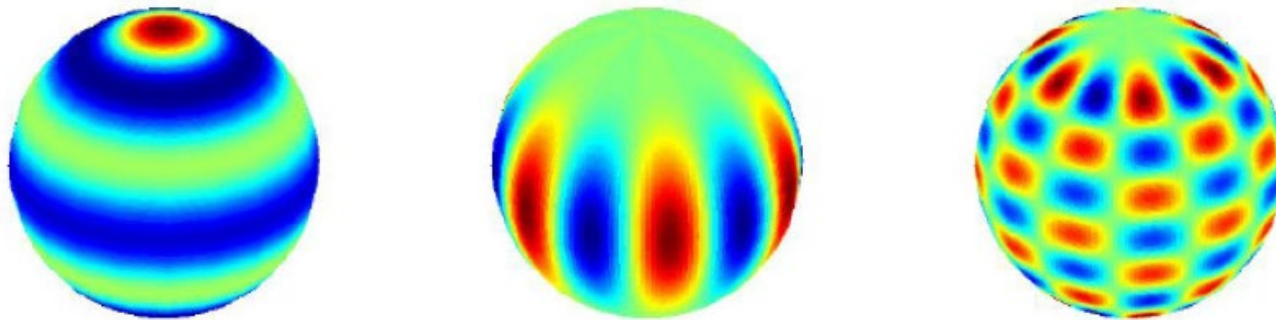
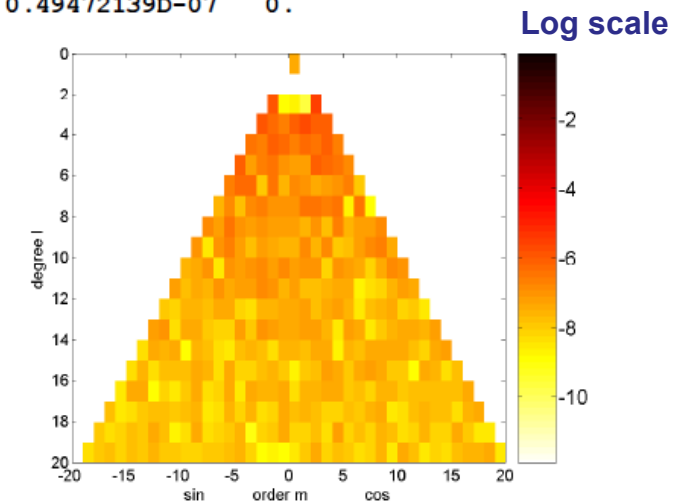


Figure 2.3: Illustration of the 3 kinds of spherical harmonics. Red areas indicative positive values, blue areas are negative. (Left:) zonal harmonics where $m = 0$, above is $P_{6,0}(\cos \theta)$; (middle:) sectorial harmonics where $l = |m|$, above is $P_{6,6}(\cos \theta) \cos(6\phi)$ and (right:) tesseral harmonics are all other cases of l and m , above we have $P_{12,6}(\cos \theta) \cos(6\phi)$.

Disturbing potential

C_nm	S_nm
-0.37013570D-07	0.
1.71110530D-09	0.
-0.20661550D-09	0.13844138D-08
0.24393835D-05	-0.14002737D-05
0.95716120D-06	0.
0.20304620D-05	0.24820041D-06
0.90478789D-06	-0.61900547D-06
0.72132175D-06	0.14143492D-05
-0.25033820D-06	0.
-0.53615738D-06	-0.47356734D-06
0.35050162D-06	0.66248002D-06
0.99085676D-06	-0.20095672D-06
-0.18851963D-06	0.30880388D-06
-0.14826667D-06	0.
0.49472139D-07	0.



Spherical harmonics expansion – gravity field change as function of height

$$V(r, \phi, \lambda) = \frac{GM}{r} \sum_{n=2}^N \sum_{m=0}^n \left(\frac{r}{R}\right)^{-n} [\bar{C}_{nm} \cos m\lambda + \bar{D}_{nm} \sin m\lambda] \bar{P}_{nm}(\sin \phi)$$
$$\delta g(\phi, \lambda) = \frac{GM}{a^2} \sum_{n=2}^N \sum_{m=0}^n (l+1) [\bar{C}_{nm} \cos m\lambda + \bar{D}_{nm} \sin m\lambda] \bar{P}_{nm}(\sin \phi)$$

Examples of global gravity field models:

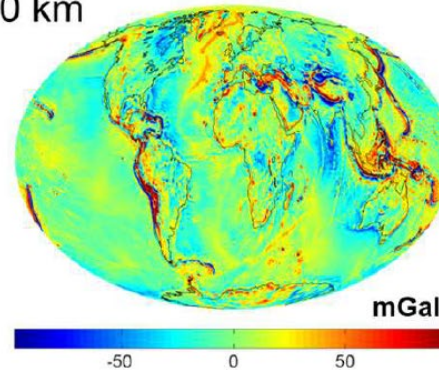
EGM2008 (N = 1960)
– satellite and surface data

GOCE-R6 (N = 300)
– GOCE/GRACE model

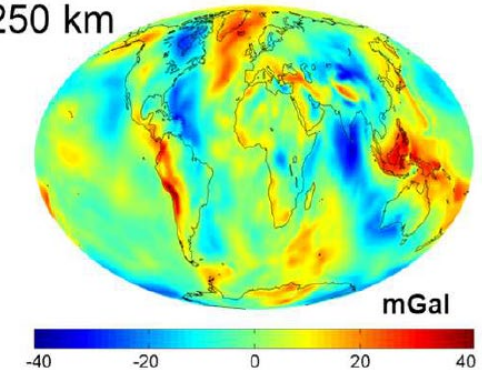
GRACE monthly models (N = 90)

See many more at
<http://icgem.gfz-potsdam.de/ICGEM/>

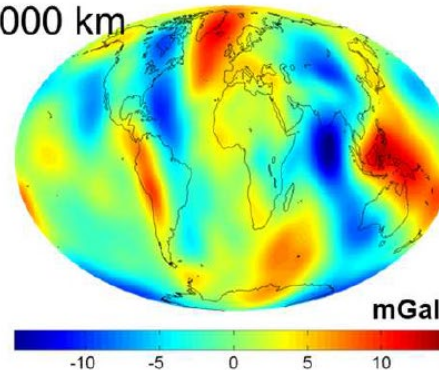
h = 0 km



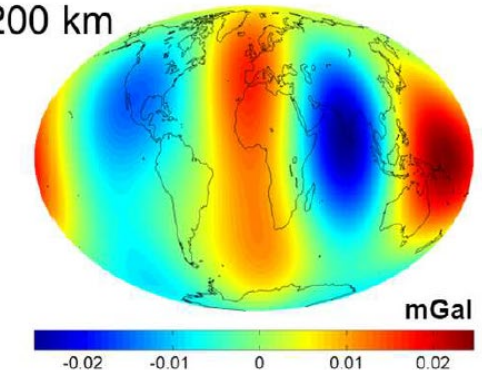
h = 250 km



h = 1000 km



h = 20200 km

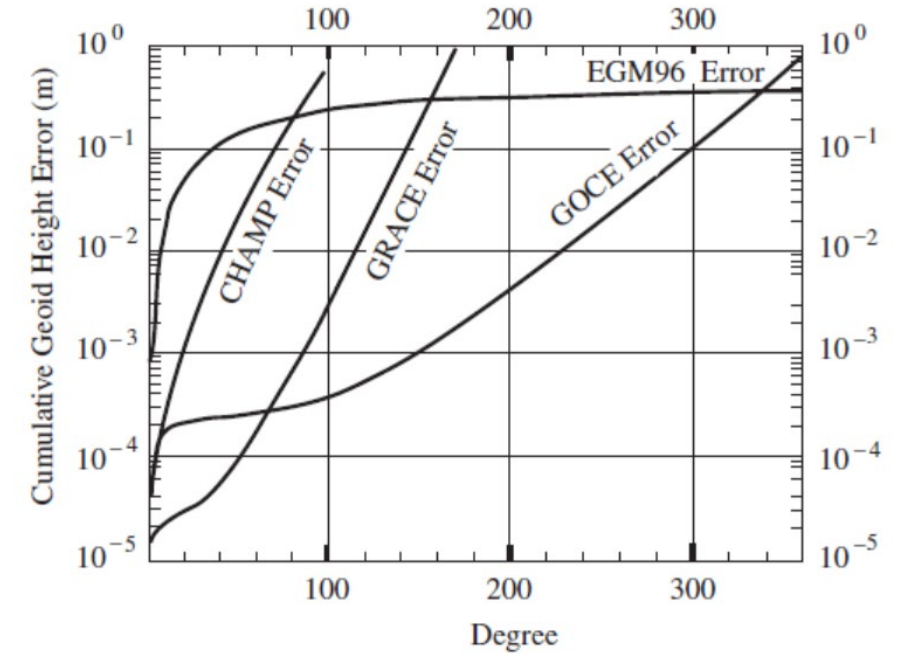
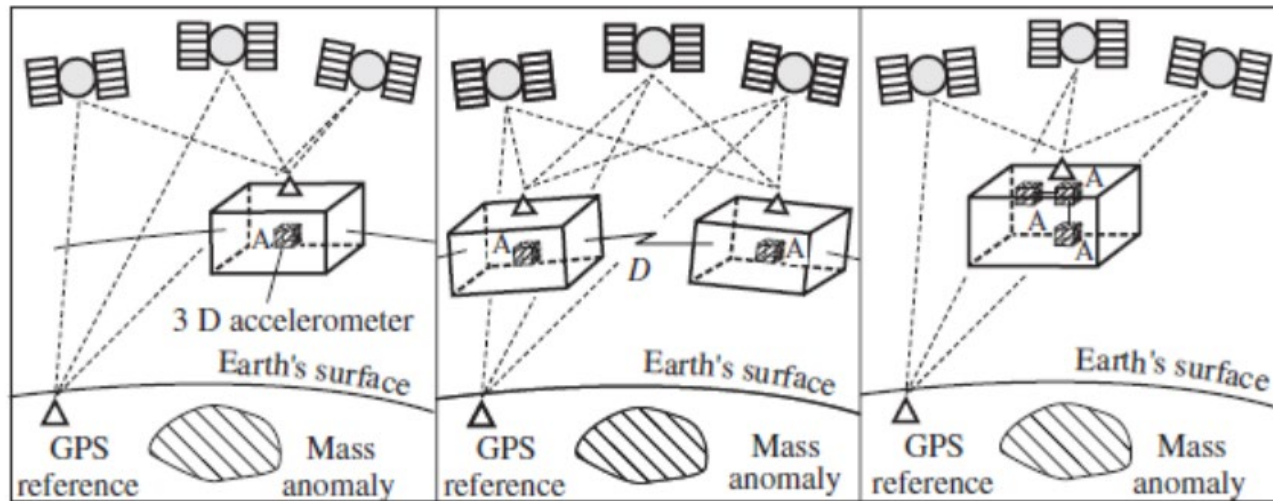


Satellite gravity: principle of measurements and error improvements

CHAMP

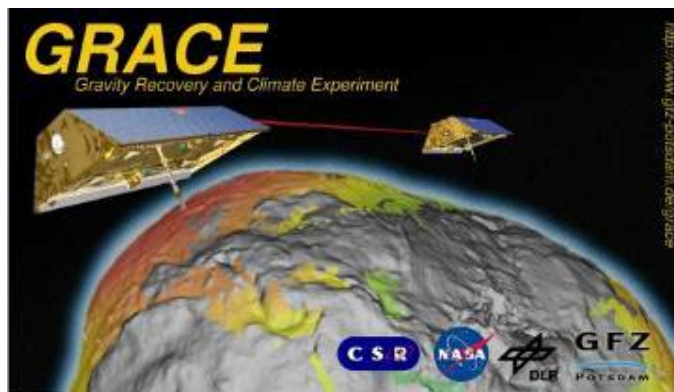
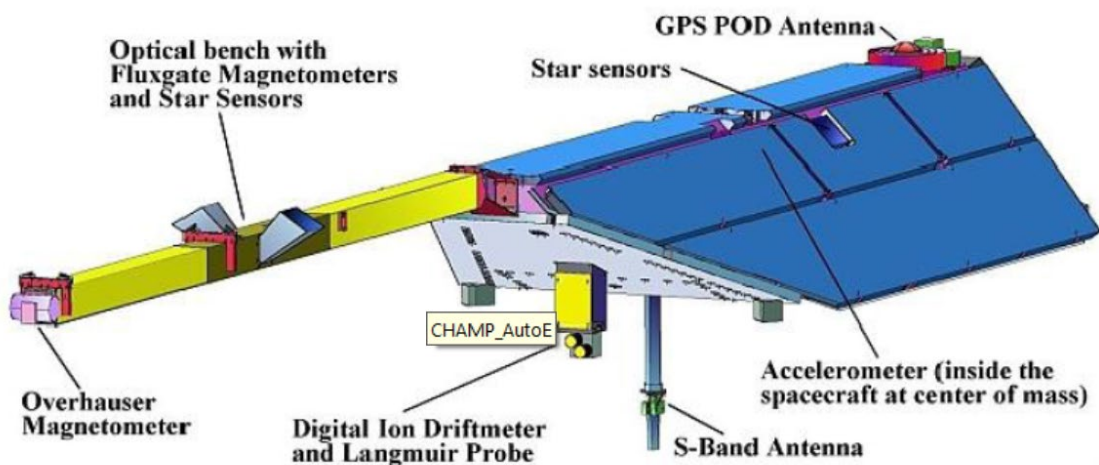
GRACE

GOCE



Error studies : $\sigma_n = \sum_{m=0}^n \left[C_{nm}^2 + S_{nm}^2 \right]$ is the degree-variance, equivalent to power spektrum in FFT

CHAMP 2000 – Challenging Micro

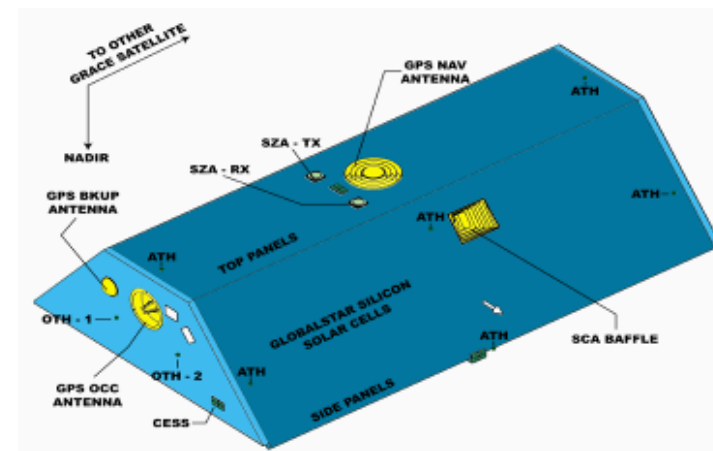


GRACE 2002-17 / GRACE-FO 2018- – Gravity and Climate Experiment

Temporal change of earth's gravity field – monthly spherical harmonic solutions

Ultra-precise geoid models to be used for:

- deep ocean changes ("ocean bottom pressure"),
- mass balance of ice sheets and glaciers,
- changes in continental storage of water and snow
- mantle and lithospheric density variations
- postglacial rebound
- geoid ... static field improve long-wavelengths



GRACE data - monthly gravity fields

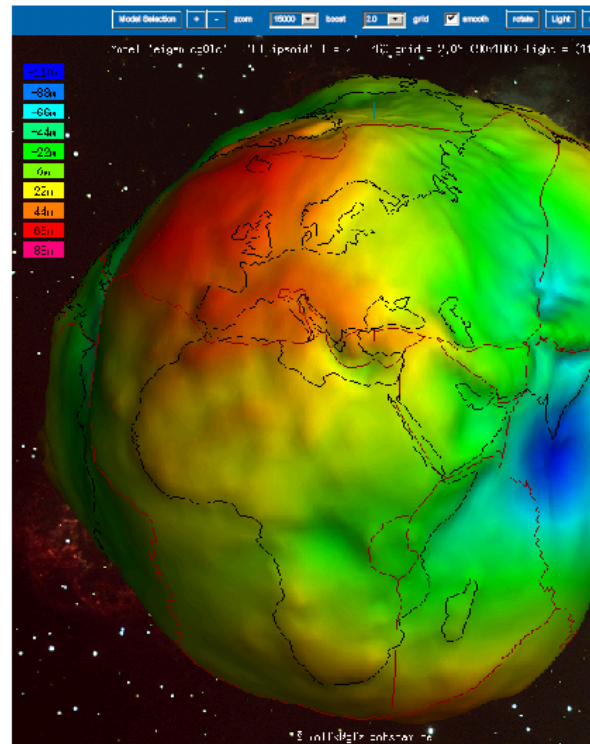
Different processing centers: CSR/University of Texas, GFZ/Germany, CNES/France, NASA JPL/USA, TU Graz/Austria ..

GRACE online information ..

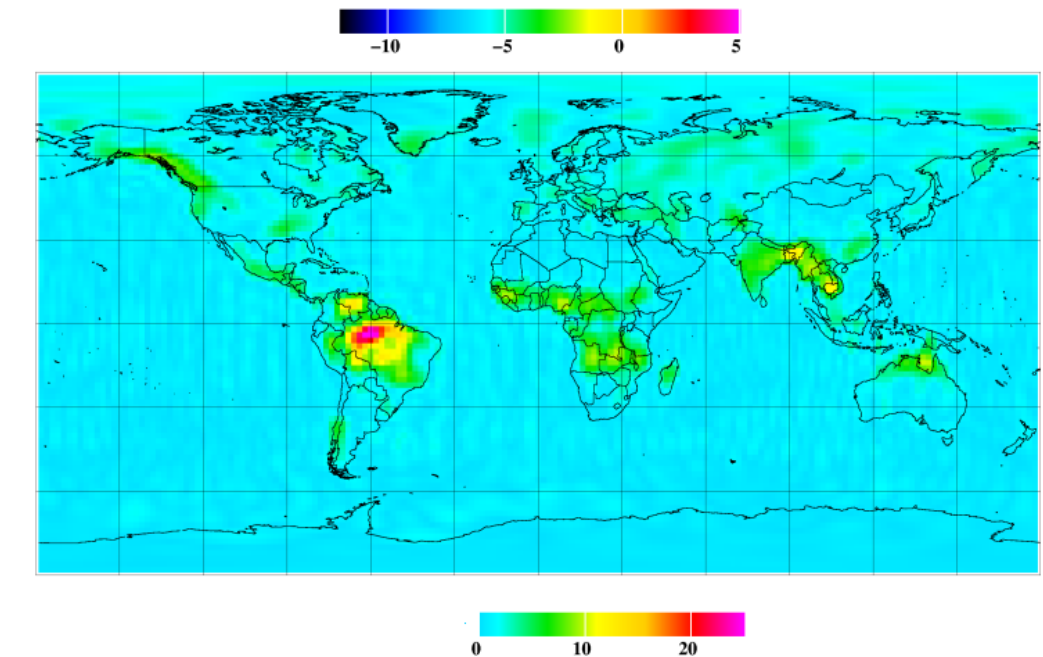
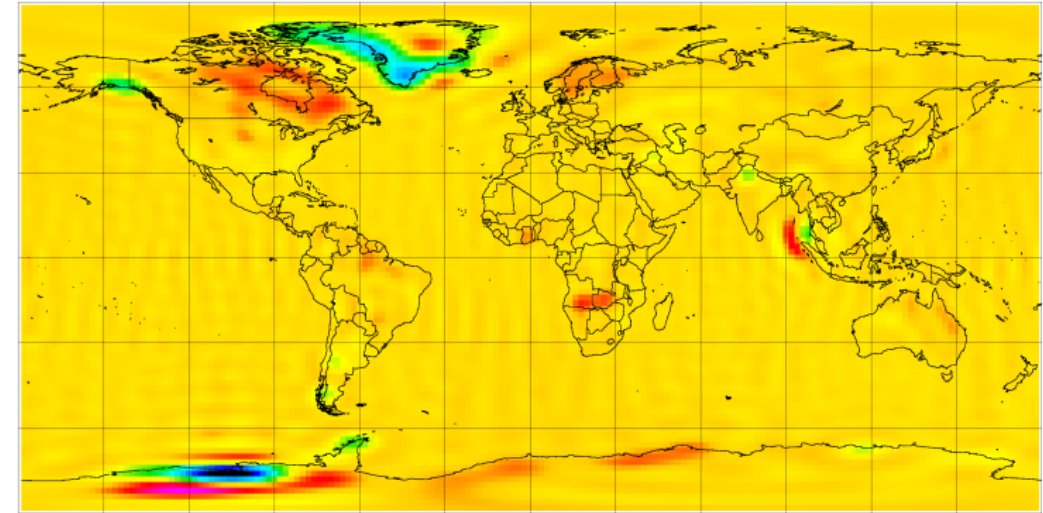
GRACE example – ICGEM

- 100+ spherical harmonic models "in stock"
- Online visualisation
- Online computation of grids
- Both static and temporal fields

www.gfz-potsdam.de/icgem/icgem.html

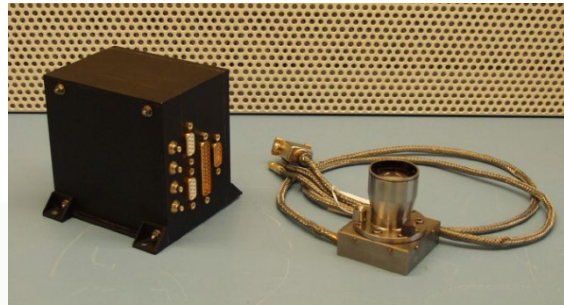


GRACE gravity changes – yearly gravity trend and seasonal change (CSR RL5 dg at 480 km, deg 60), μGal

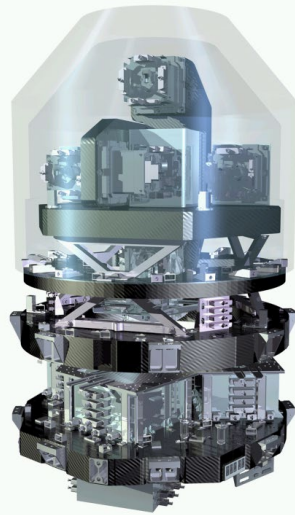


GOCE – ESA satellite gradiometry 2009-13

- 6-component measurement of tensor
 $V_{xx}, V_{yy}, V_{zz}, V_{xy}, V_{xz}, V_{yz}$
- New technology: drag-free satellite and electrostatic accelerometers
- Very low orbit (260 km) ... Lowered to 224 km in last phase of mission => higher resolution



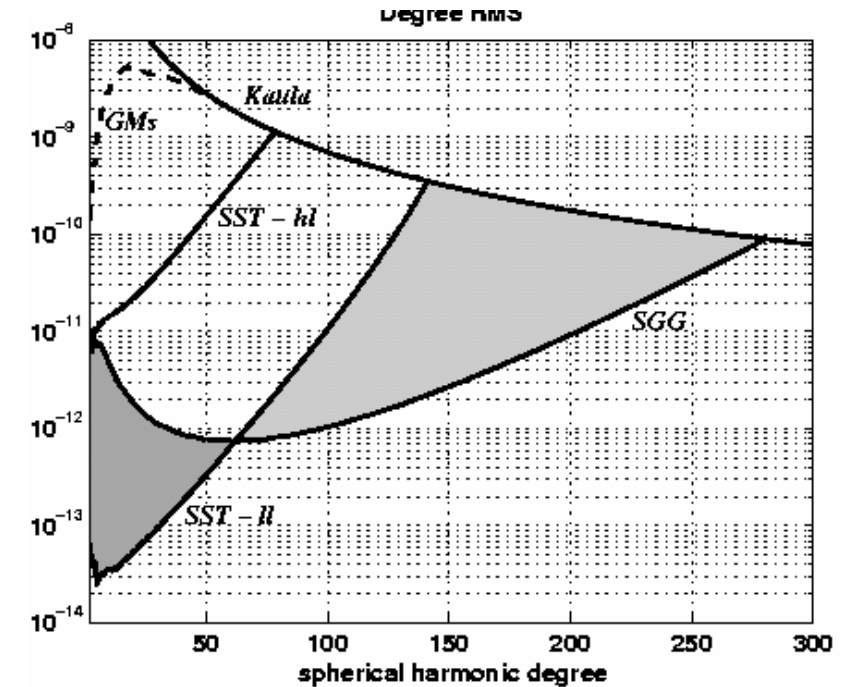
Star camera (DTU Space)



3-axis gradiometer (ONERA)



Error spectrum for CHAMP (SST-hl),
GRACE (SST-II) and GOCE (SGG)

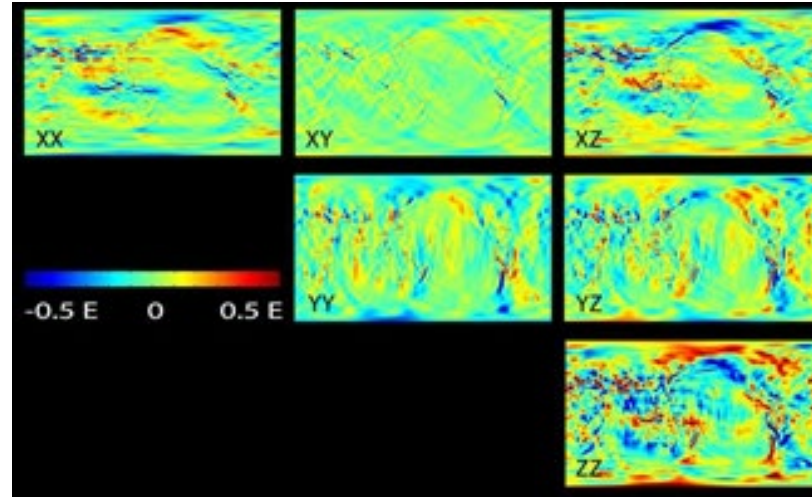


Several GOCE models – latest
Release 6 – 2016
Complete mission data 2009-13:
DIR – fields with GOCE+GRACE
TIM – GOCE only, degree 300

GOCE gravity gradients

$$V_{ij} = \frac{\partial^2 V}{\partial i \partial j}$$

$$V'' = \begin{pmatrix} V_{XX} & V_{XY} & V_{XZ} \\ V_{YX} & V_{YY} & V_{YZ} \\ V_{ZX} & V_{ZY} & V_{ZZ} \end{pmatrix}$$



Only 5 independent components (symmetry + Laplace equation)

Measurement units: Eotvos, 1 E = 10^{-9}s^{-2}

$$V_{XY} = V_{YX}, \quad V_{XZ} = V_{ZX}, \quad V_{YZ} = V_{ZY}$$

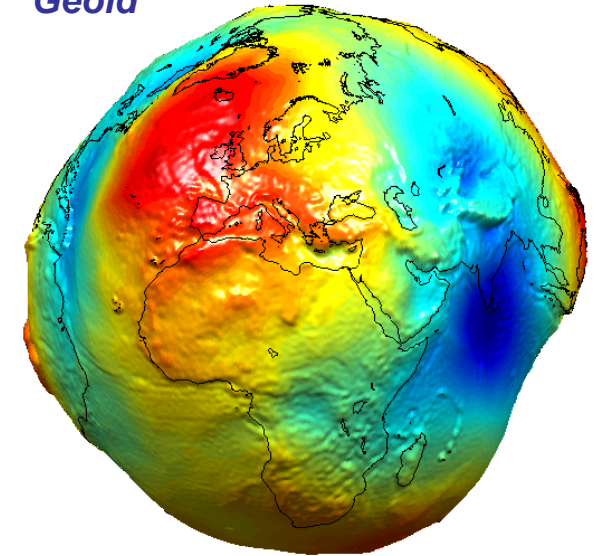
$$V_{XX} + V_{YY} + V_{ZZ} = 0$$

L1 data: gradients local "strapdown" (forward, left, cross system)

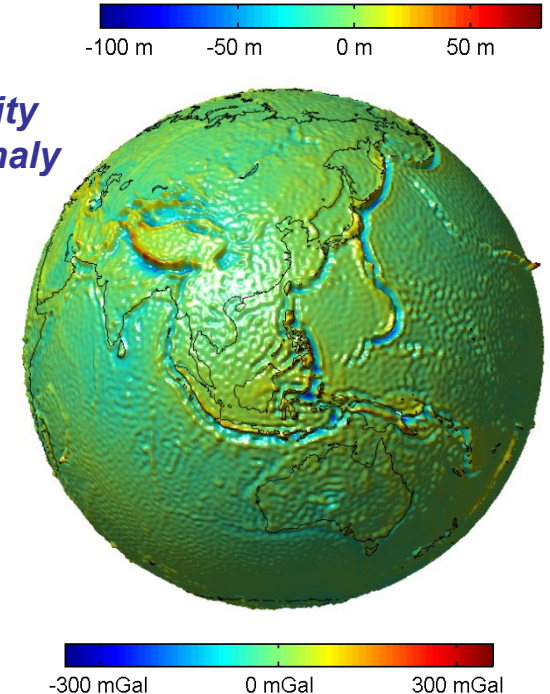
L1B: N,E,U system (complicated by different quality of measurements)

L2: spherical harmonics (ESA models: "R5 direct" or "R5 timewise" ..)

Geoid



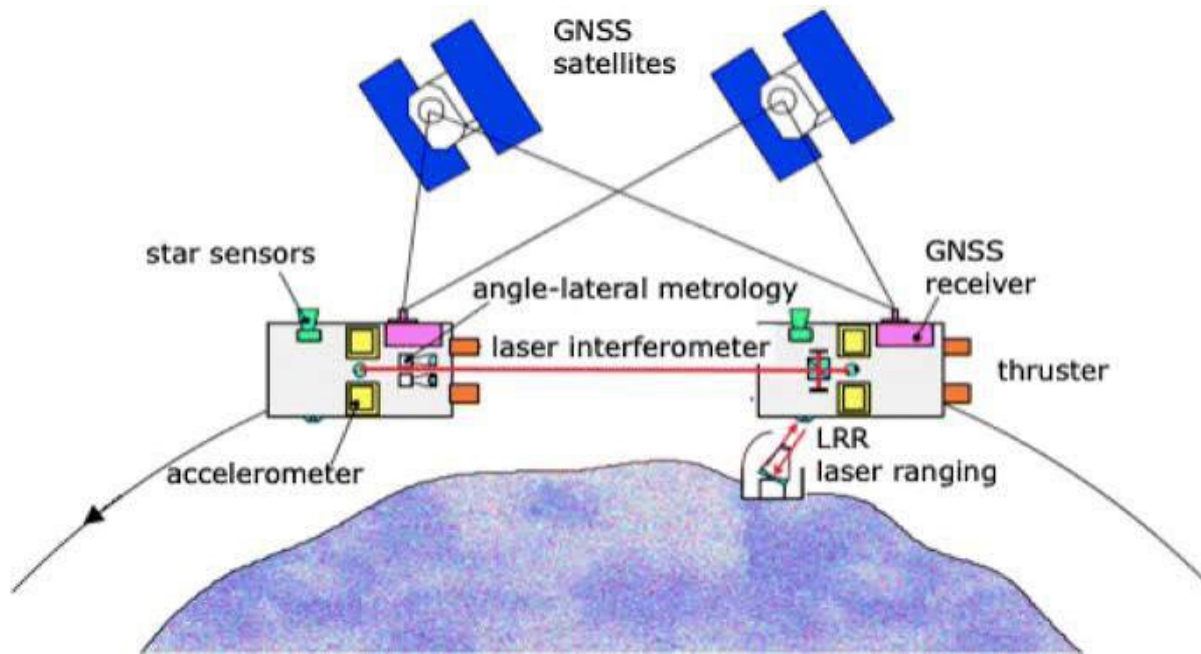
Gravity anomaly



NGGM – Next Generation Gravity Mission (ESA)

When flying together with GRACE-FO/GRACE-C = "MAGIC" satellite constellation

Laser links only – lower orbit to give higher resolution .. Two satellite pairs allow solutions every 5 day

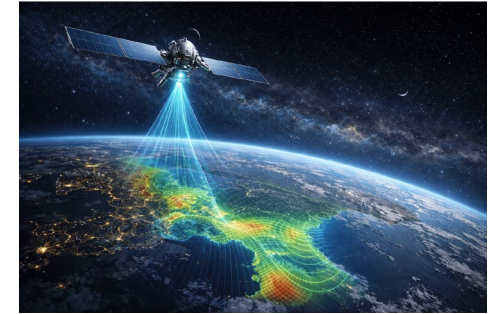


Quantum gravimetry satellite missions ... CARIOQA and NASA-Inflection

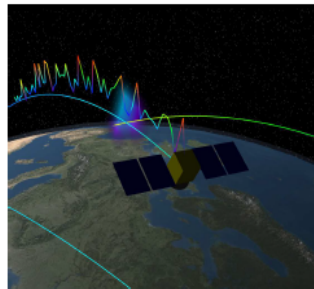
CARIOQA EU project to develop a quantum pathfinder mission – .. Lead CNES, DLR ..
Building on EU-DEFIS desktop report 2020-21, launch 2031-32? *Funded via HE calls..*

NASA-JPL to launch gravity gradiometer 2030? Block-band lattice quantum technologies?

NASA-Inflection



CARIOQA Framework



CARIOQA:

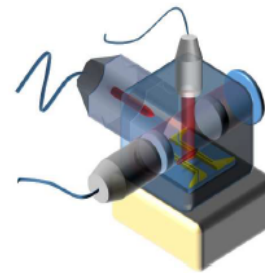
- Cold Atom Rubidium Interferometer in Orbit for Quantum Accelerometry
- Mission for demonstrating the operation of a Quantum Accelerometer on a satellite.
- Scientific data feedback: Earth's gravity needs validation and Thermosphere modelling.

Instrument guidelines :

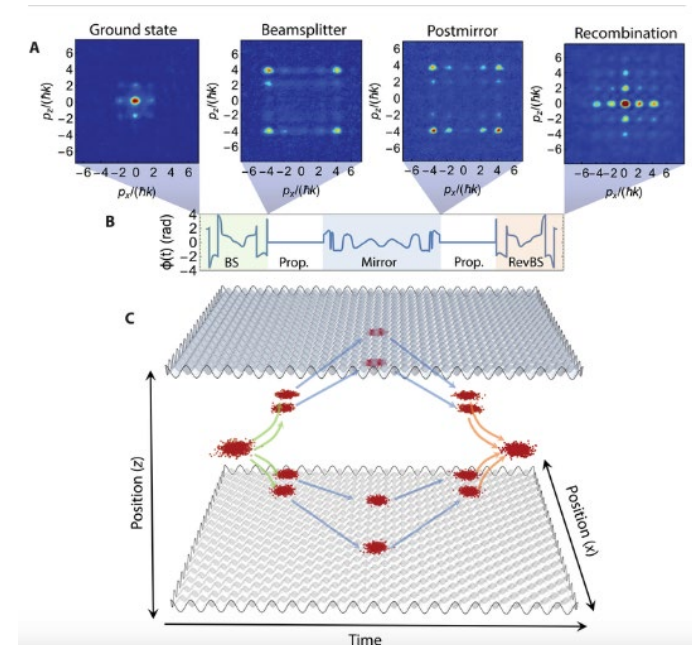
- Atom accelerometer
- Sensitivity: $< 10^{-9} \text{ m.s}^{-2}.\text{Hz}^{-1/2}$
- Mass: $< 100 \text{ kg}$
- Consumption: $< 100 \text{ W}$

Mission :

- Satellite platform: COTS (If possible)
- Development: ~ 5 years
- Mission lifetime: 3 to 5 years



(T Laveque, CNES)



JILA, Science 2025

Gravity data collection – global cooperation

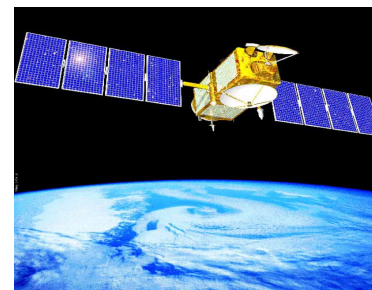
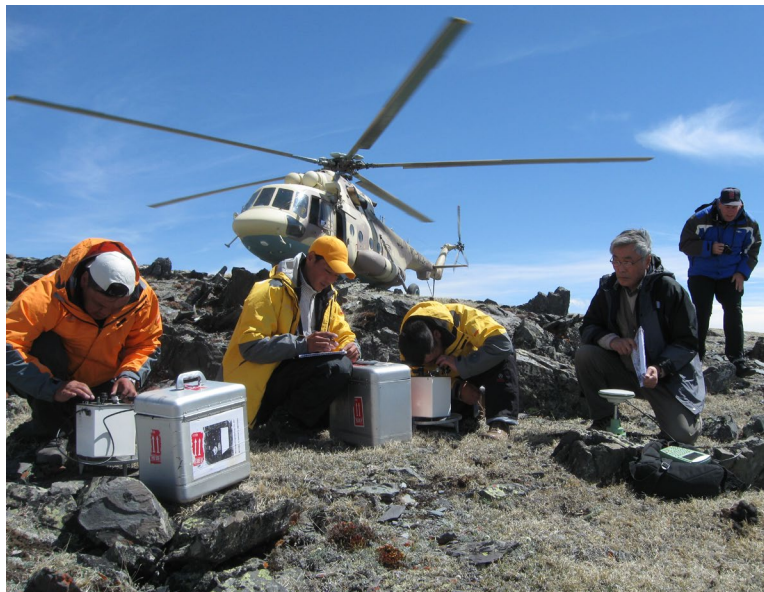
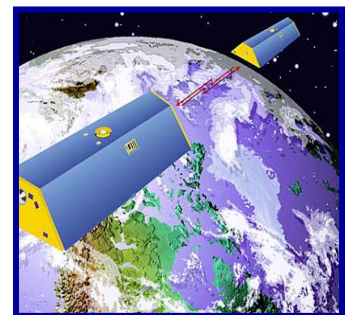
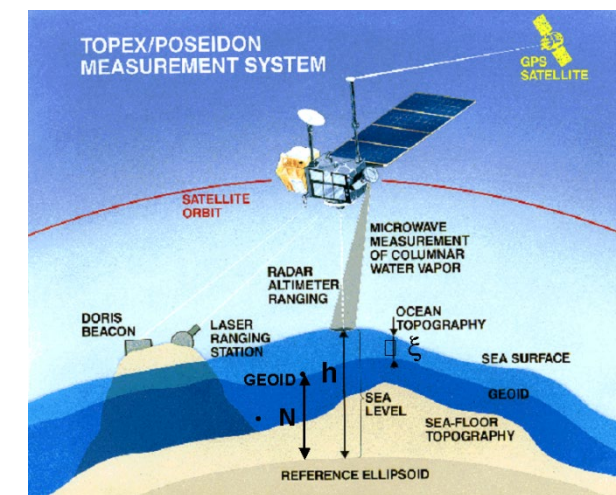
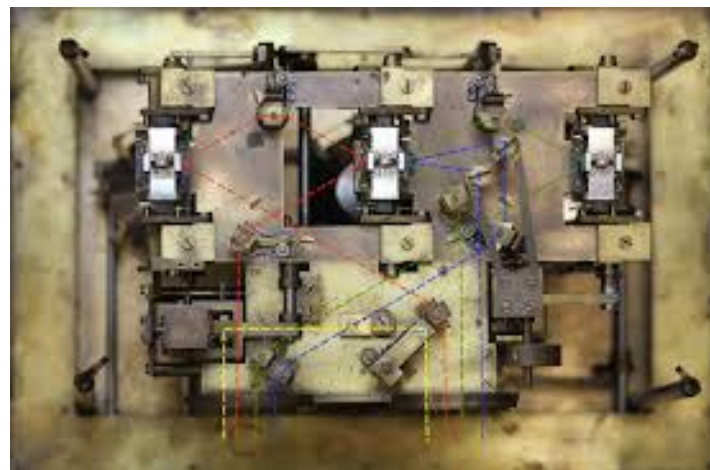
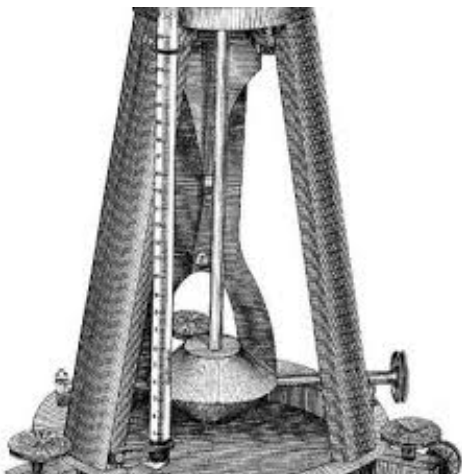


Photo: © NH 06/72-R/N USNS Henderson (T-ESB-9)





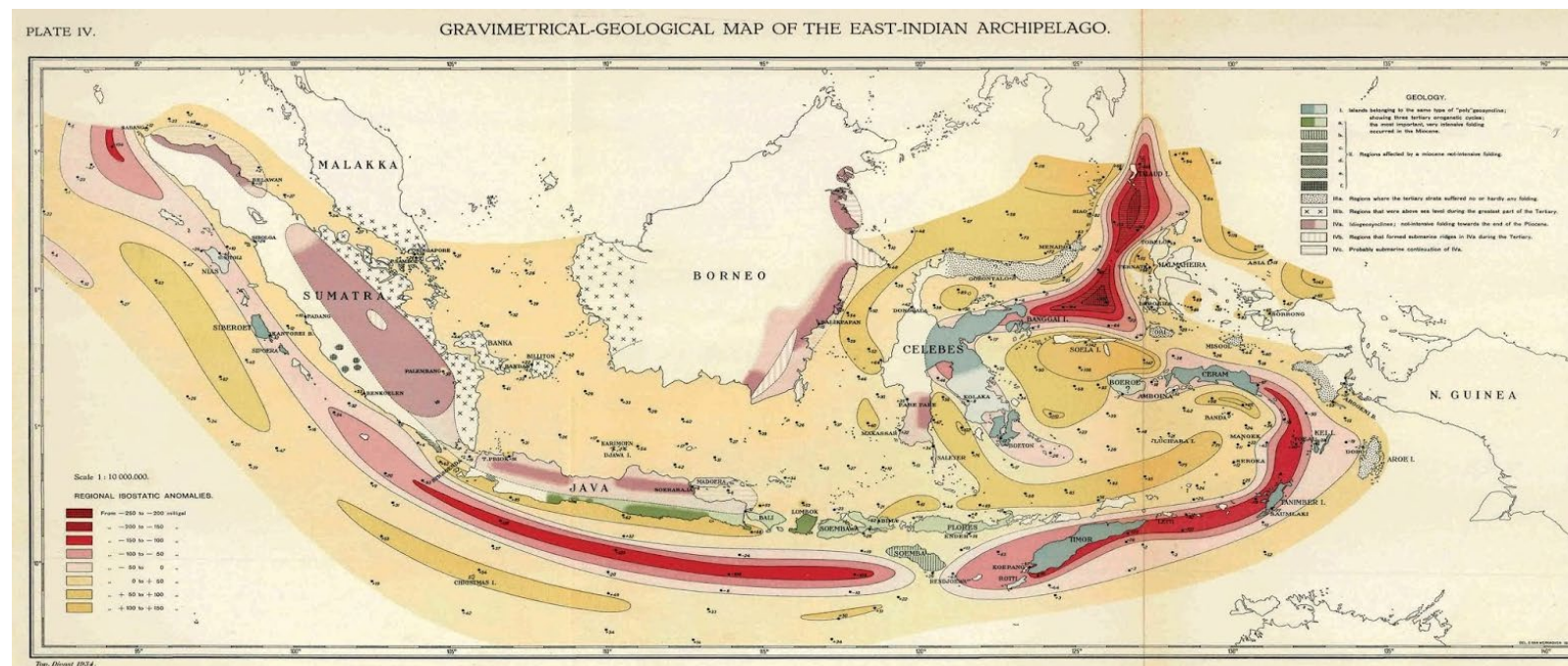
Gravity measurements – historical: Pendulums

1920's:

- First spring gravimeters
- First mobile measurements (Vening-Meinesz, submarines).



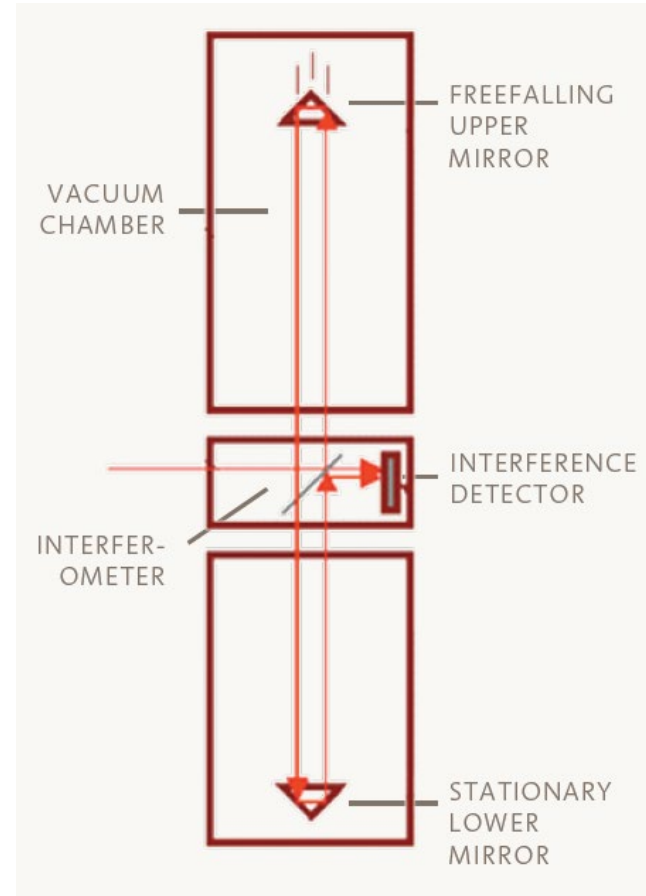
Vening Meinesz with his gravimeter



Classical gravity measurements – absolute and relative

Micro-g Lacoste (A10-019)

- Principle: Free fall
- Accuracy $\sim 70\text{nm/s}^2$
- ~ 600 free fall for a measurement
- Measurement time $\sim 1\text{-}2$ hr

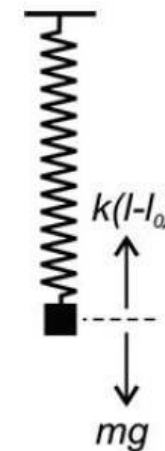


Scintrex CG6

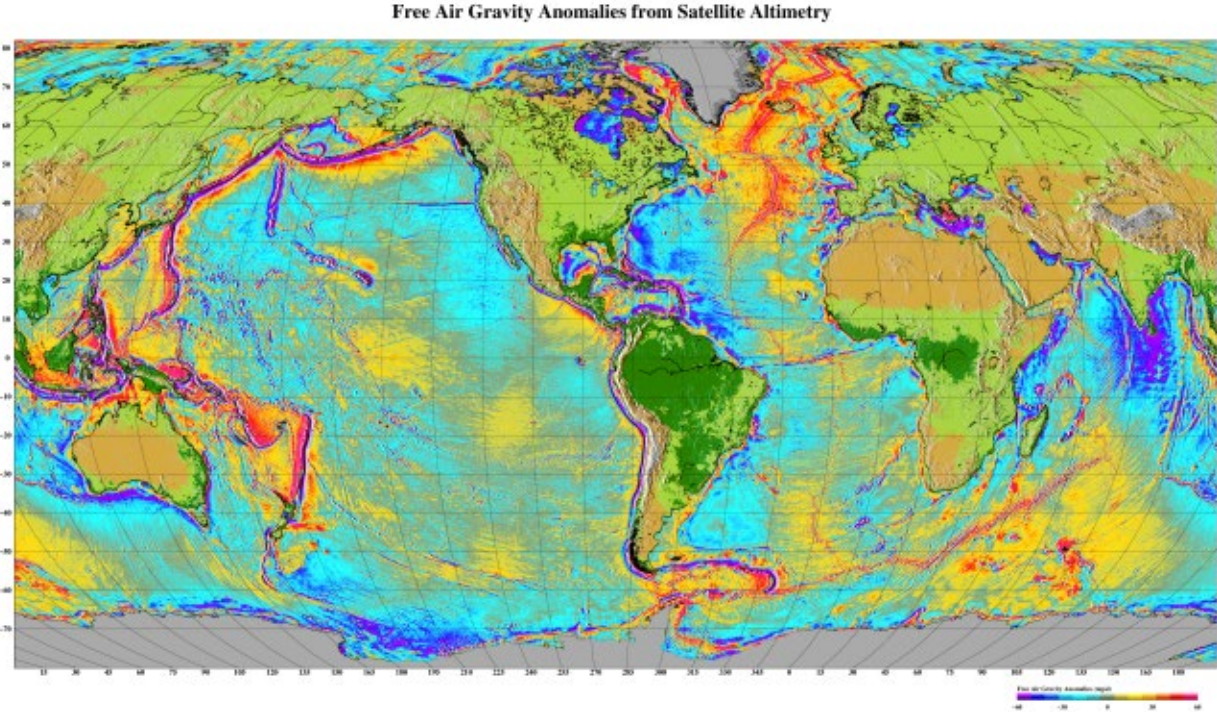
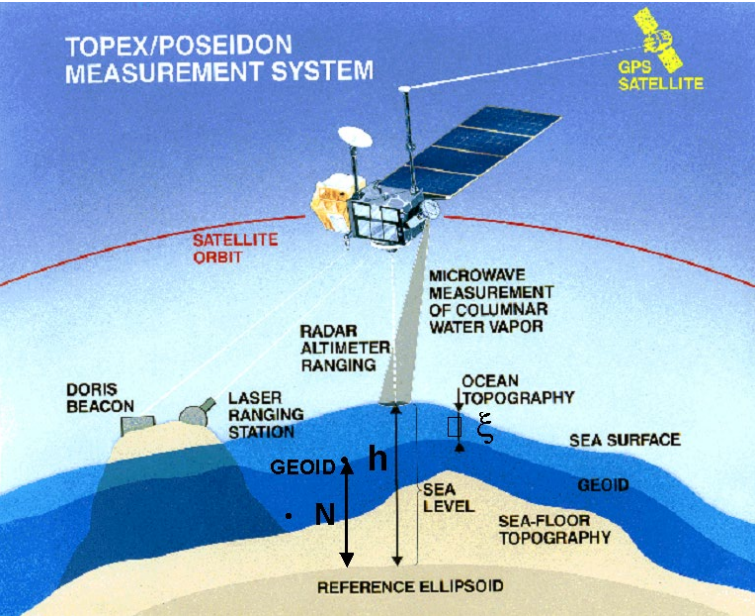
Relative gravity instruments



- Portable (handbag-size)
- Easy to use
- Measures in a few minutes



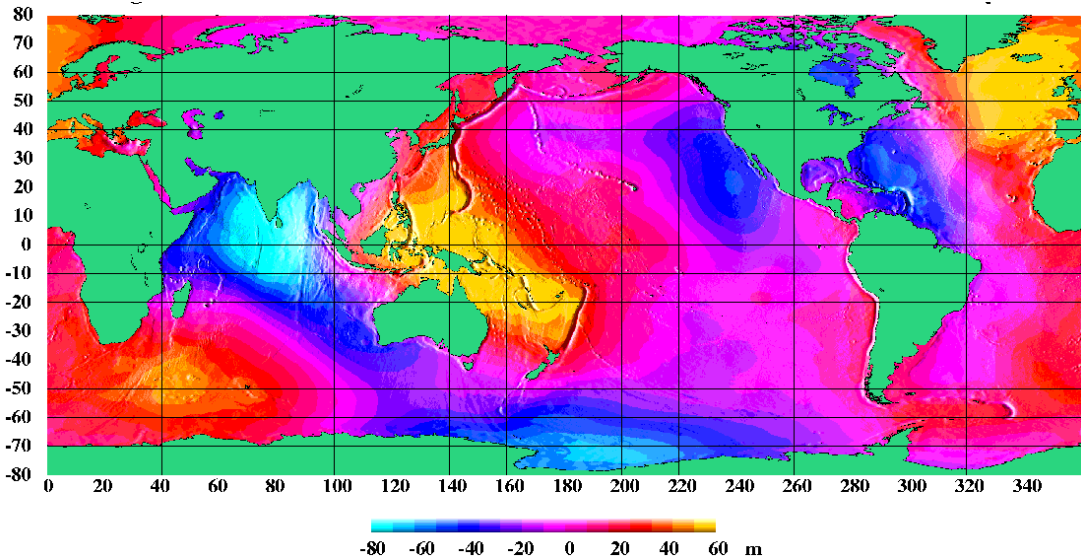
Satellite altimetry – marine high-resolution gravity



Global gravity anomalies (DTU Space)

Mean sea surface from space measured by radar altimetry + GOCE

=> detailed *g*

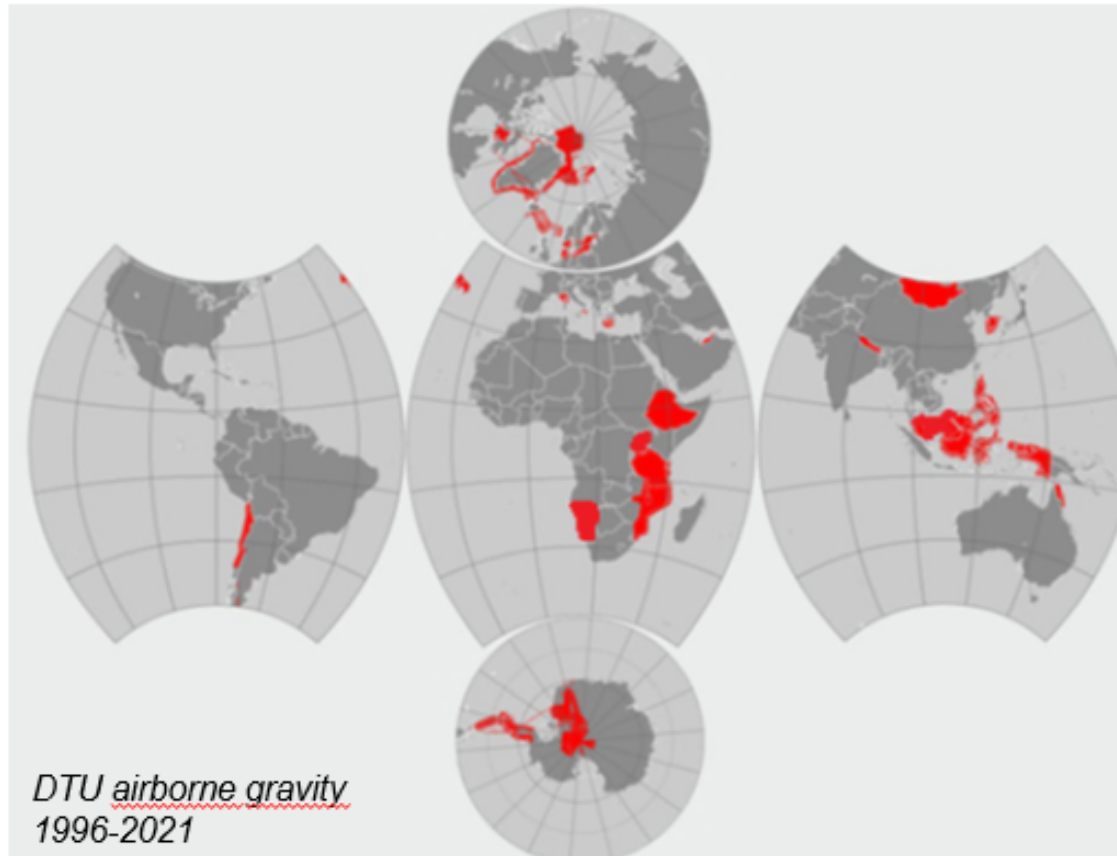
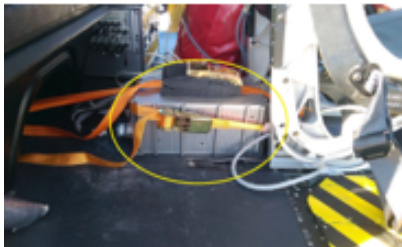


Geodetic airborne gravimetry – filling the medium wave-length voids

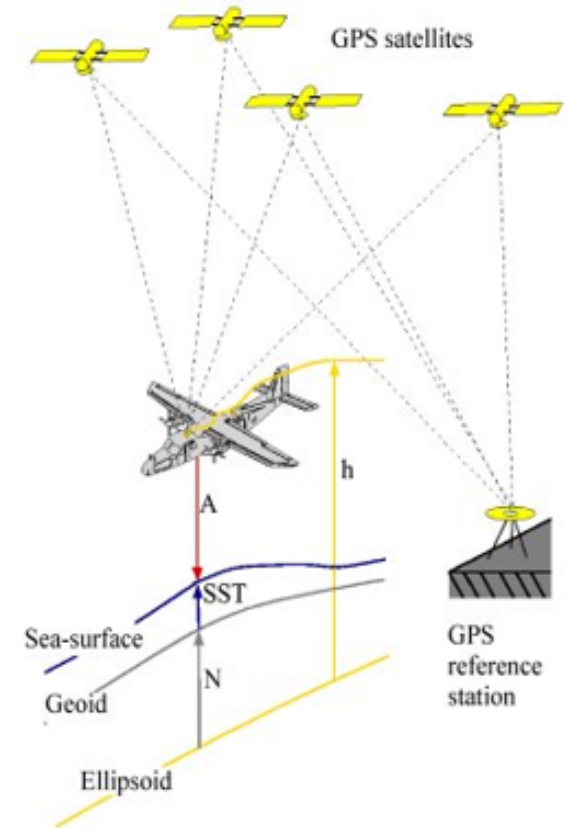
Challenges: Sensors – conventional gravimeters have nonlinearities, sensitive to turbulence

IMU sensors – more linearity, but bias drift ... flying both instrument types give better performance

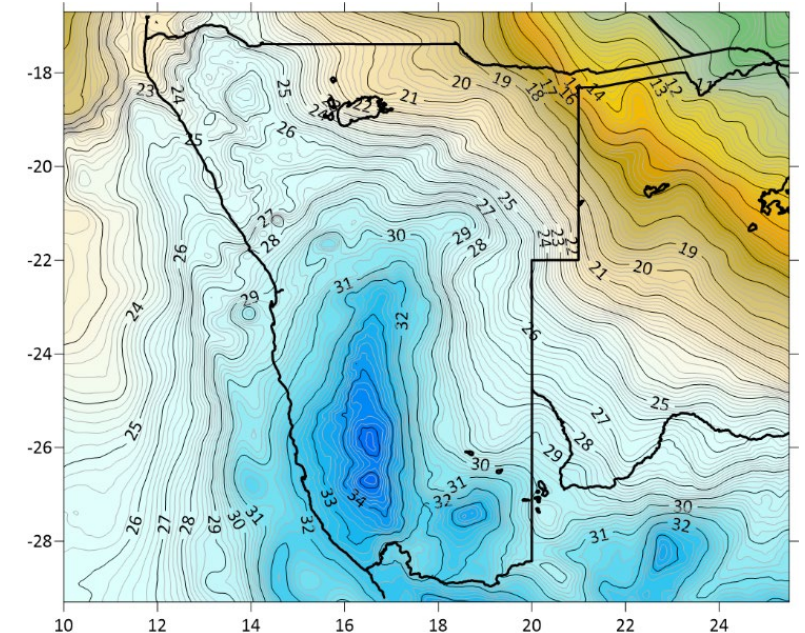
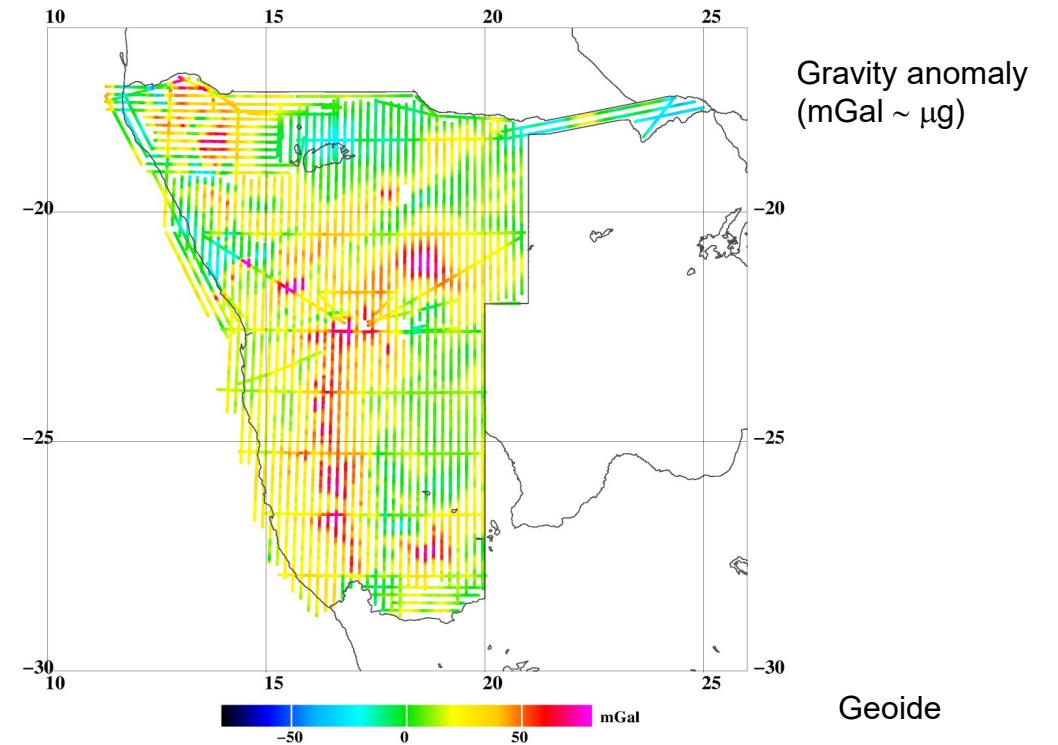
Current trends: Thermally controlled IMUs (iMAR, Sander Geophysics), Drones, Quantum ...



*DTU airborne gravity
1996-2021*

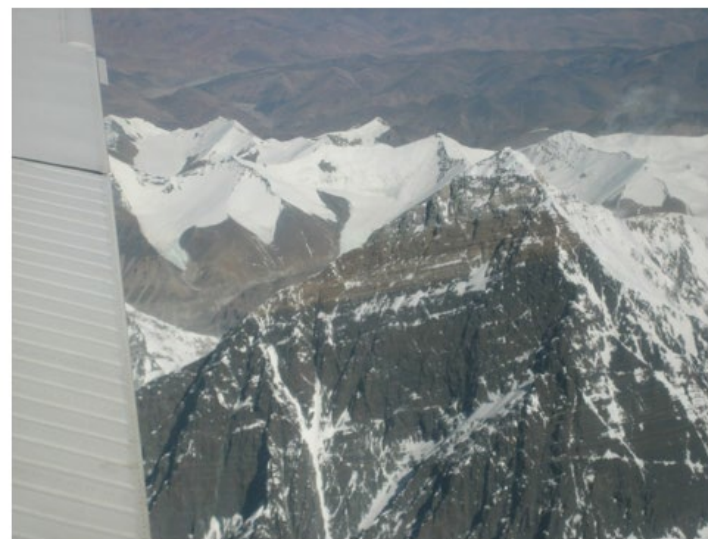


Example: Namibia 2020-21 ... covid-19, weather, aircraft problems, Instruments and bureaucracy challenges at +40°C



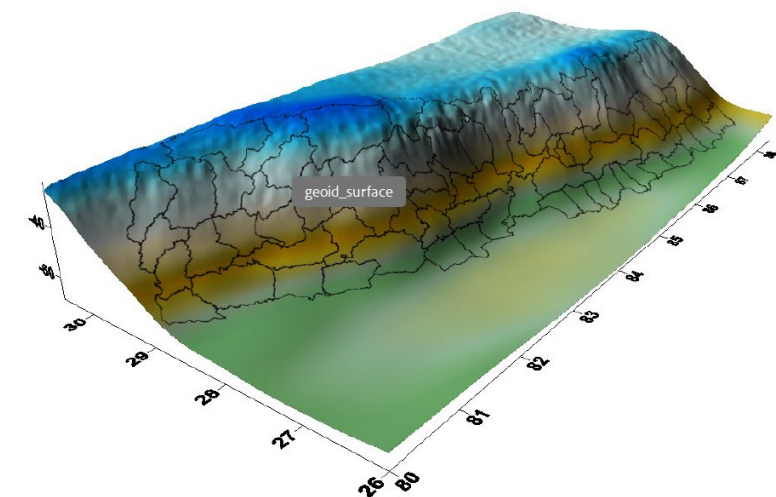
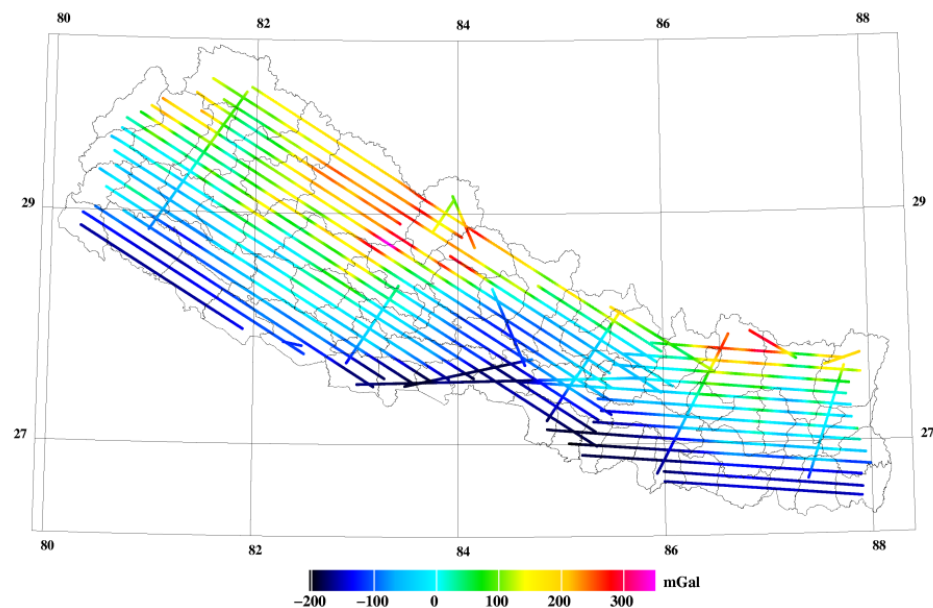
Example: Nepal ... most challenging flights, most rough gravity field in the world (flown 2010 .. Bhutan 2026)

Mt Everest summit from south



Heights of Mt Everest (meter)

Survey of Nepal Official value	8848
China (Y. Chen, 2005)	8847.93
Nat. Geographic Society (Washburn et al, 1999 - EGM96 based)	8850
Ellipsoidal height ITRF (Chen, 2005; Poretti et al, 2004)	8821.40 ±.03
New geoid	-27.54
Geoid-quasigeoid sep.	1.81
EGM08 geoid	-28.49
Height from new geoid	8848.9



Other developments - drones

Use for climate and environment monitoring ..

GNSS, IMU, lidar, *gravimetry*

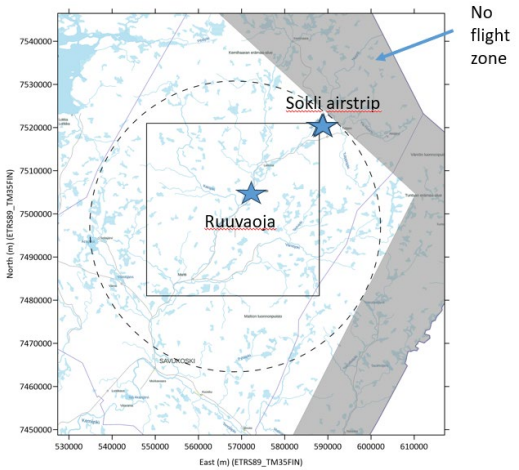
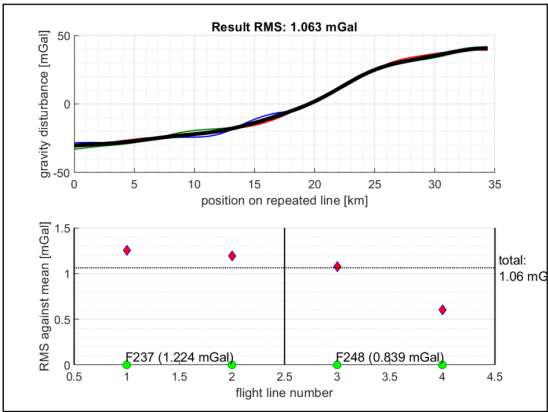


Drone gravimetry?

DroneSOM – EU project, Sokli, Finland, 2023; Sweden and Germany 2024



First drone tests 2020 – Irish Sea / BVLOS (70 km line)

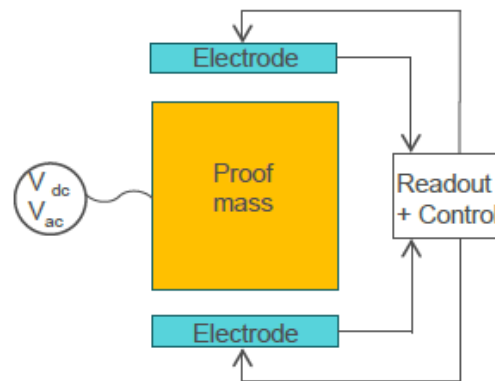
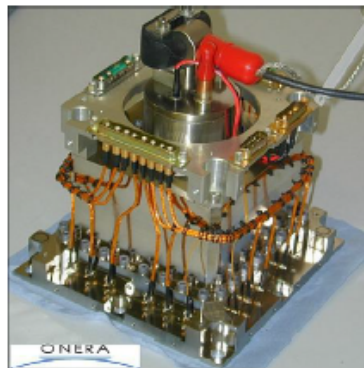


Quantum vs classical sensors

Classical accelerometer

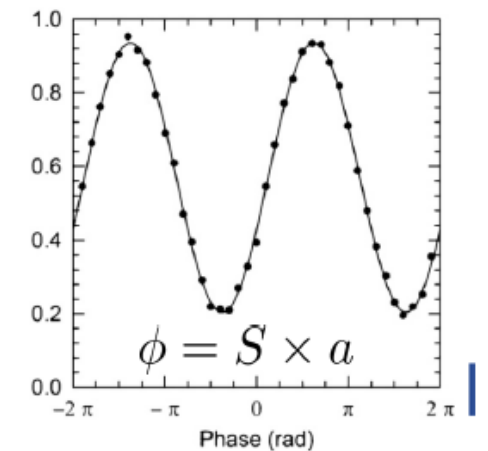
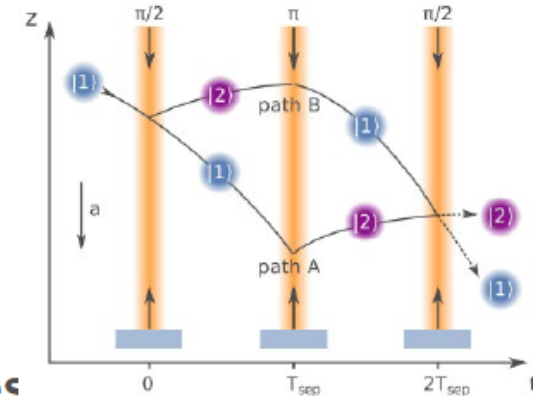
- High maturity
- High resolution and bandwidth
- **Drift and bias**
- **Calibration = loss of continuity and measurement time**

GOCE single accelerometer head

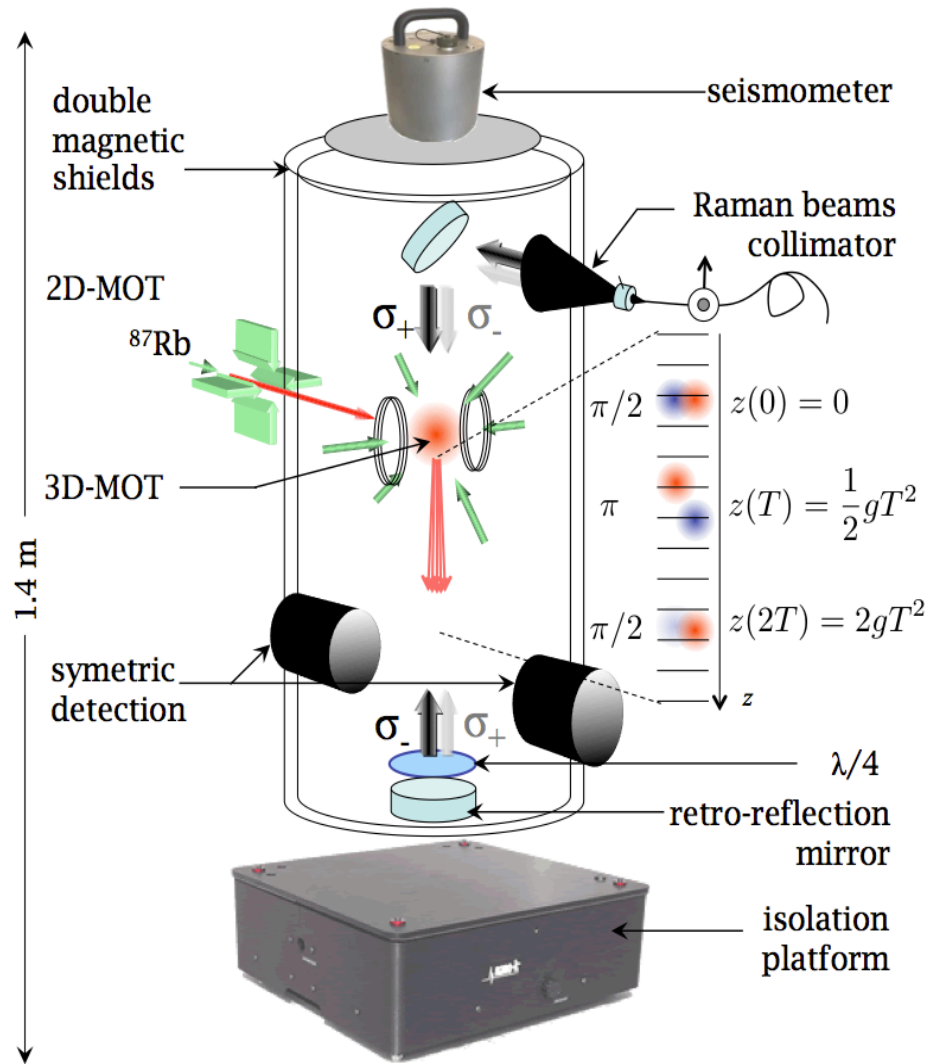


Quantum accelerometer

- High stability (no drift)
- High accuracy (no bias)
- High resolution
- Scalable dynamic range
- **Low bandwidth, dead times**
- **Phase ambiguity**



Atom interferometer – quantum gravimeter (Rb-87)



Builds on mass – photon dualism

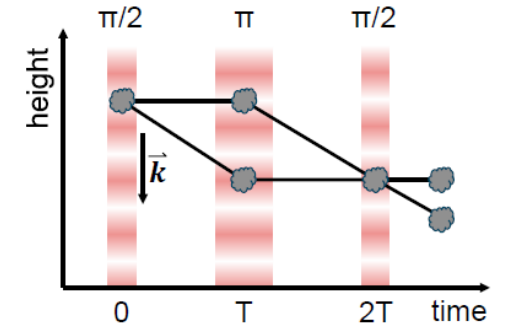
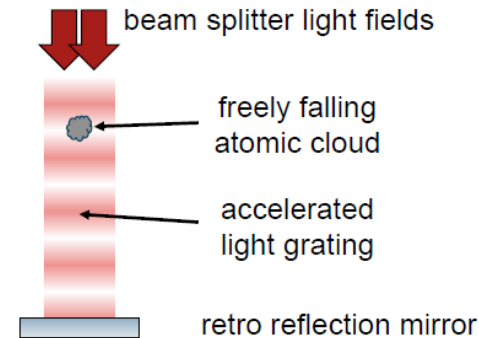
L. de Broglie's (1924)



$$\lambda_{dB} = \frac{h}{p} \quad [1]$$

p relativistic impulse
 λ_{dB} wavelength of the particle
 h Planck-constant

Atoms in free fall, Mach-Zehnder like $\pi/2 - \pi - \pi/2$ pulse geometry:



Interferometer phase, ϕ_i imprinted at pulse i:

Acceleration $\vec{a}$, effective wave vector $\vec{k}$:

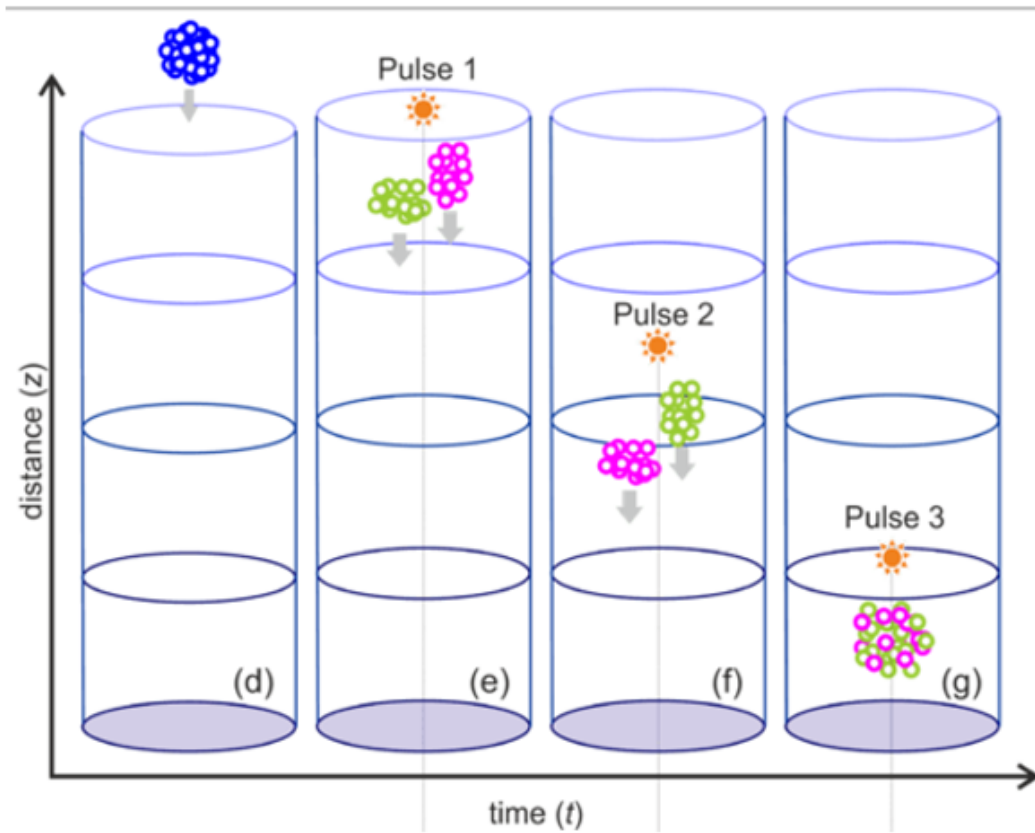
Forward drift velocity $\vec{v}$, rotation $\vec{\Omega}$, enclosed area $\vec{A}$:

$$\phi_{tot} = \phi_1 - 2\phi_2 + \phi_3$$

$$\phi_{acc} = \vec{k} \cdot \vec{a} T^2$$

$$\phi_{rot} = 2(\vec{k} \times \vec{v}) \cdot \vec{\Omega} T^2 \sim \vec{A} \cdot \vec{\Omega}$$

Quantum Gravimetry: Atom Interferometry



A cloud of cold atoms is prepared and dropped

$|i, p\rangle$

First Raman pulse acts as a beam splitter

$|i, p\rangle$

$|e, p + \hbar k_{\text{eff}}\rangle$

Second Raman pulse acts as a mirror

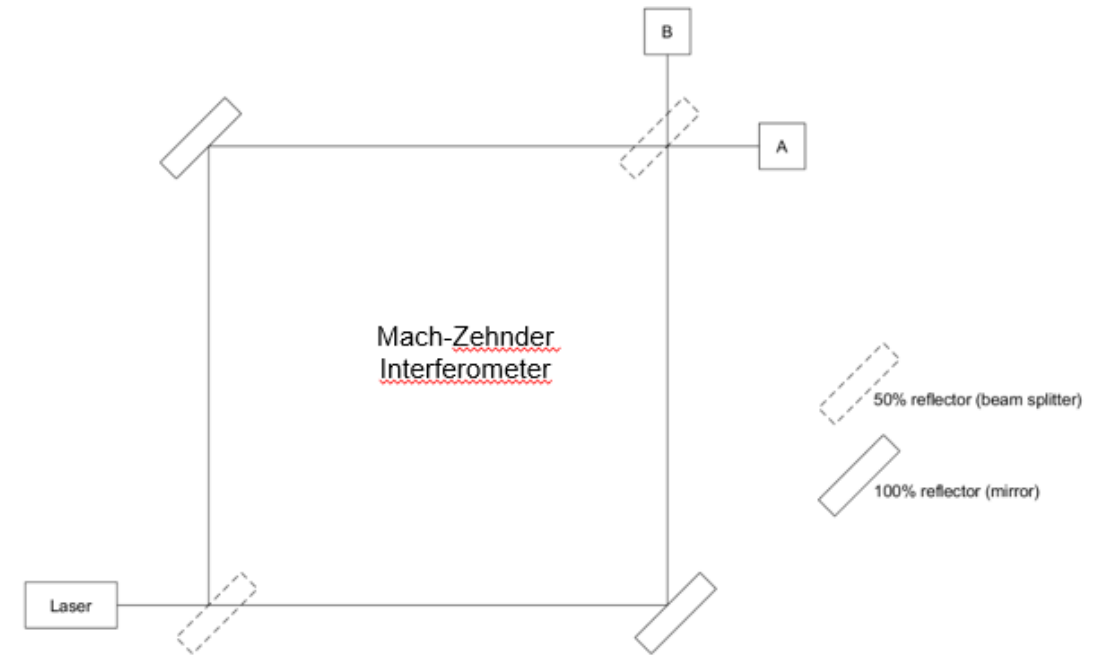
$|i, p\rangle$

$|e, p + \hbar k_{\text{eff}}\rangle$

Third Raman pulse acts as a beam splitter

$|i, p\rangle$

$|i, p\rangle$



The phase of the atom cloud depends on the vertical position, z_i , and the time, t_i

$$\Phi_i = k_{\text{eff}} z_i - \omega_{\text{eff}} t_i$$

The total interferometer phase shift (ratio of atoms in two states)

$$\Delta\Phi = (\Phi_1^A - \Phi_2^A) - (\Phi_2^B - \Phi_3^A)$$

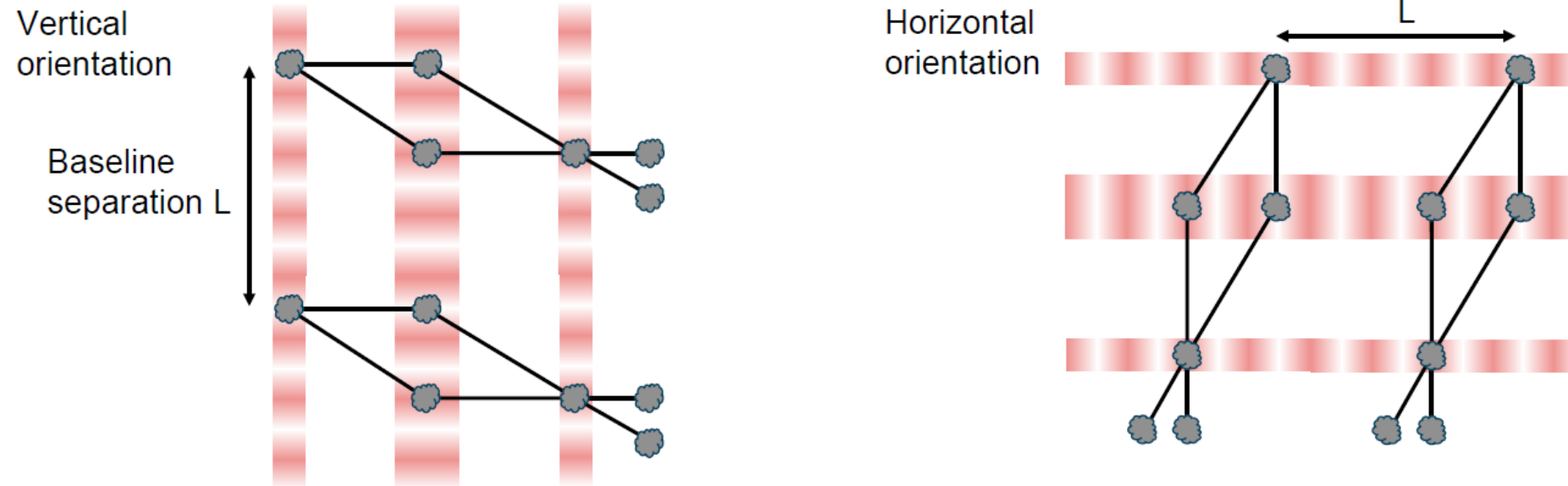
In case of gravity along vertical axis

$$\Delta\Phi = k_{\text{eff}} g T^2$$

Atom interferometer – quantum gradiometer (Rb-87)

Measurement of gravity gradient components .. gravity gradients measure field directly in orbit

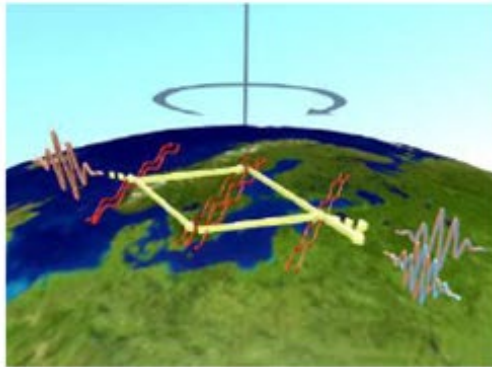
Combination of two Mach-Zehnder like atom interferometers, simultaneous interrogation with the same light field → extraction of mean gravity gradient between the two interferometers from the differential signal



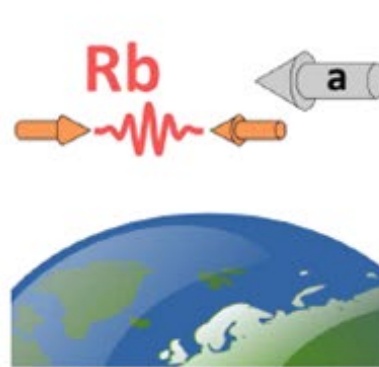
[McGuirk et al., PRA 65, 033608 (2002); Fixler et al., Science 315, 74 (2007); Biedermann et al., PRA 91, 033629 (2015); Chiow et al., PRA 93, PRA 93, 013602 (2016); Rosi et al., Nature 510, 518 (2014); Asenbaum et al. PRL 118, 183602 (2017)]

M Weigelt, DLR

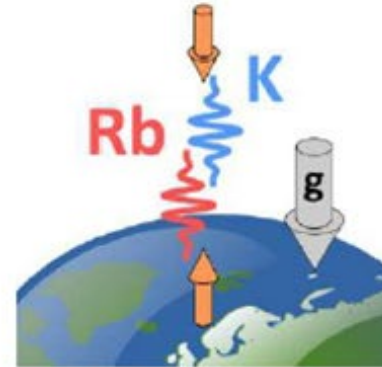
Quantum developments – relevant to gravity field sensing / PNT / fundamental physics



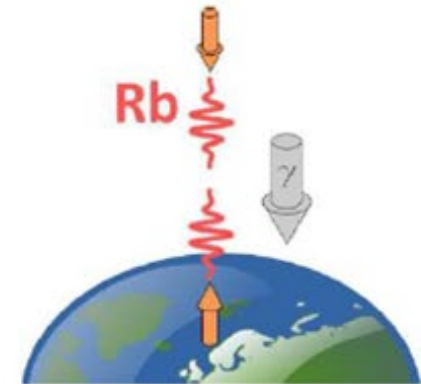
Rotation¹⁾



Acceleration



Universality of free fall

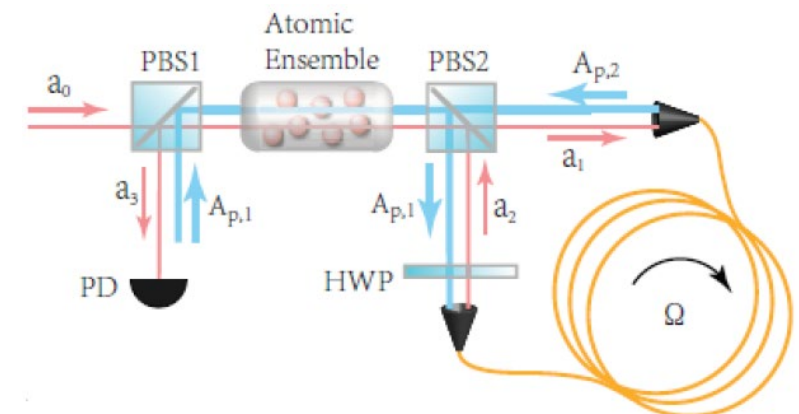


Gradients

S. Abend, IQO

A large variety of applications:

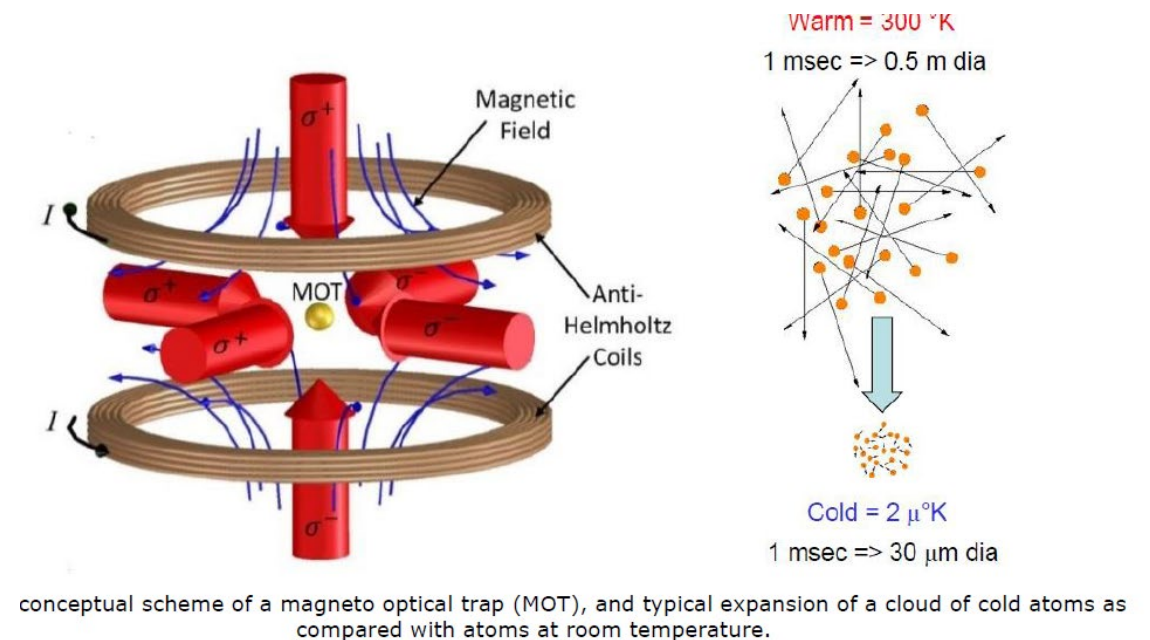
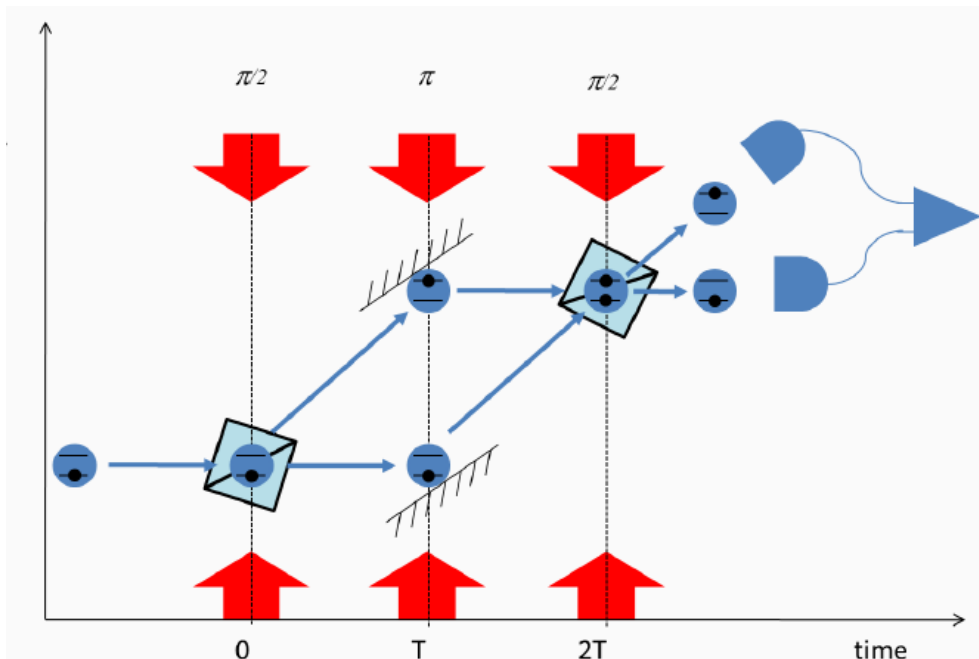
- Highly accurate gravimeters, gyroscopes, tiltmeters
- Practical applications: navigation, geodesy, civil engineering
- Measurement of fundamental constants: h/m , G
- Test of fundamental physics: dark energy and matter
- Proposals for gravity wave detection



Fibre optic quantum gyro interferometer

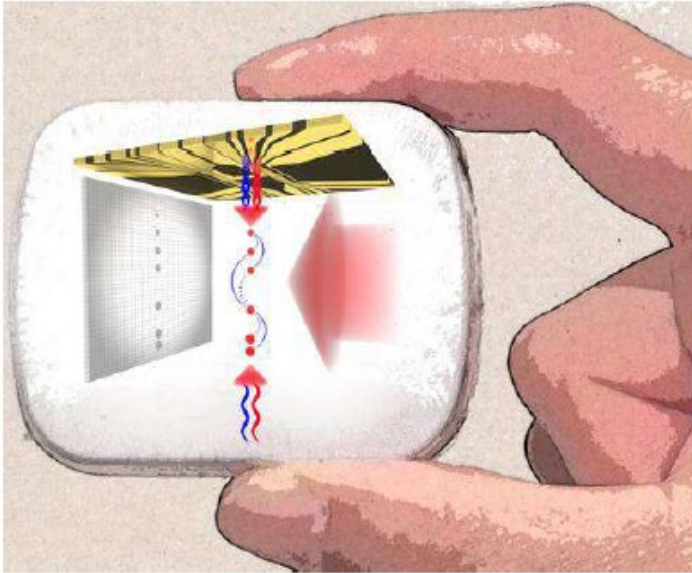
Atom interferometer – matter-wave gravimeter principles – ONERA "girafe"

- Typical 10^6 Be-87 atoms per cycle .. Magneto-optical trap few mm dimension, raman laser "push" $\pm \pi/2$
- Ultra-high vacuum with laser cooling, down to μK – "Molasses" (colder: Bose Einstein condensate)
- ONERA "Girafe" typical repeat period $\sim 10\text{-}20$ ms (interrogation) .. much longer interrogation possible in space
- Interference phase pattern output relates to $g \Rightarrow$ direct measurement of gravity
- Phase ambiguities and interrogation breaks $\Rightarrow$ use of a standard IMU for hybrid processing



Next generation atom-chip devices

Measuring Gravity with an Atom Chip



[November 11, 2016, Physics 9, 131]

Towards miniaturized Quantum Sensors:

- Novel robust production methods
- Advanced integrated functionality
- Reduced overall complexity

International developments

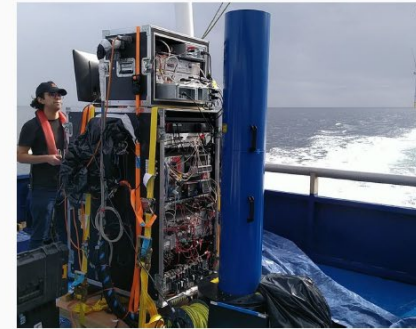
France: ONERA, Exail ..

UK, Australia, Singapore, China, US/DARPA ..

Atom-interferometry on moving platforms

Two maritime trials in 2023 (North Sea) and 2024 (Arctic Circle with Royal Navy)

Sensor operated in various sea states, and maintained original performance on return



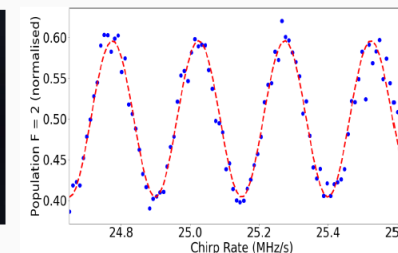
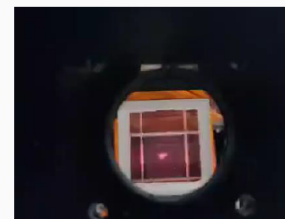
UNIVERSITY OF
BIRMINGHAM



Miniaturised demonstrators

Compact atomic gravity gradiometers for operation in motion

Approaches for suppressing inertial effects informed by maritime trials

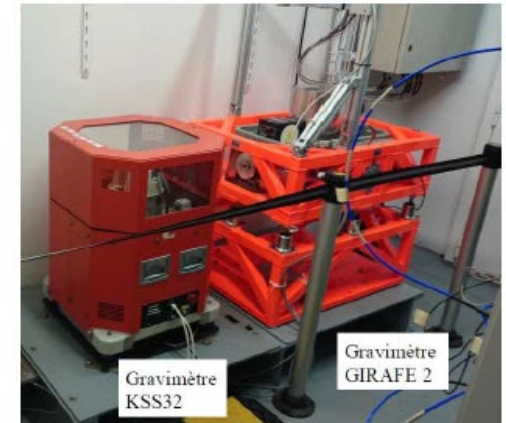
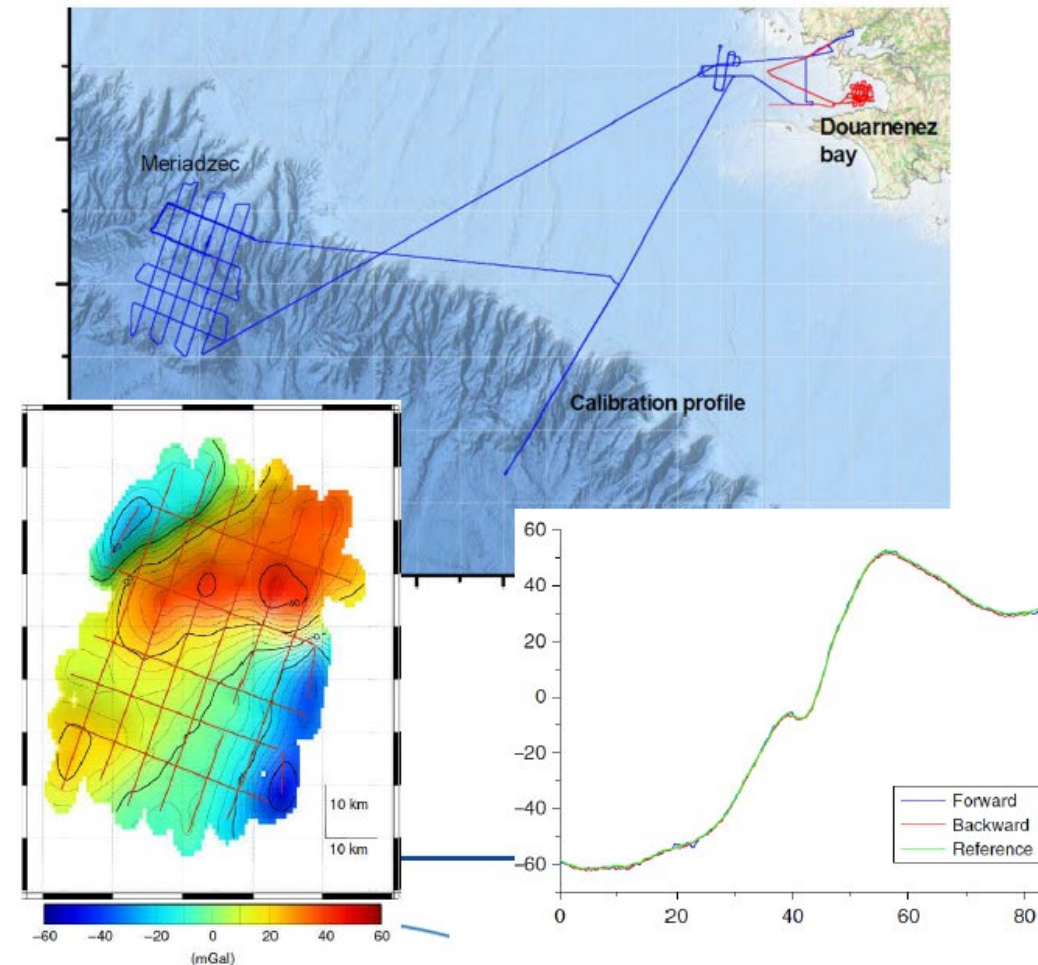


UoB high-bandwidth gravimeter, up to 100 Hz, fringes at 30 Hz



First practical applications of Atom Interferometry gravimetry – ONERA / GIRAFE

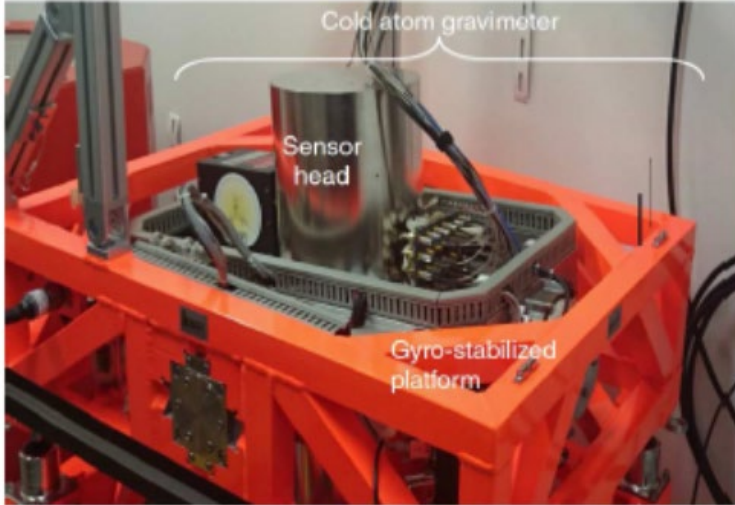
6 campaigns (1 week → 10 month) from 2015 to 2024



Precision
0.1 – 0.5 mGal

Y. Bidel et al., Absolute marine gravimetry with matter-wave interferometry, Nature Communications 9, 627 (2018).

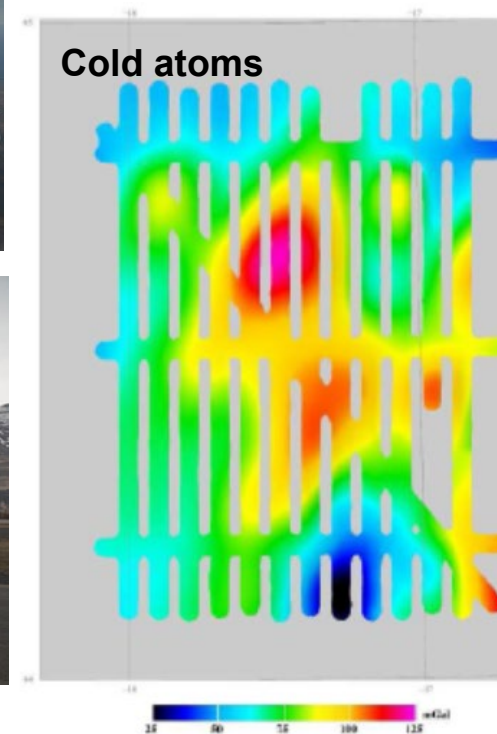
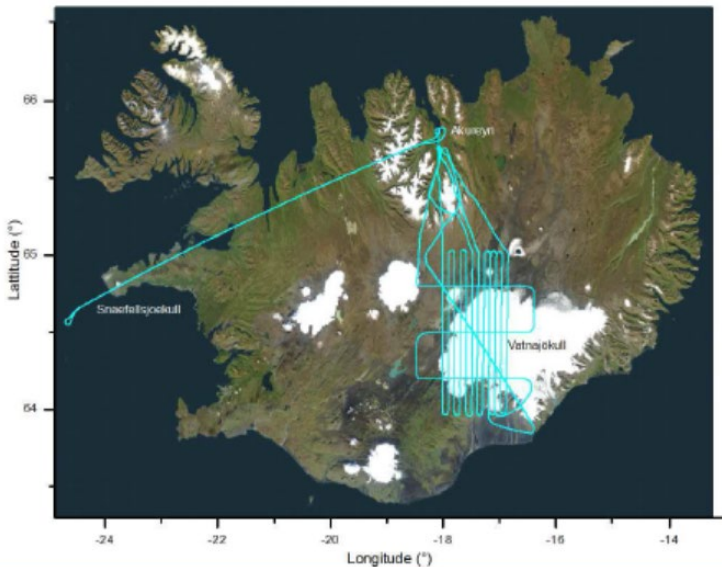
Airborne Quantum Gravimetry campaigns – DTU Space / ONERA / ESA



Flight campaign ONERA "GIRAFFE": Iceland 2017 /
Toulouse: Bay of Biscay, Pyrenees 2019
Validation: state-of-the-art strapdown IMU gravimeter /
upward continued ground gravity

2017: ~ 2 mGal rms
accuracy

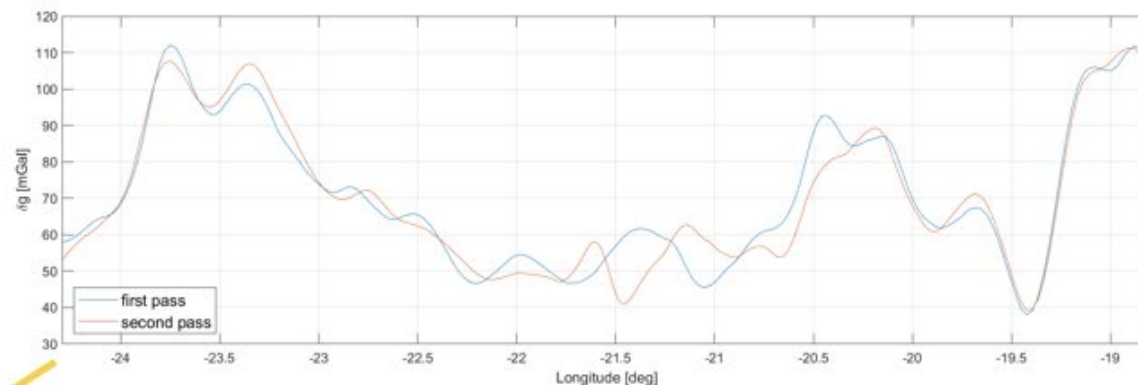
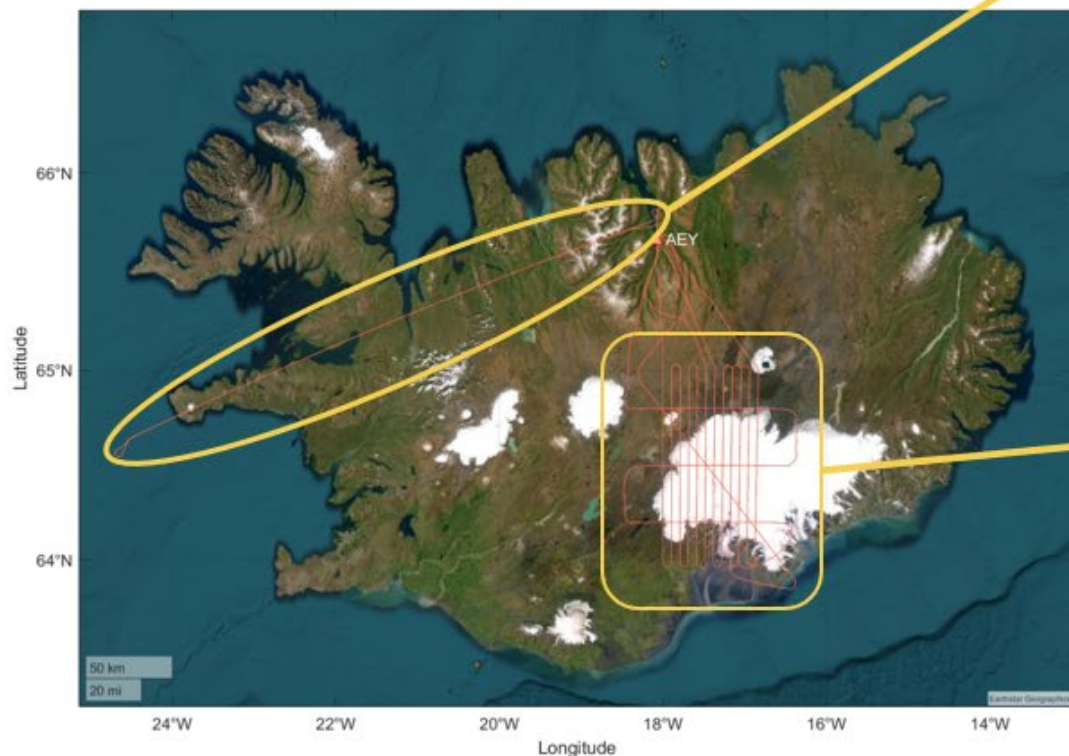
2023: < 1 mGal rms
accuracy, GPS limited





Absolute airborne gravimetry with a cold atom sensor

Yannick Bidet¹ · Nassim Zahzam¹ · Alexandre Bresson¹ · Cédric Blanchard¹ · Malo Cadoret² · Arne V. Olesen³ · René Forsberg³

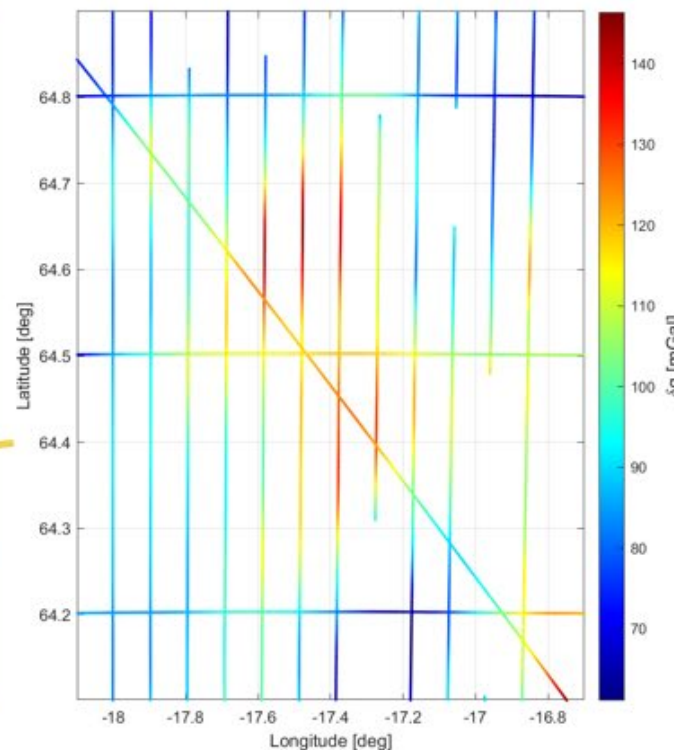


Repeat line statistics

Mean	Std	
0.6	5.5	mGal

Vatnajökull cross-over statistics

Crossings	47	
Mean	1.2	mGal
Std. dev.	3.7	mGal
Min/max	-6.4/8.3	mGal
RMS	3.9	mGal
RMSE	2.7	mGal

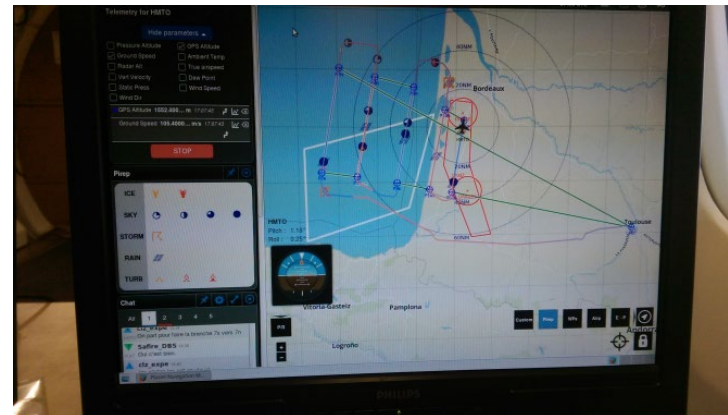




Toulouse 2019 (ONERA/DTU/BGI)

Two classic
g-sensors flown

4 flights
– 2 months!!





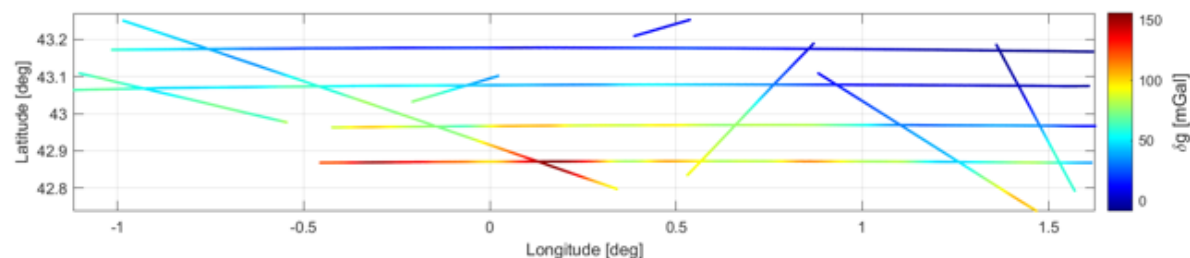
The 2019 Campaign in France

Evaluation

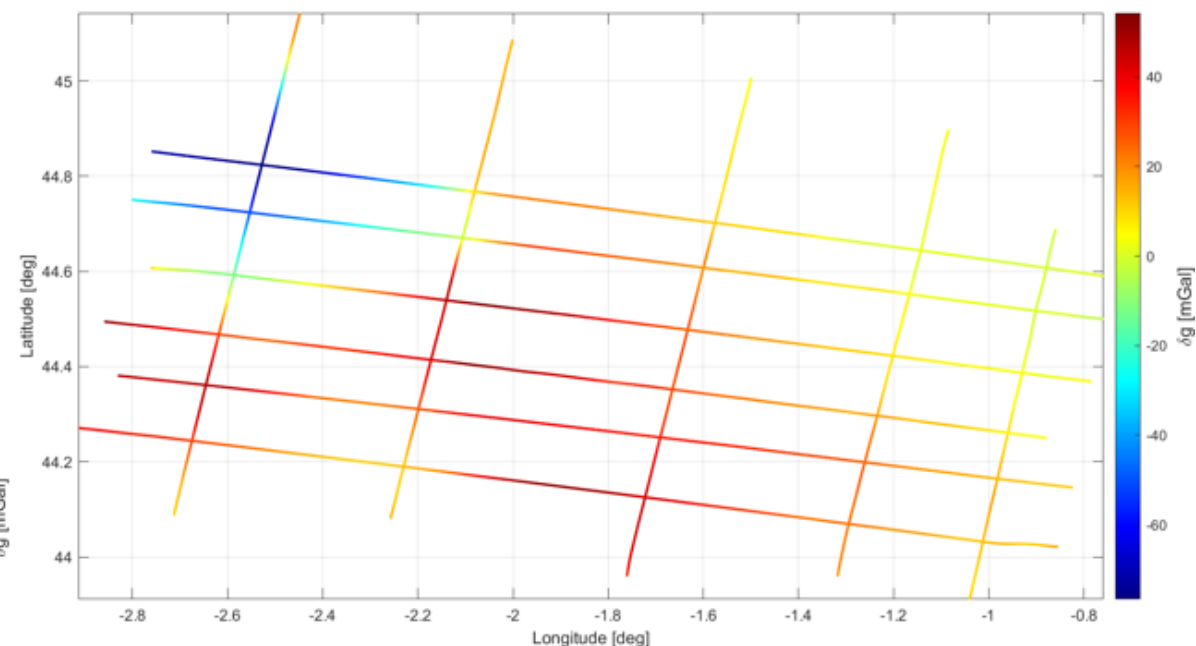
Cross-over statistics

	GIRAFE	iMAR	
Crossings	17	18	
Mean	-1.0	-0.8	mGal
Std. dev.	1.4	1.8	mGal
Min/max	-3.4/1.0	-4.8/2.4	mGal
RMS	1.7	1.9	mGal
RMSE	1.2	1.4	mGal

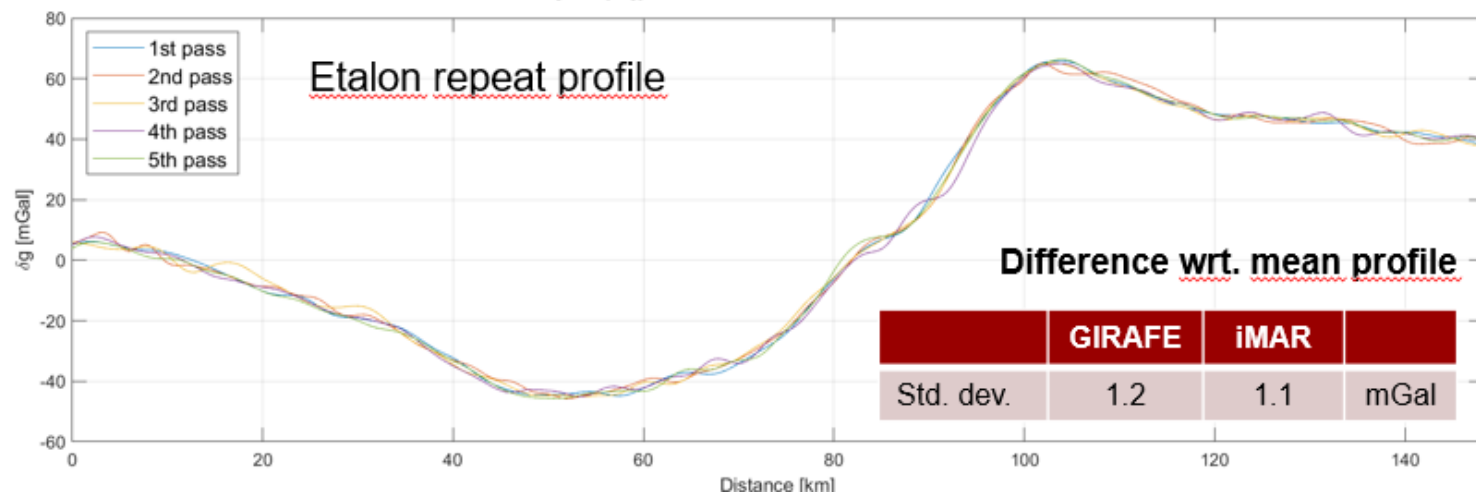
Pyrenees



Bay of Biscay



Etalon repeat profile



Difference wrt. mean profile

	GIRAFE	iMAR	
Std. dev.	1.2	1.1	mGal

Cross-over statistics

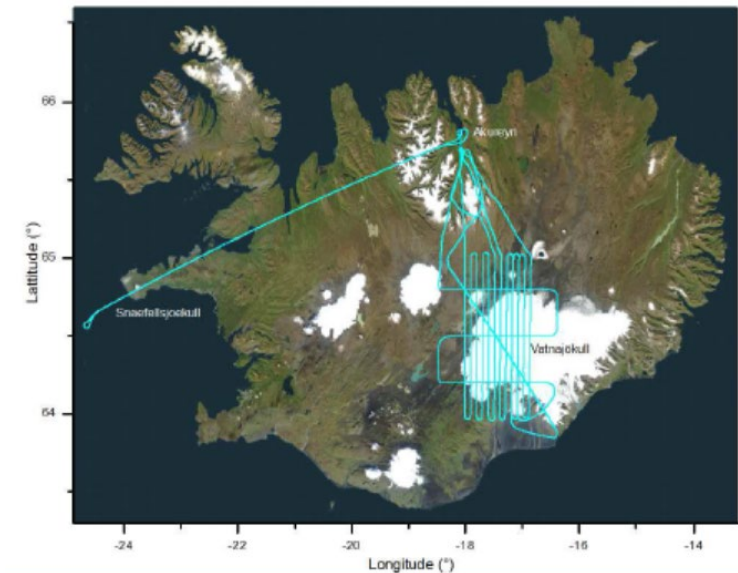
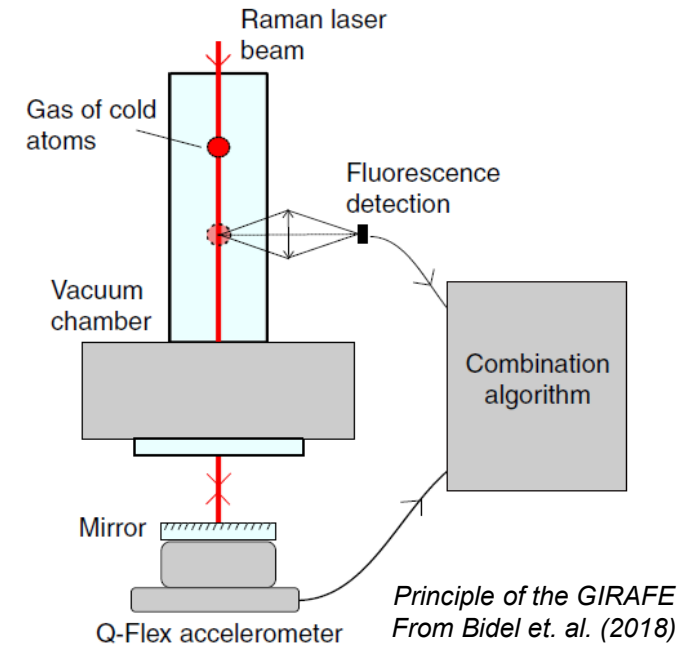
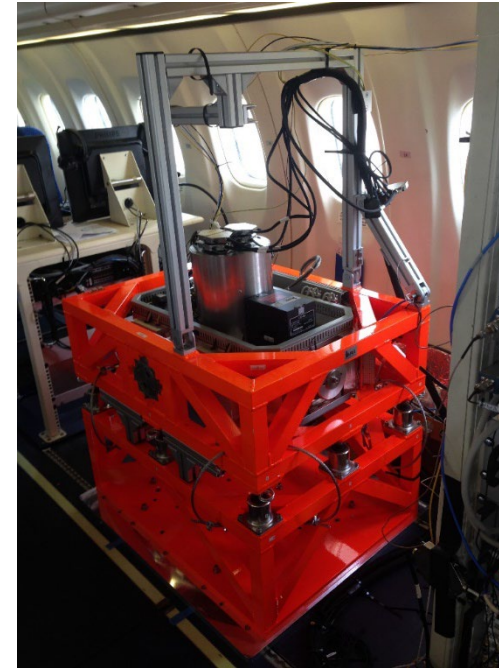
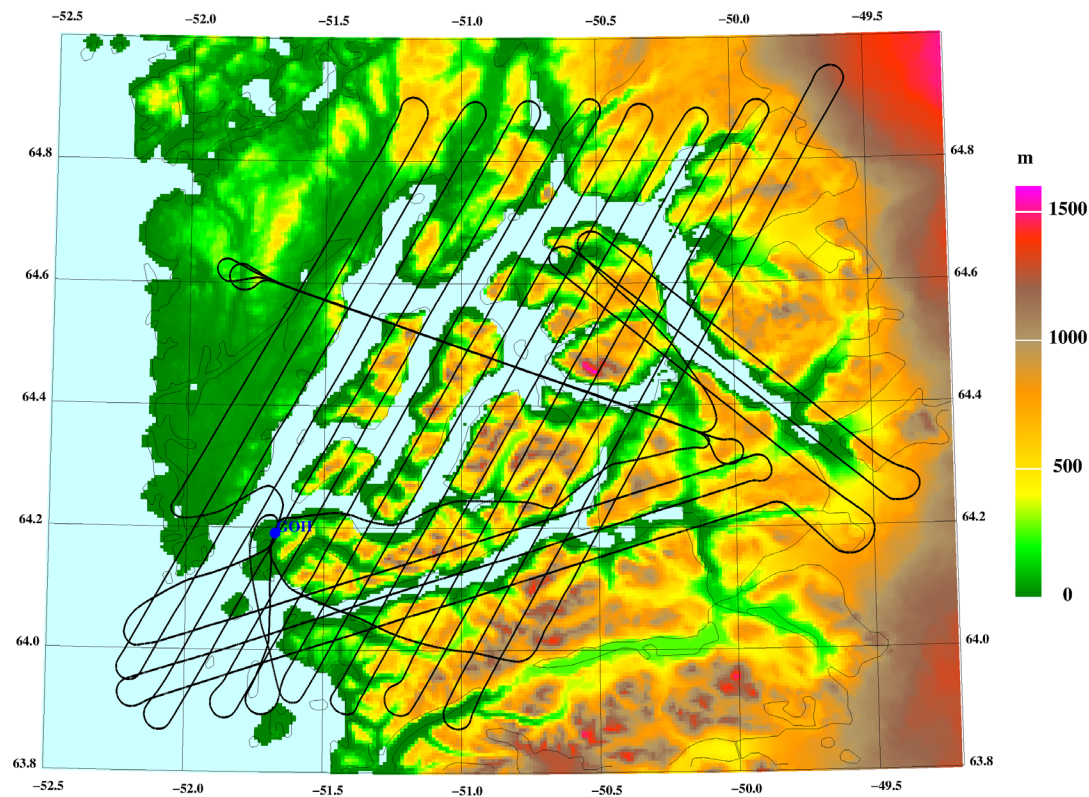
	GIRAFE	iMAR	
Crossings	30	26	
Mean	0.4	-0.2	mGal
Std. dev.	1.8	1.9	mGal
Min/max	-2.8/4.0	-4.9/2.9	mGal
RMS	1.8	1.9	mGal
RMSE	1.3	1.4	mGal



Greenland/Iceland quantum gravimetry survey June 2023

Purpose:

- Enhanced GIRAPE cold atom gravimeter (ONERA)
- 1 cm geoid of Nuuk region (SDFI)?
- Iceland gravity changes? (ESA NGGM)
- Navigation errors if GNSS jamming





Installation

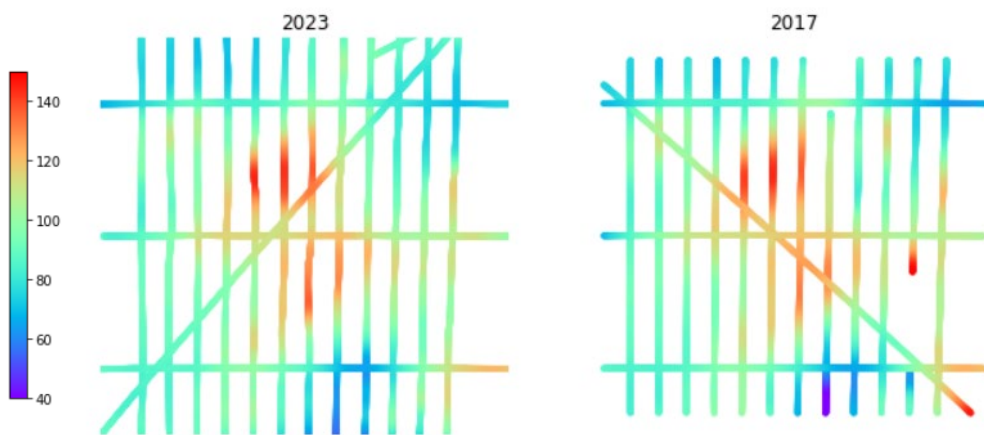
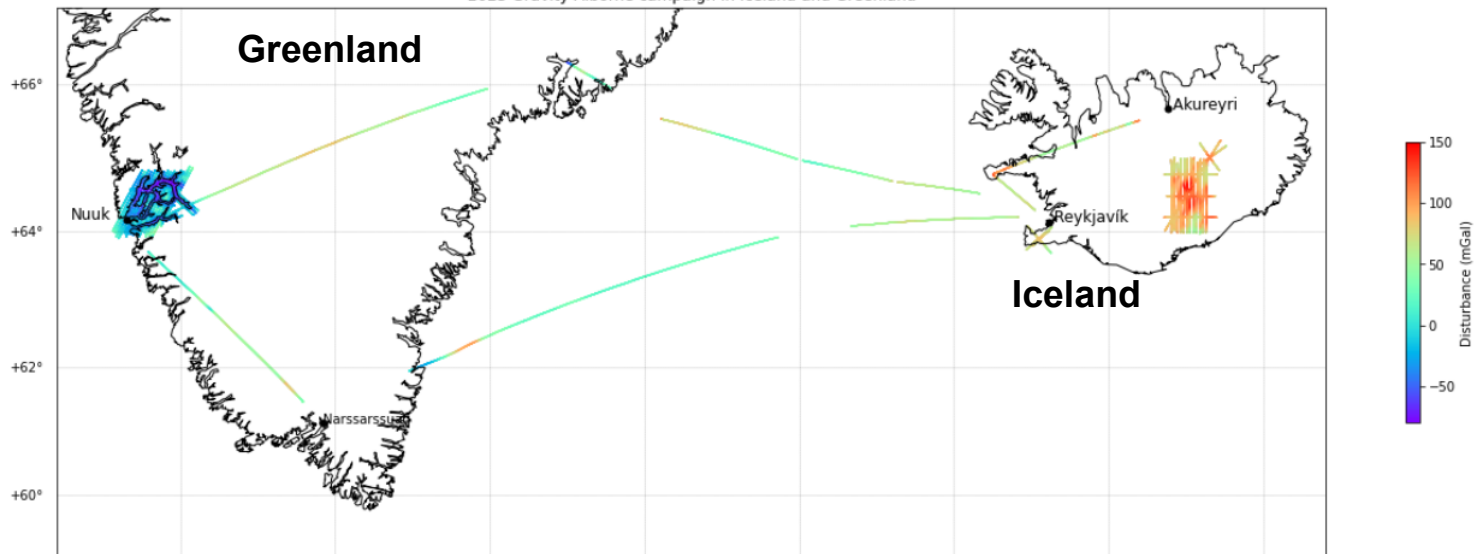


Hardware	Sub-weight	weight
GIRAFE		240.5 kg
Base plate	21.5 kg	
Platform	168.5 kg	
Cables	13.5 kg	
Sensor	37.0 kg	
iMAR		10.0 kg
Base plate	2.5 kg	
Gravimeter	7.5 kg	
Rack 1		97.5 kg
Rack 2		113.0 kg
Rack 3		94.0 kg
Power setup		34.0 kg
12 AH box	11.0 kg	
20 AH box	13.0 kg	
Battery charger	2.0 kg	
Inverter	8.0 kg	
Navigation setup		6.0 kg
PC	4.0 kg	
GNSS receivers	2.0 kg	
Total weight		595.0 kg



Grønland/Island 2023 airborne quantum campaign

2023 Gravity Airborne campaign in Iceland and Greenland



Gravity anomalies Vatnajökull ice cap

ICELAND:

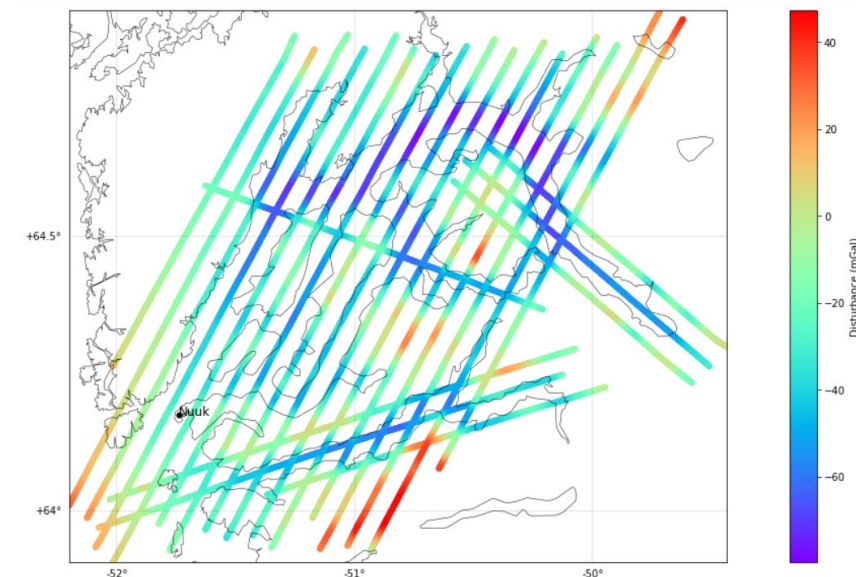
2017 : estimated error = 2.8 mGal

2023 : estimated error = 1.1 mGal

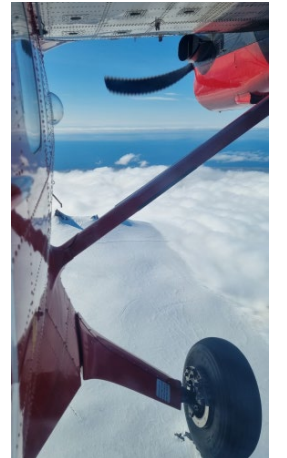
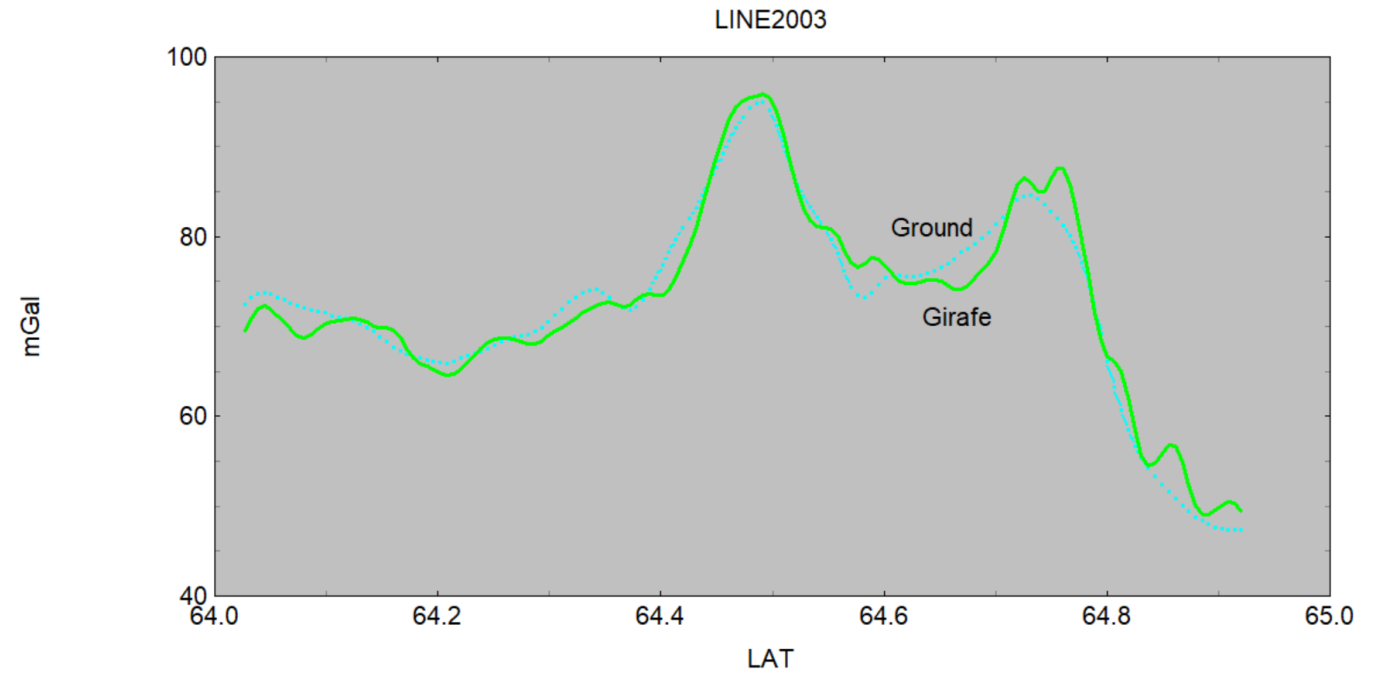
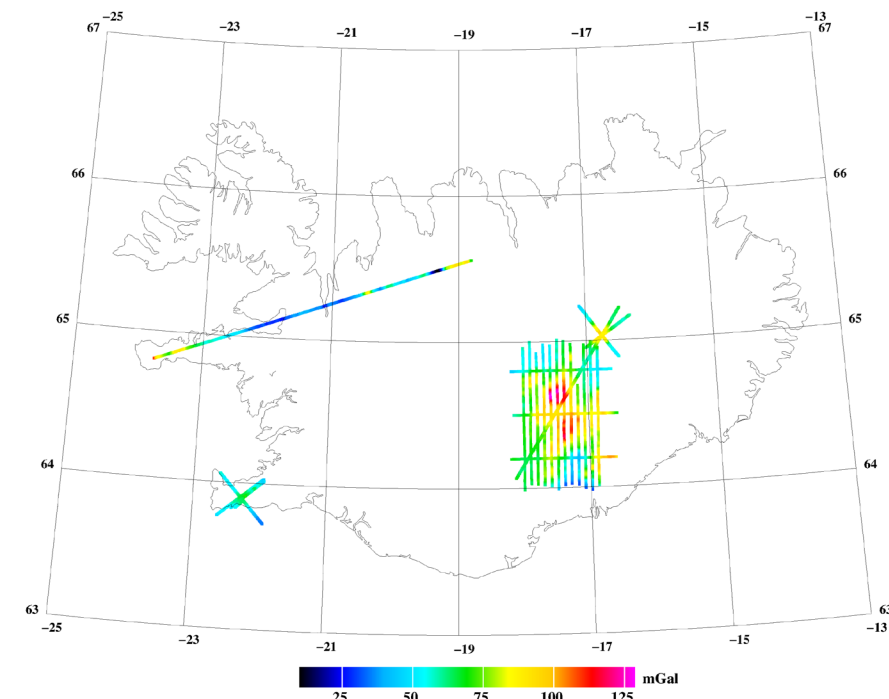
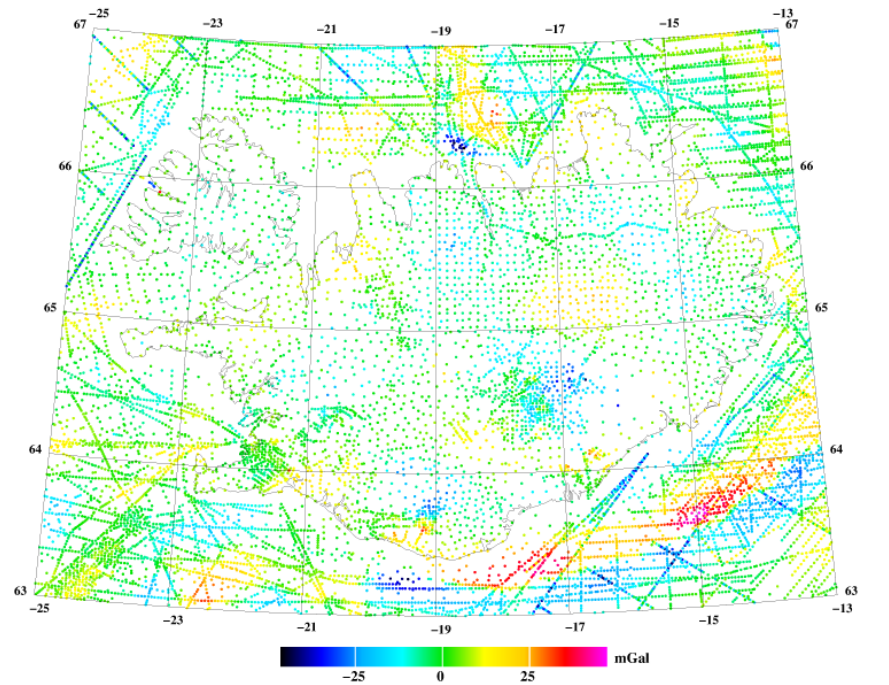
NUUK:

estimated error = 2.1 mGal

Fjord effects ...

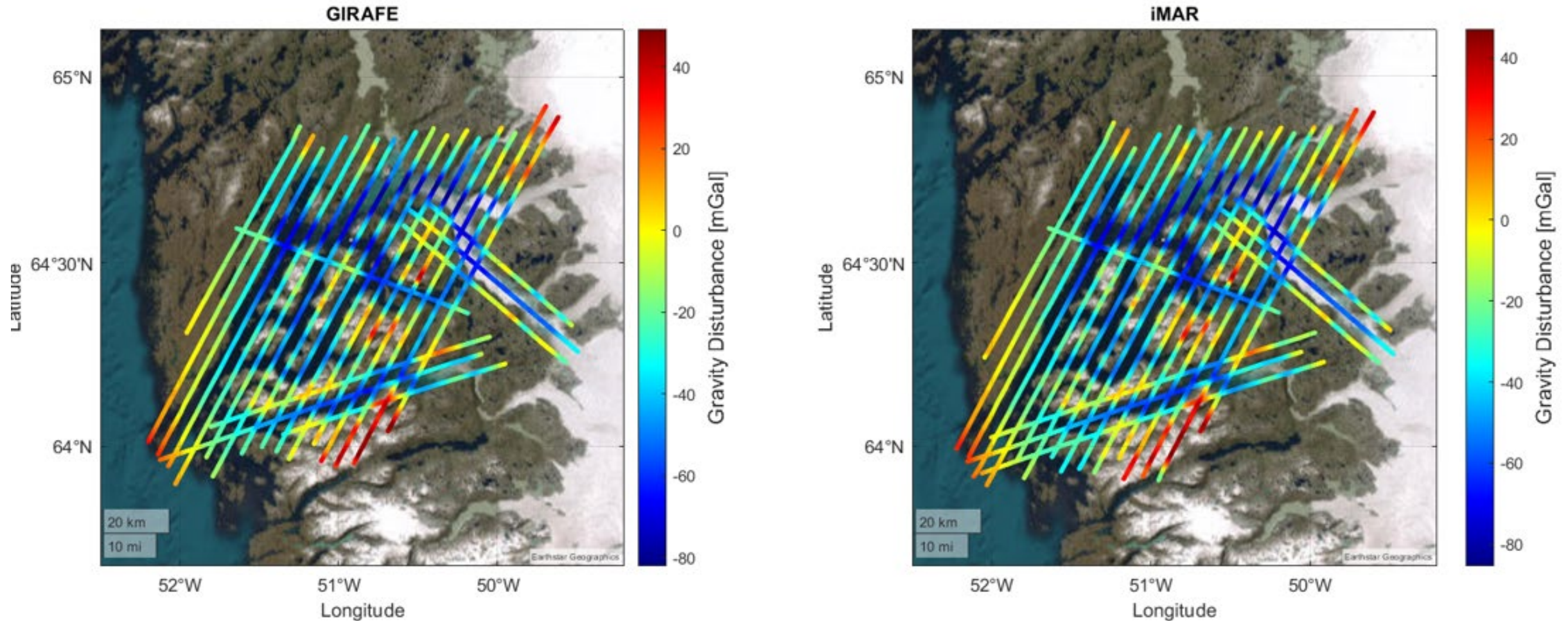


Iceland 2023: comparisom with gravity data on ice and land

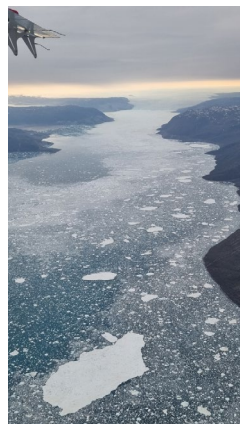
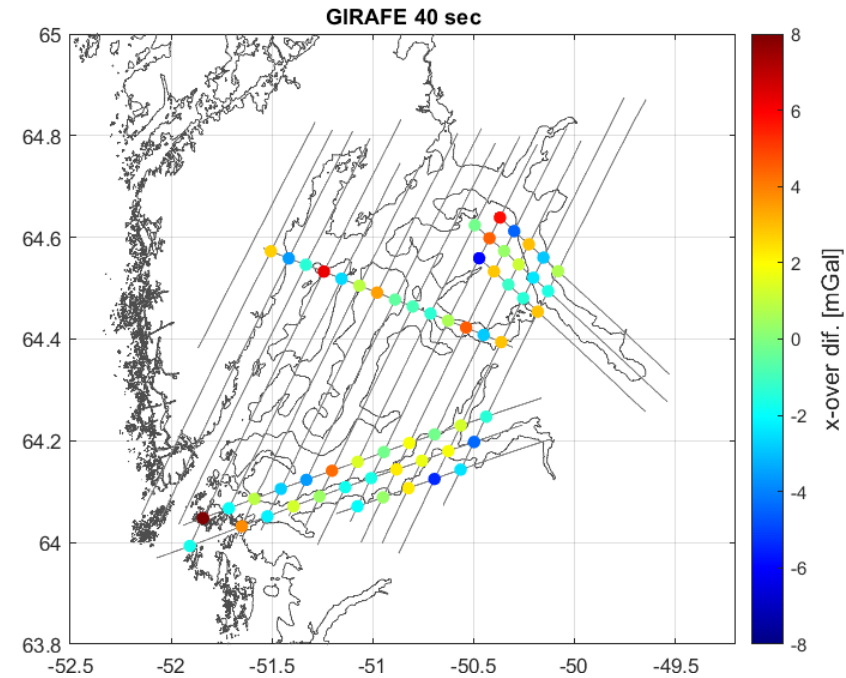
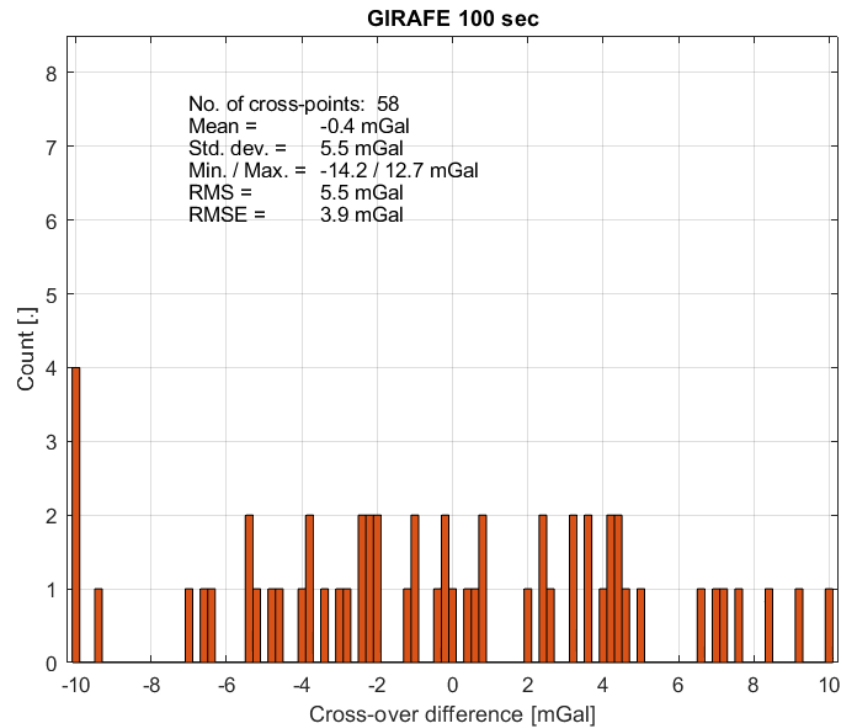
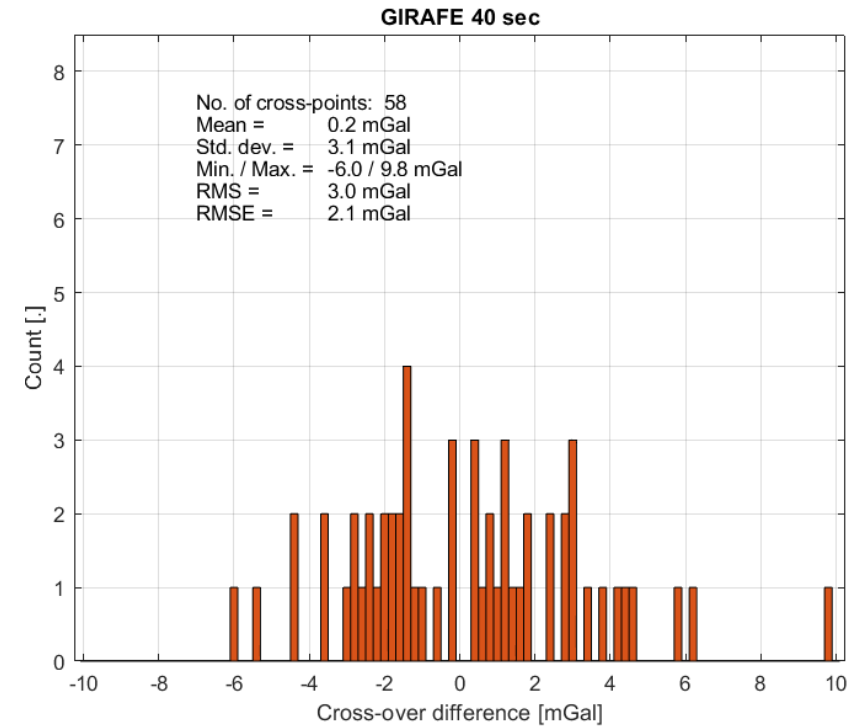


Greenland: comparison of quantum and iMAR IMU airborne

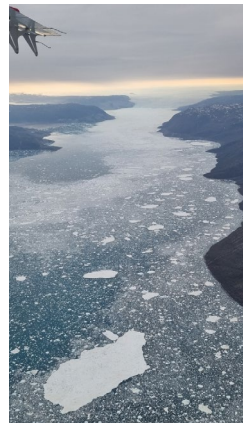
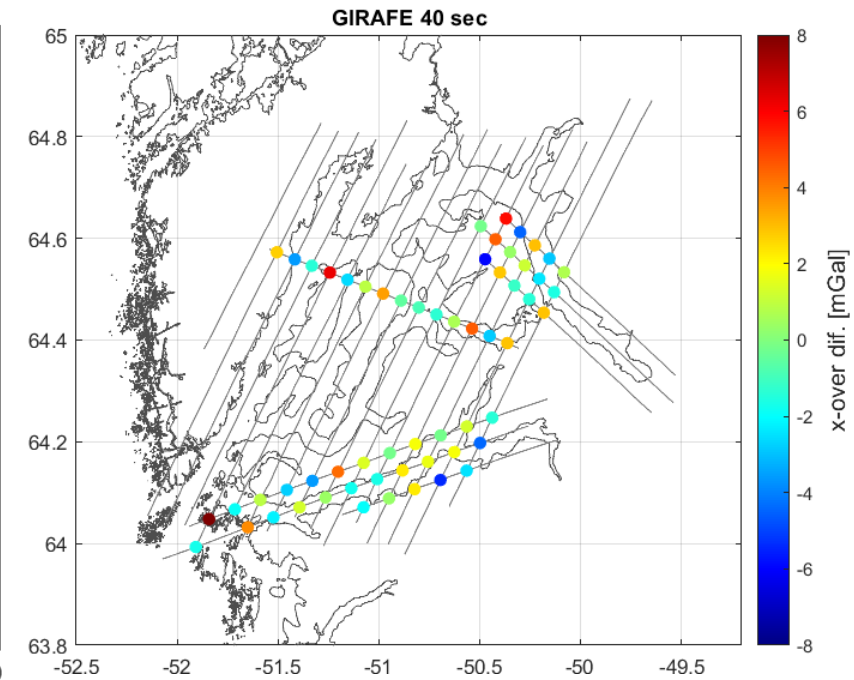
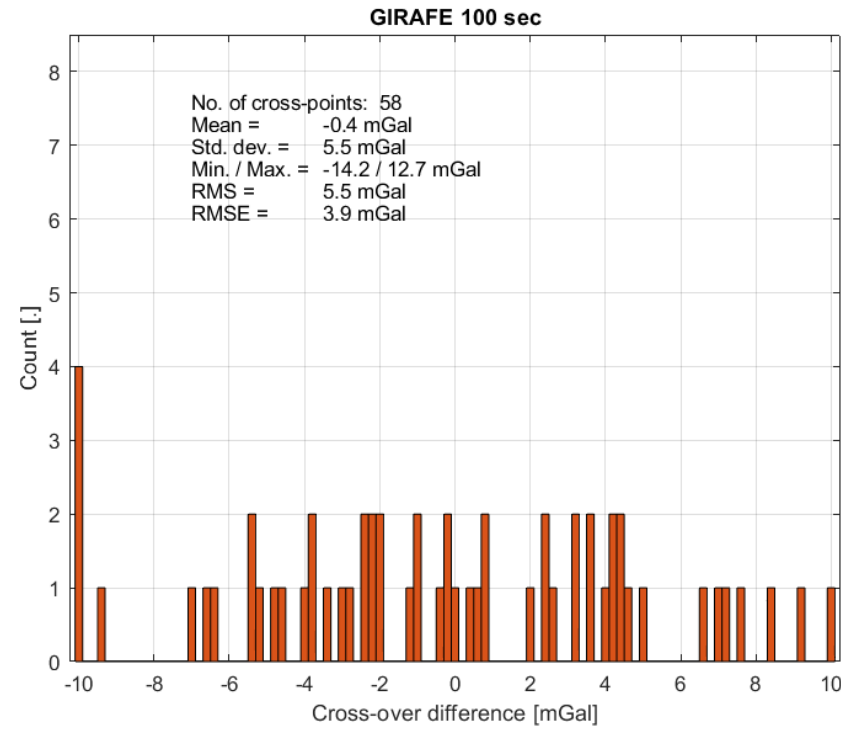
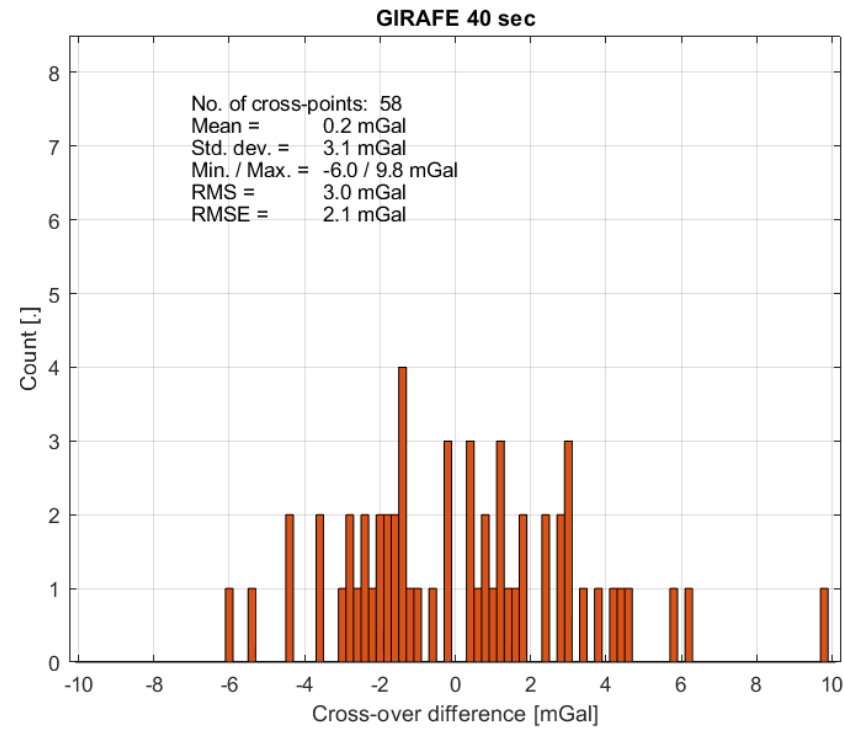
Fjords at km+ depths are generate strong gravity lows



Nuuk 2023 quantum gravimetry cross-overs ... *Impact of (too much?) filtering*

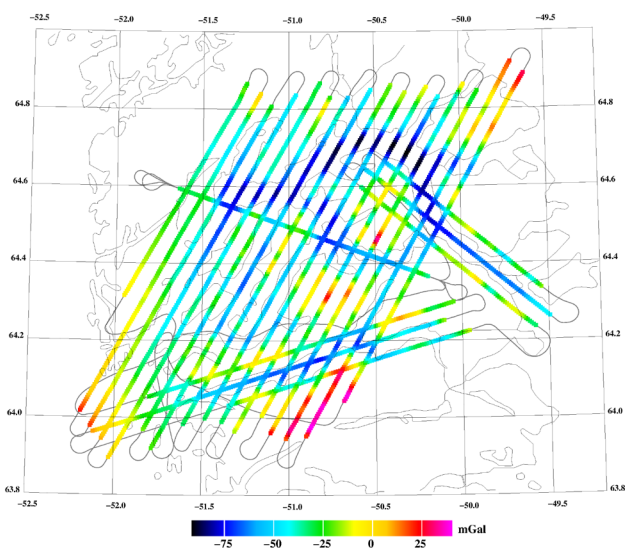


Nuuk quantum gravimetry ... *cross-over validation tricky due to large signals and (too-much?) filtering*

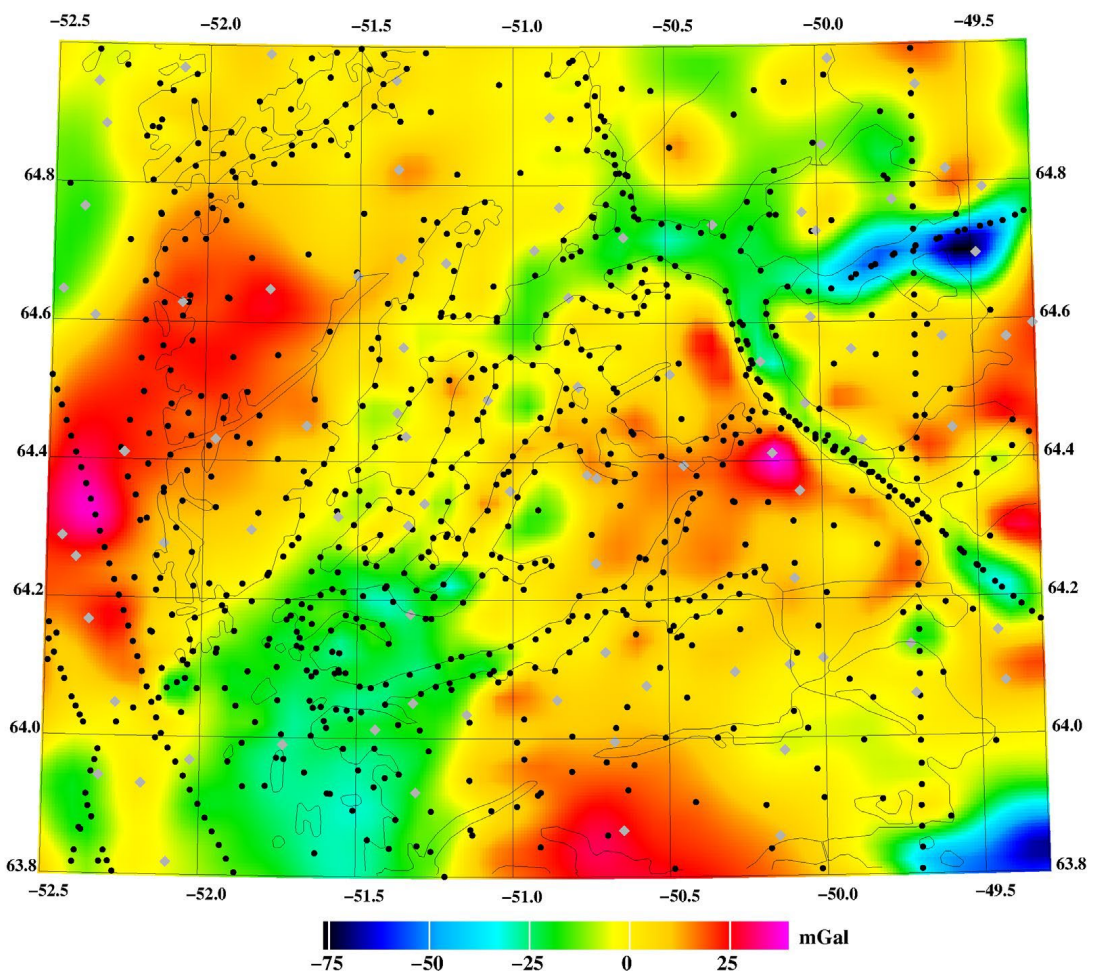
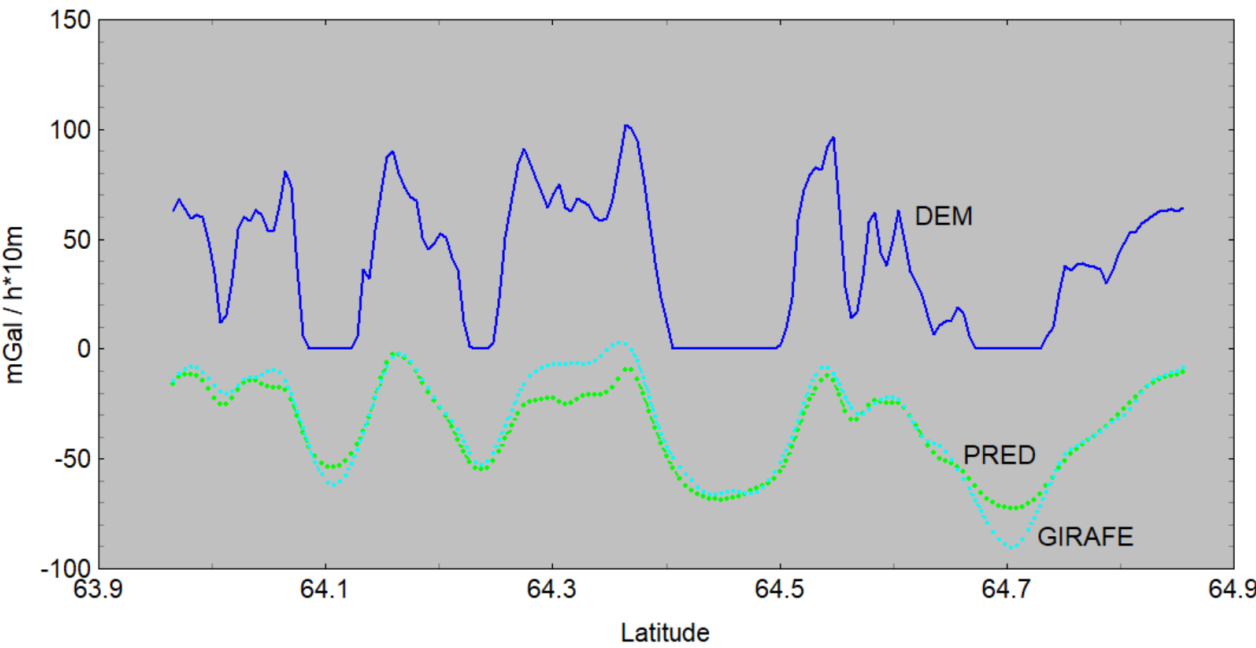


Nuuk upward continuation

Predicting gravity anomaly and deflections of the vertical along the quantum flight tracks ..

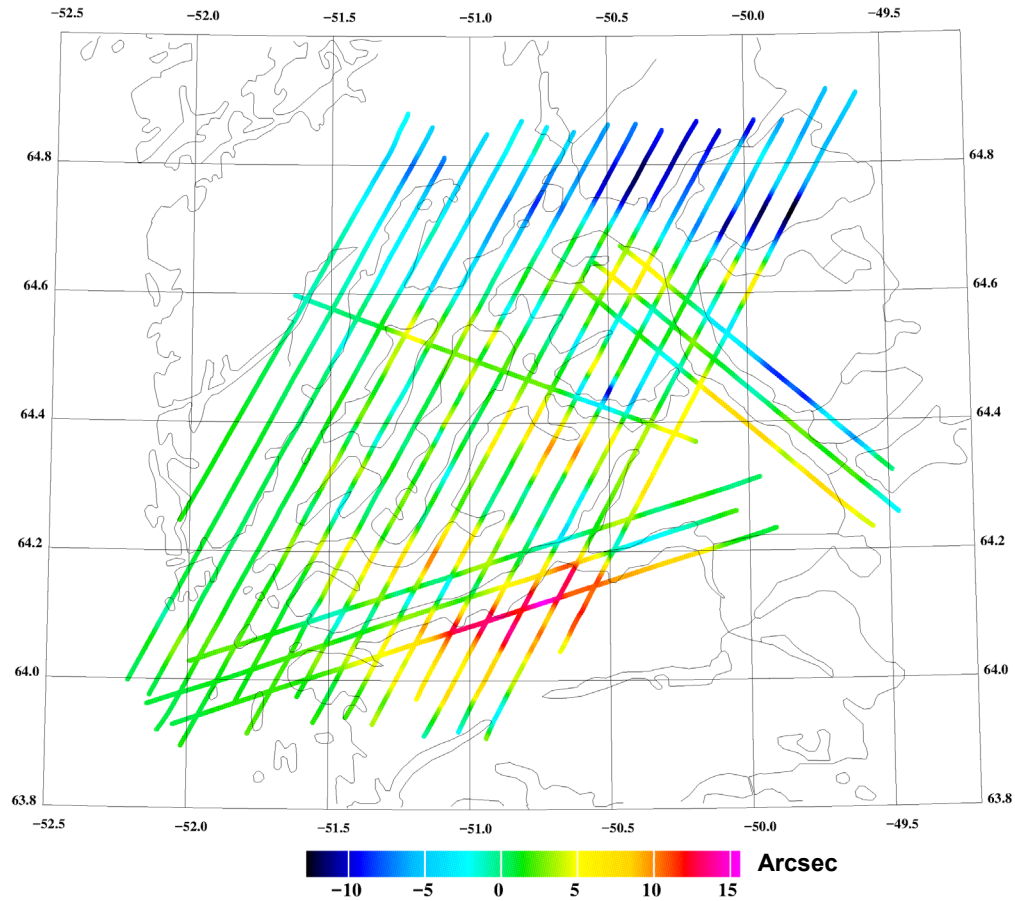


LINE 8006 - free-air anomaly and terrain

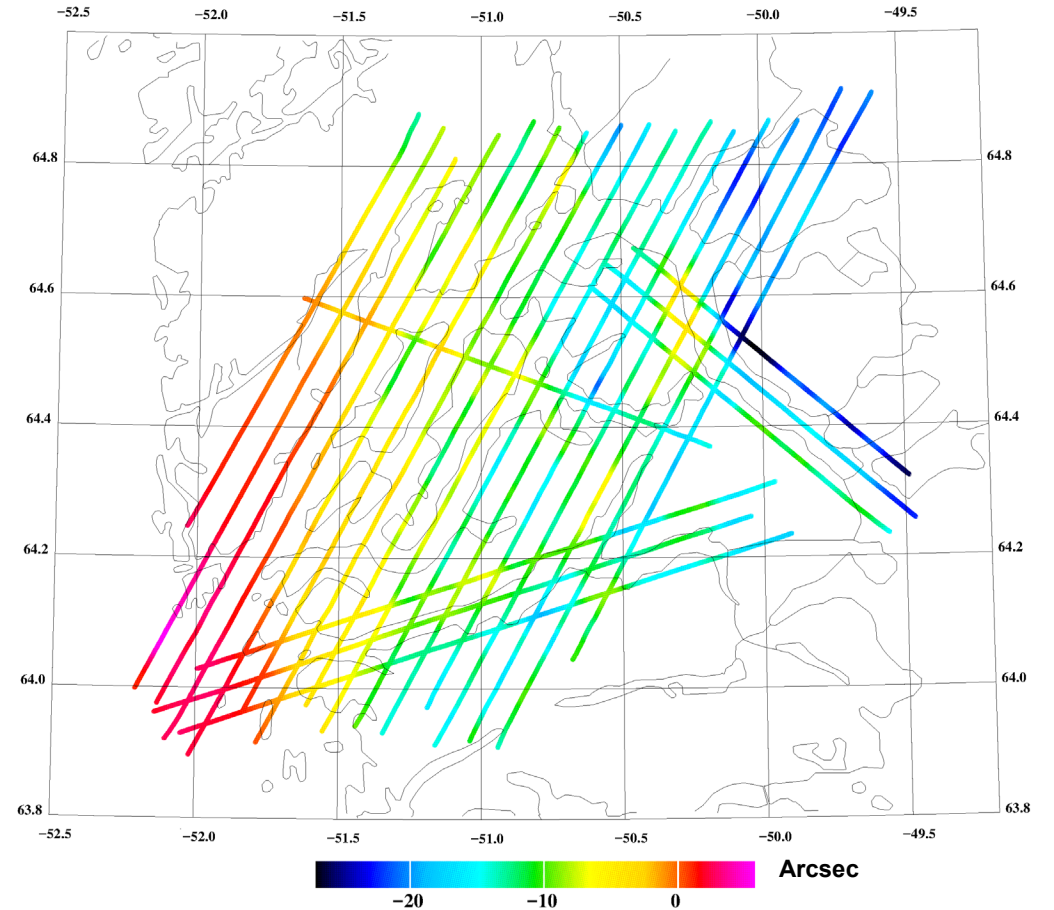


Nuuk predicted horizontal gravity (deflections of vertical)

ζ (North-South)



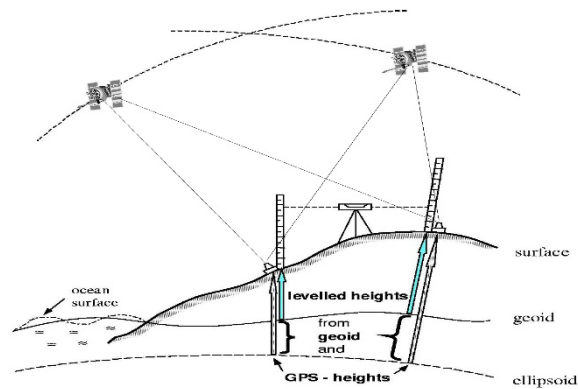
η (East-West)



1 Arcsec ~ 4.8 mGal

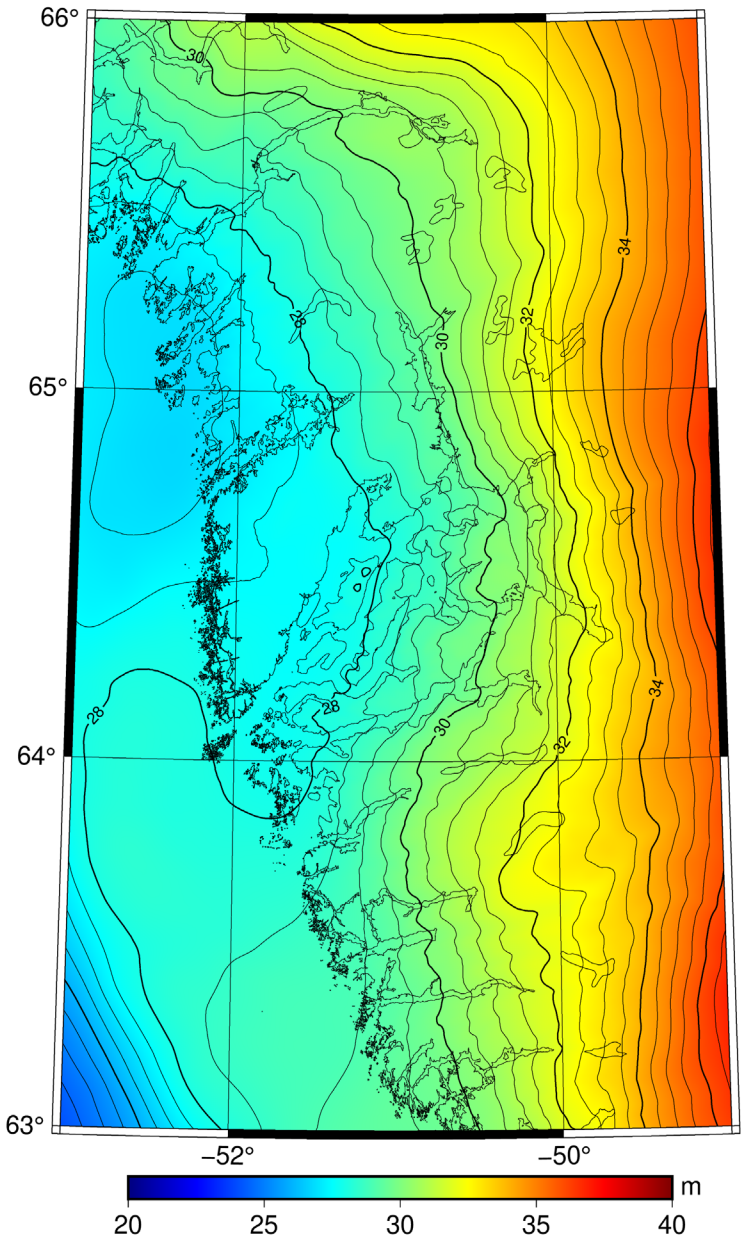
Nuuk quantum geoid ζ
Changes after quantum

Hergeir Teitsson,
Umhvørvisstovan,
Faeroe Islands

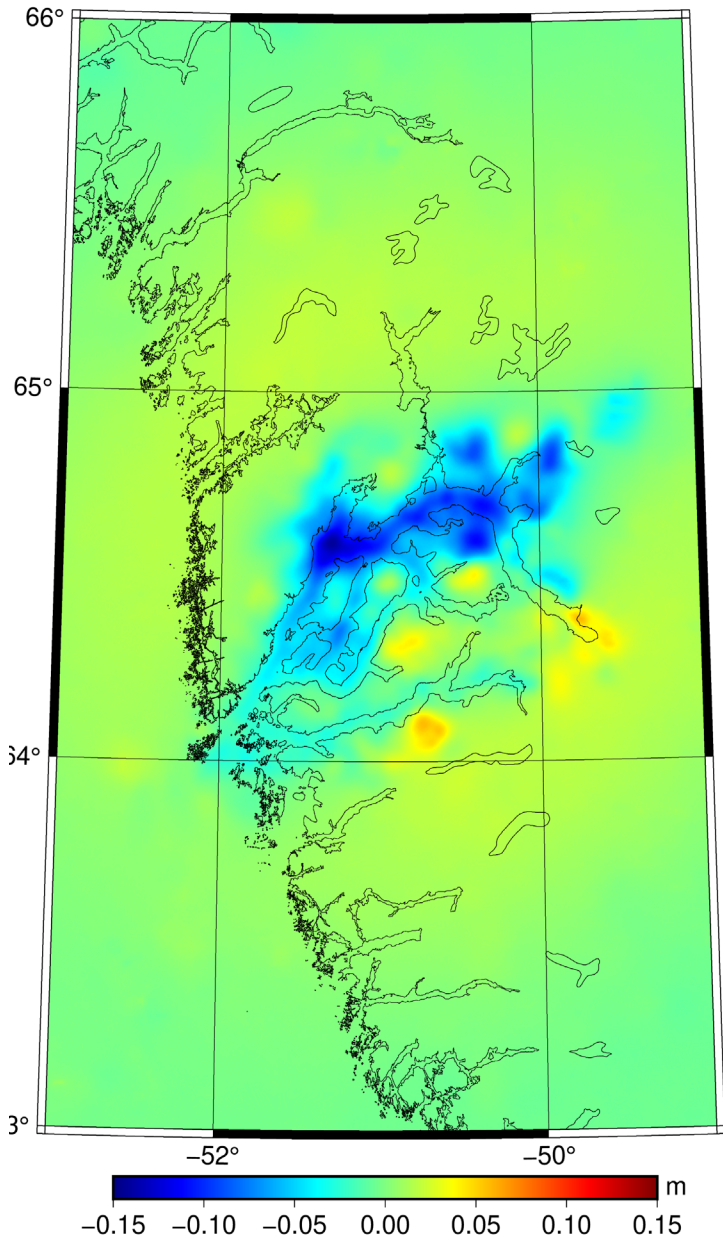


New height system
of Greenland - *quantum*

ζ Nuuk2 /w girafe2023

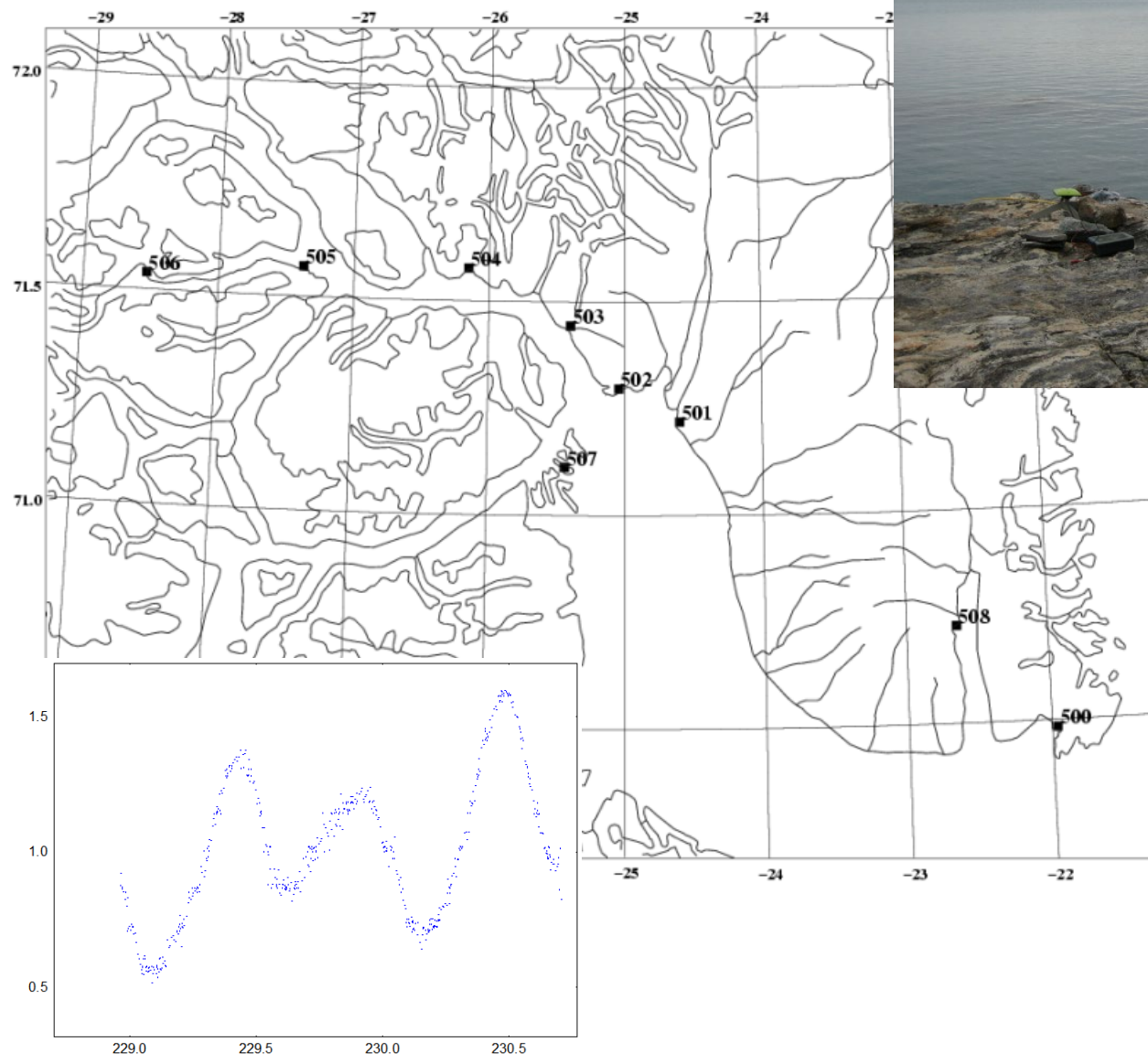


ζ difference girafe2



Geoid validation

GNSS and tide gauge measurements ~ 5 cm



Ongoing developments – 2nd generation quantum gravimeter – strapdown/hybrid

(EDF ADEQUADE project – next airborne campaign 2027)



Goal : Develop the second generation of airborne quantum gravimeter

- **More precise** : 1mGal $\rightarrow$ 0.1 mGal
- **Smaller** : strapdown (no gyrostabilized platform)
486 l, 200 kg $\rightarrow$ 27 l, 48 kg

Size and mass reduction of the gravimeter

486 l
200 kg



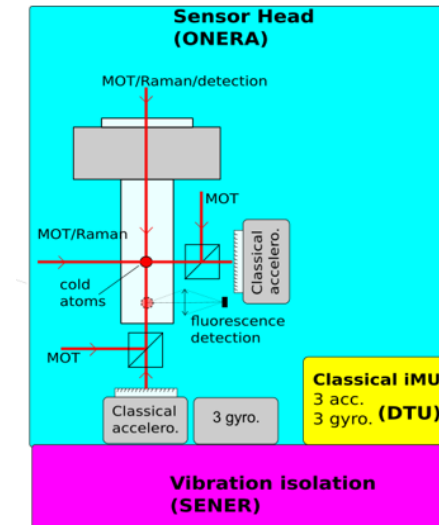
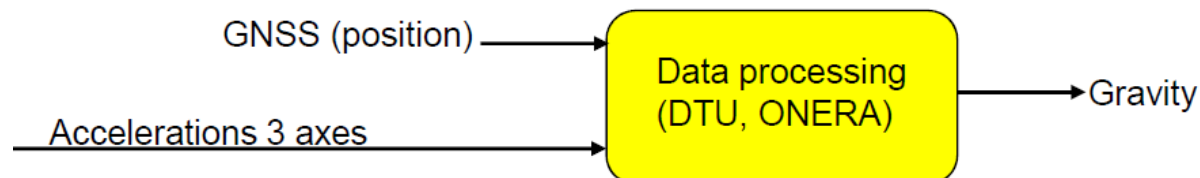
27 l
48 kg



Partners : ONERA, LTE, EXAIL, DTU, SENER, SAFRAN

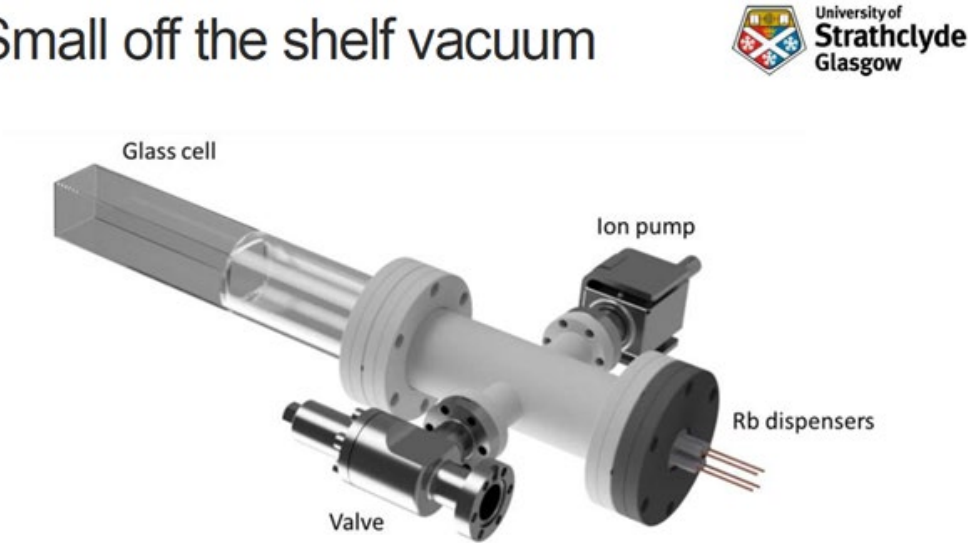
Tasks :

- Study technics and methods to improve the precision and to allow strapdown operation (LTE, ONERA, DTU)
- Definition, realization and test of a strapdown gravimeter (ONERA, EXAIL, DTU, SENER, LTE, SAFRAN)



Other developments of quantum gravity:
UK, US, Germany, Australia, Singapore ..

Small off the shelf vacuum



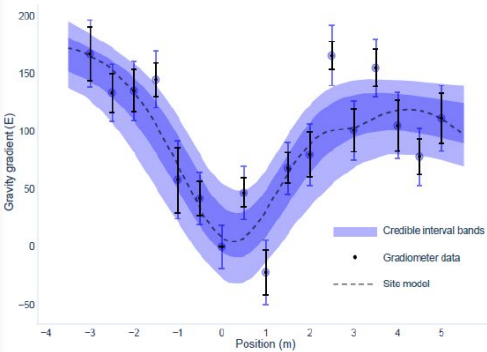
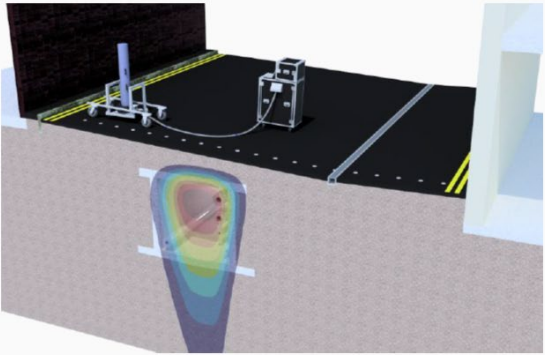
Atom-interferometry on moving platforms

Two maritime trials in 2023 (North Sea) and 2024 (Arctic Circle with Royal Navy)
Sensor operated in various sea states, and maintained original performance on return



Underground inspection

Gravity gradient detection of a multi-utility tunnel under a road



Stray, B. et al. Nature 602, 590–594 (2022)

New spin-out 2023



New EU Quantum project 2025-2029



The EQUIP-G Project

European Quantum Infrastructure Project for Gravimetry



EQUIP-G

01/06/2025 – 31/05/2029



Funded by
the European Union

Aquisition of new quantum gravimeters within EU

- 7 terrestrial quantum gravimeters
- 2 terrestrial quantum gradiometers
- 1 moving-base (airborne) quantum gravimeter

10 Use Case Demonstrations

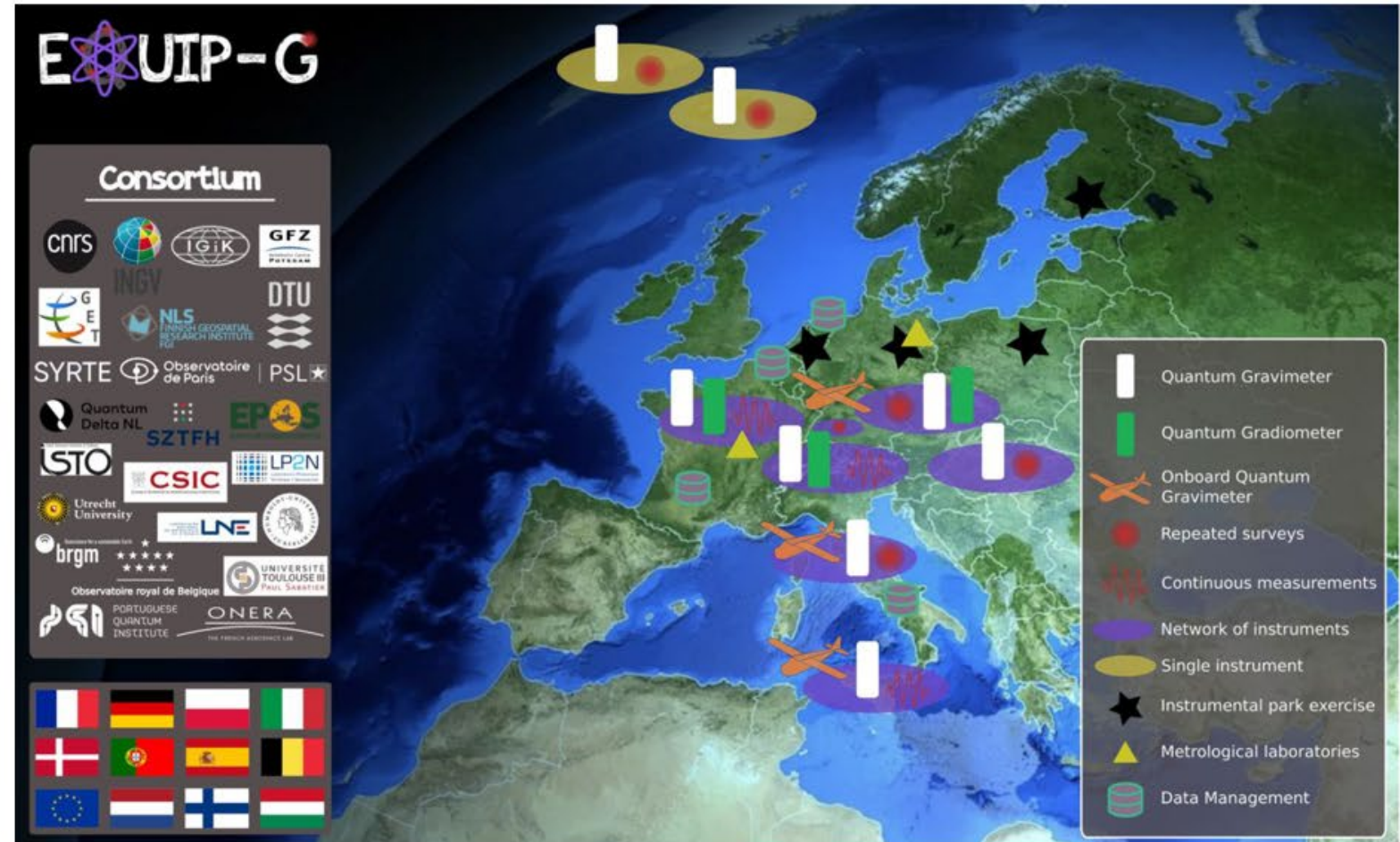
Hydrology, nuclear waste, vulcanology climatology, geothermic, geodesy, airborne mapping

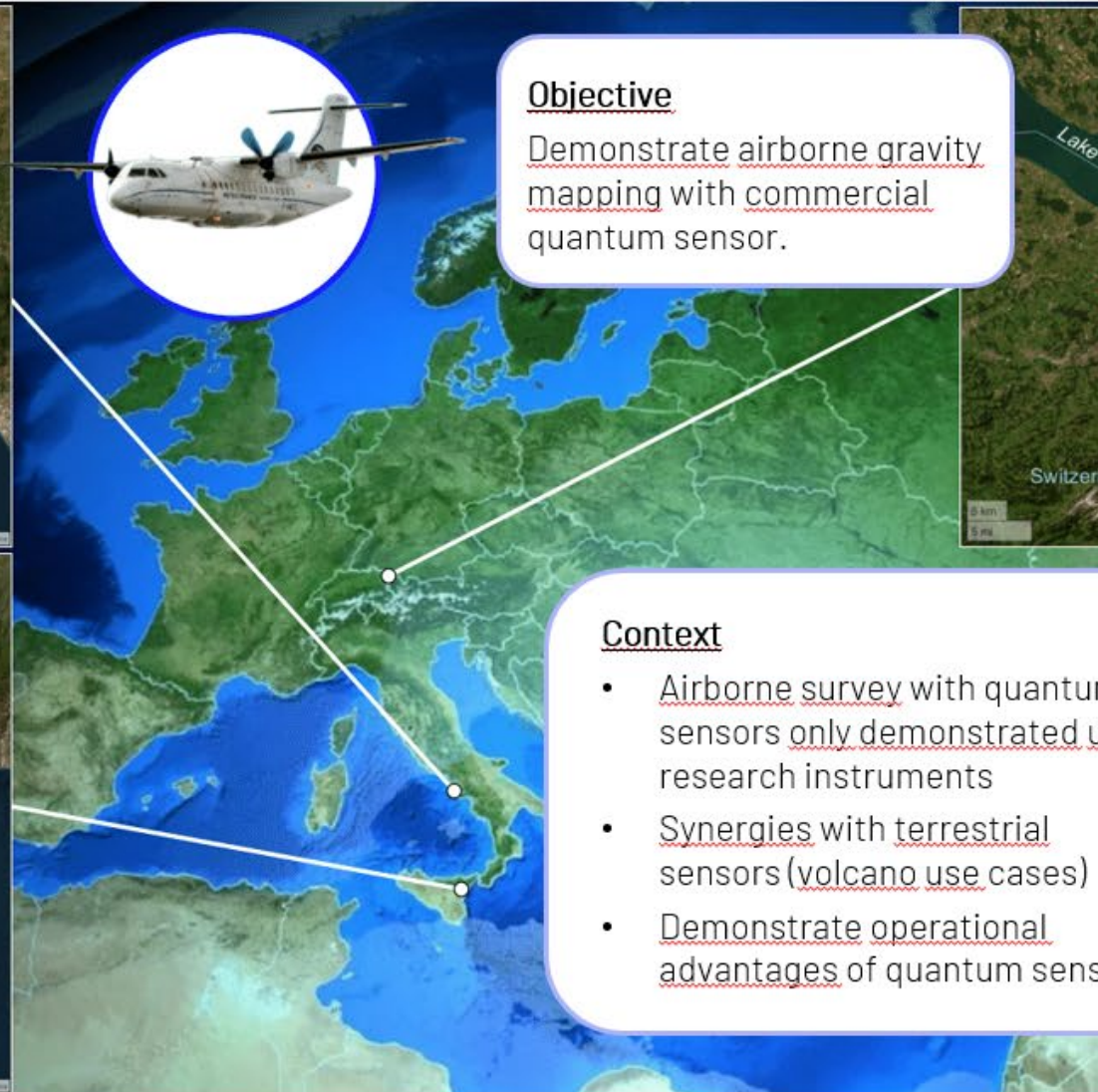
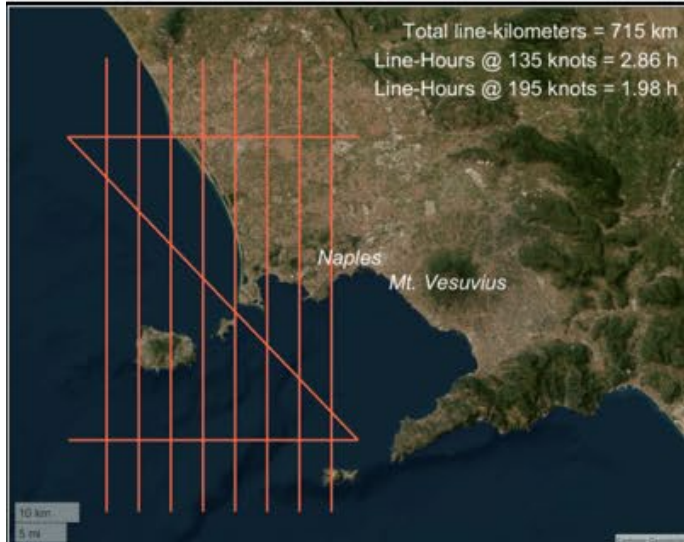
Creating an instrument sharing facility

- Loan of quantum gravimeter
- 2 open calls for instrument loan

Community building

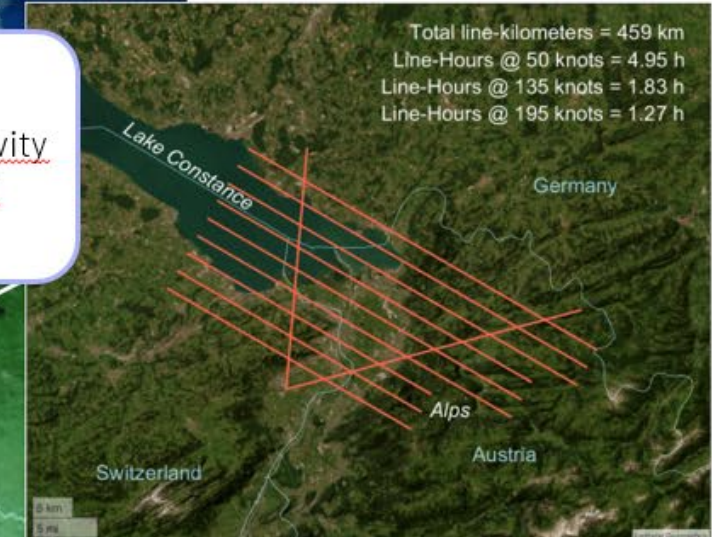
- 2 EU-wide workshops
- 2 instrument training workshops
- Platforms, conferences, organizations, etc.





Objective

Demonstrate airborne gravity mapping with commercial quantum sensor.



Context

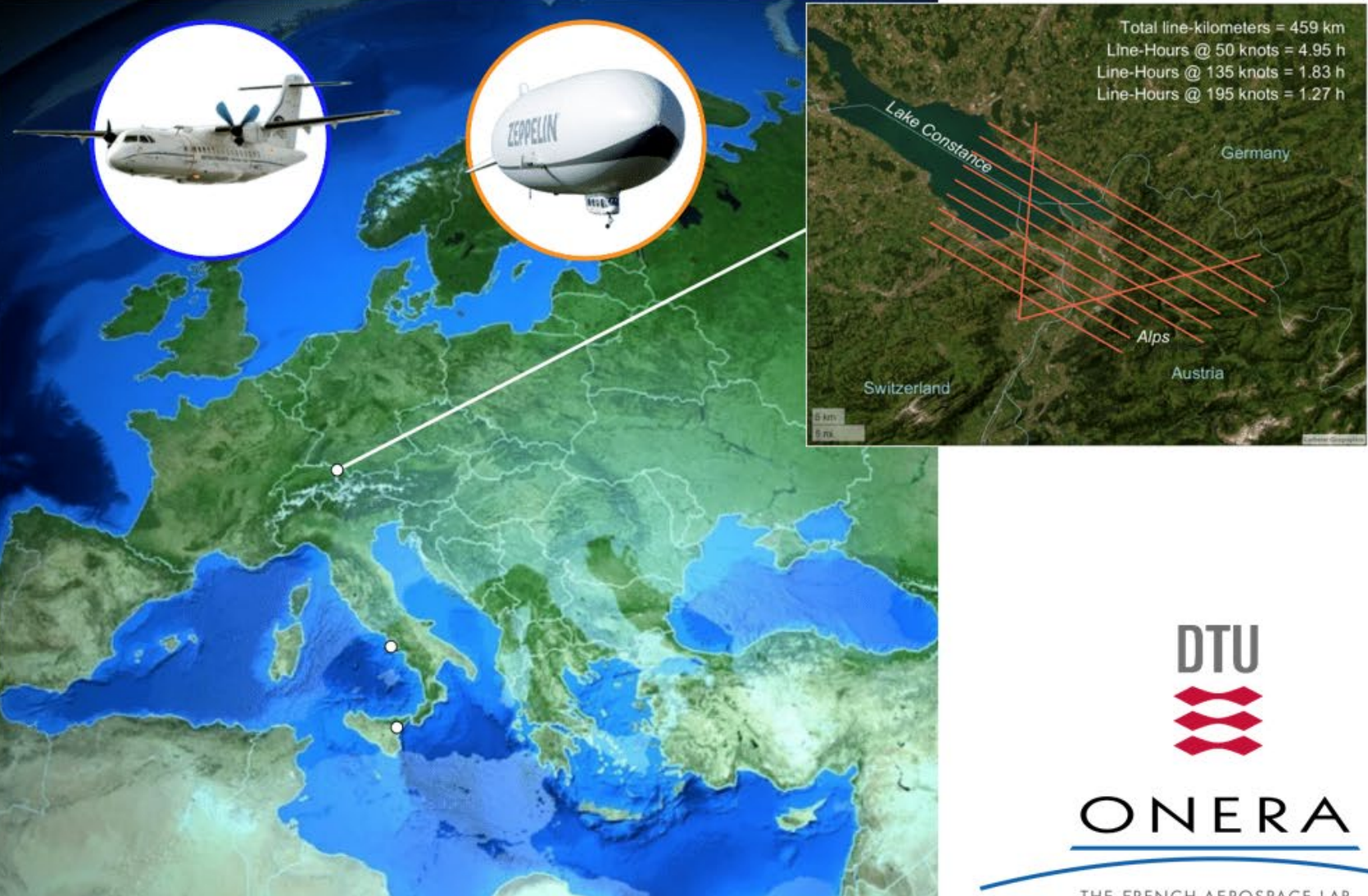
- Airborne survey with quantum sensors only demonstrated using research instruments
- Synergies with terrestrial sensors (volcano use cases)
- Demonstrate operational advantages of quantum sensors

Objective

High resolution gravity mapping
using innovative (airship) platform.

Context

- Airship offers new innovative platform
- Can move slower than aircraft
=> higher spatial resolution
- Direct comparison with fixed-wing (survey lines are repeated)
- Platform offers new possibilities (stationary and vertical profiles)



Back to space quantum gravimetry



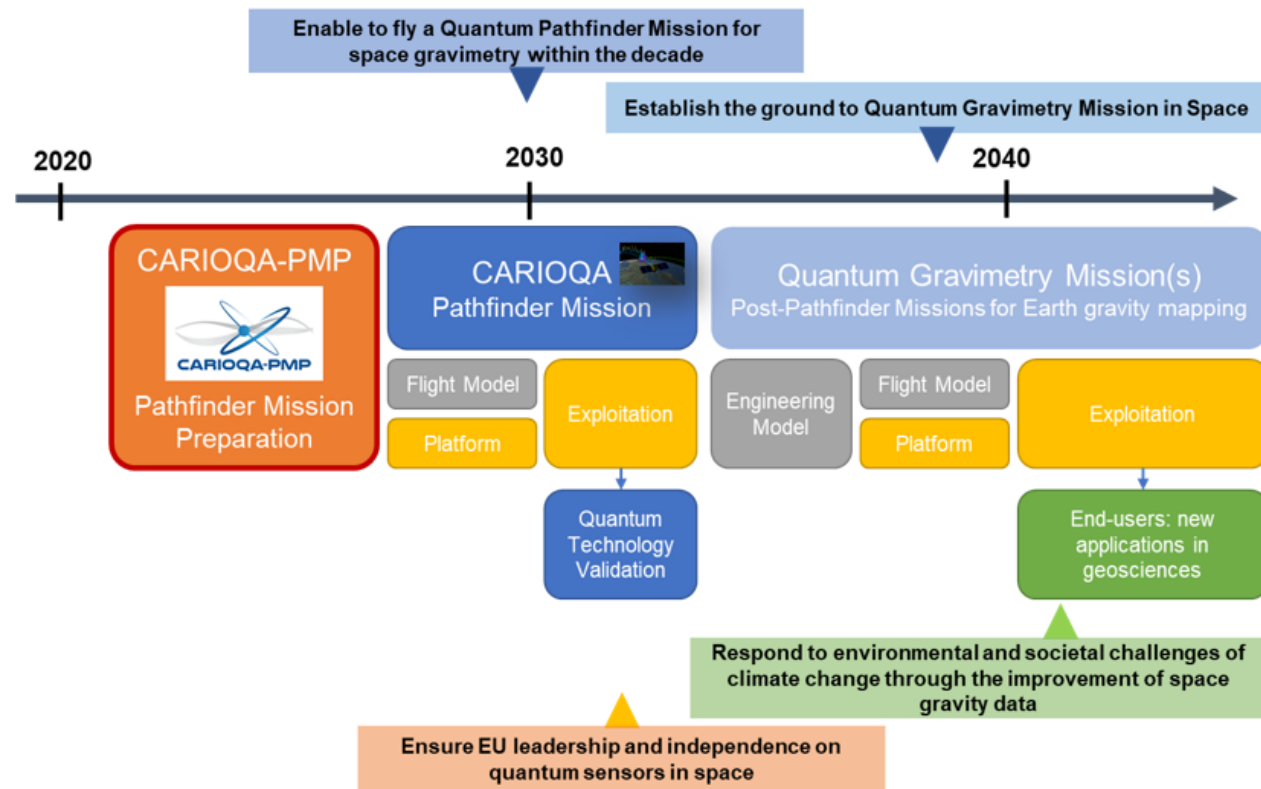
The CARIOQA-PMP Project

Cold Atom Rubidium Interferometer in Orbit for Quantum Accelerometry - Pathfinder Mission Preparation

01/12/2022 - 31/03/2026

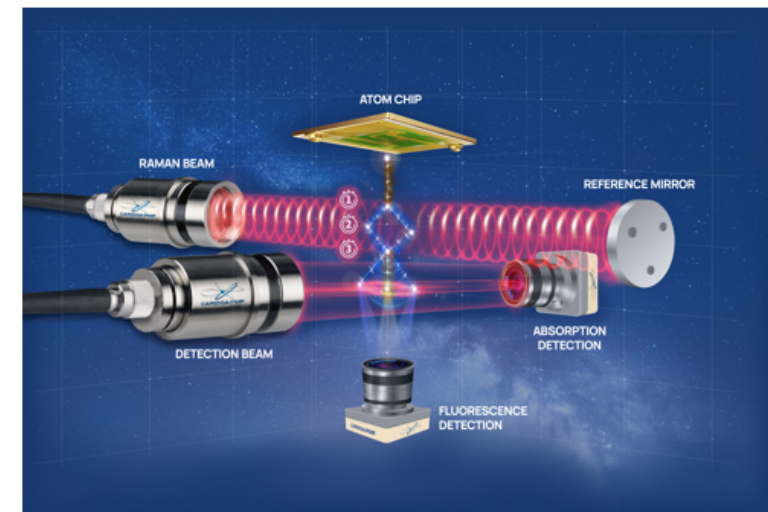


Funded by
the European Union



DTU Involvement:

- Quantum gravimetry state-of-the-art analysis
- Quantum pathfinder mission scenario building
- Development of simulation tools for enhanced quantum sensor satellite missions and performance estimation (lead beneficiary)
- Quantum space gravimetry mission scenario building for Earth observation
- Designing a quantum space gravimetry mission



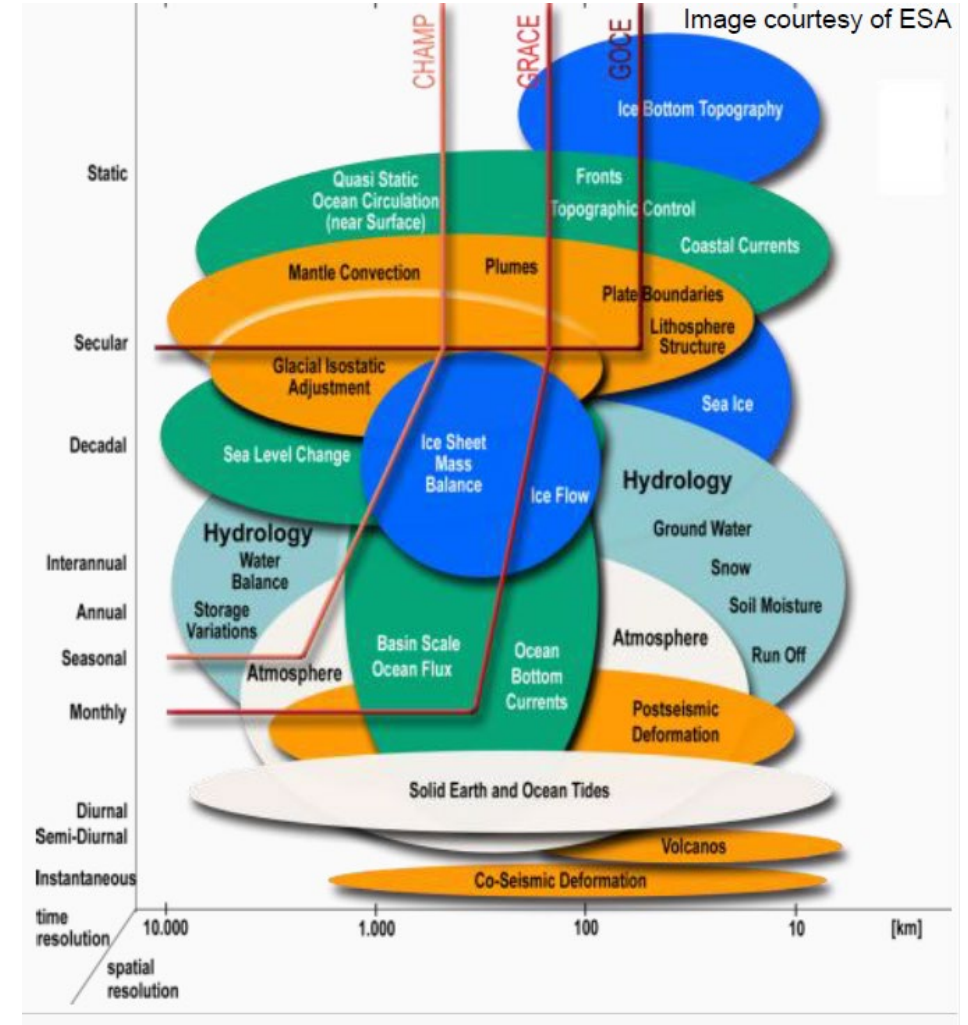
Challenges and limitations of quantum space gravimetry

- Orbit heights ... need to be low (300-350 km)
- Accelerometer better than $\sim 10^{-10} \text{ m/s}^2$ for CHAMP-like pathfinder mission
- Gradiometer $\sim$ broadband accuracy better than 10^{-12} /s^2 (GOCE-like mission)

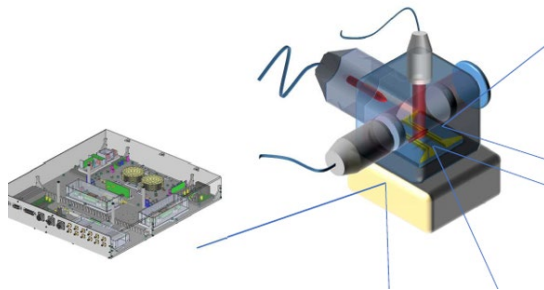
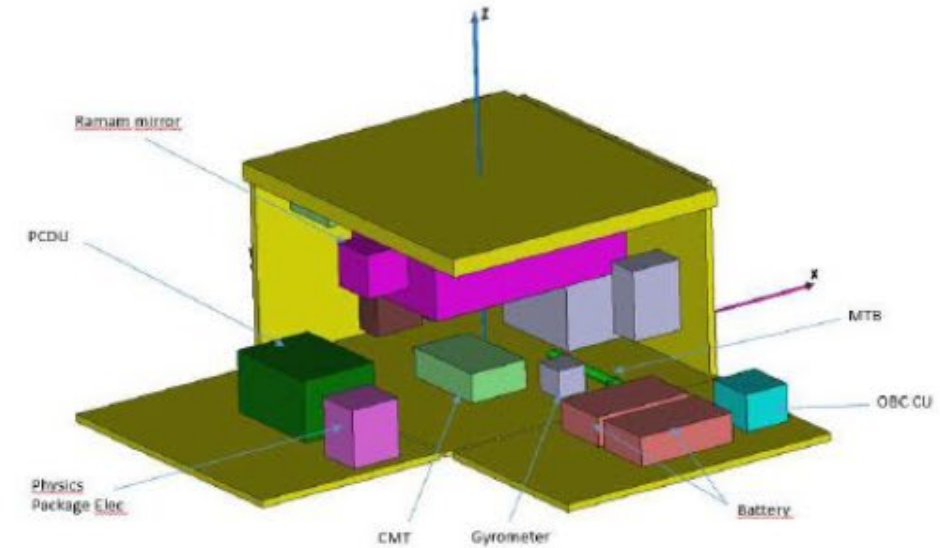
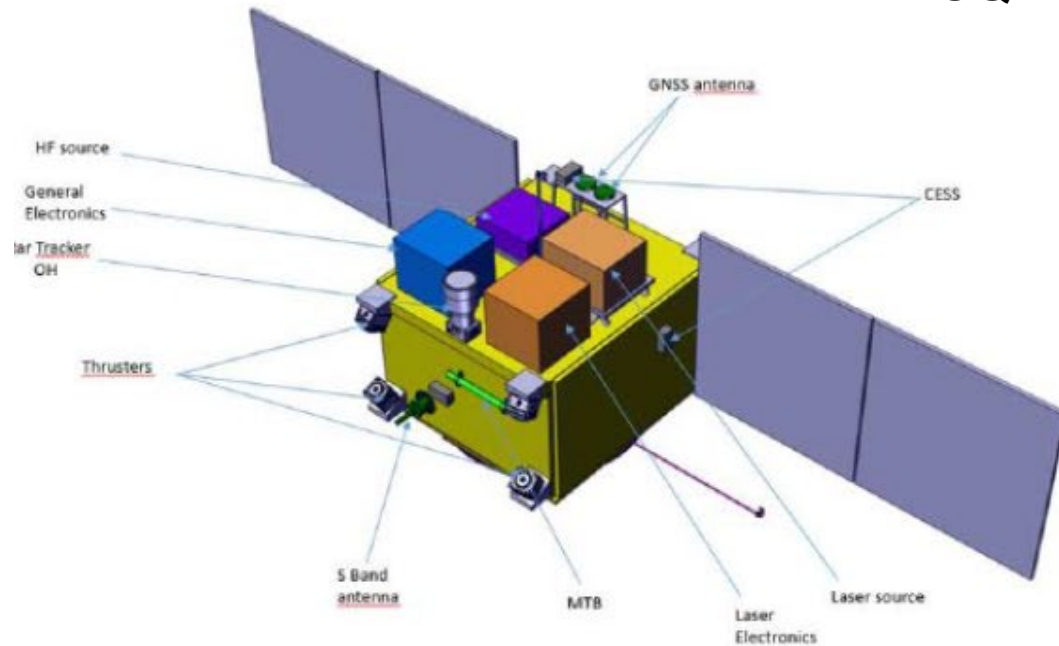
Constellation mission vision 2045+: frequent repeats in time and space, providing near-real time warnings ... floods, earthquakes, climate events ..

- Space agencies (ESA, NASA, DLR/CNES and EU) active in supporting new quantum sensor developments

EU CARIOQA = Cold Atom Rubidium Interferometer in Orbit for Quantum Accelerometry)



CARIOQA preliminary design (CNES)



Laser systems, ultracold Rb source, orbit, systems integration, interrogation time ..

Instrument	Applications	Sensitivity
Absolute quantum gravimeter (SYRTE)	Ground/Laboratory	$5 \times 10^{-8} \text{ m.s}^{-2}.\text{Hz}^{-1/2}$
Commercial absolute quantum gravimeter (IXBLUE/Muquans)	Ground/Field applications	$5 \times 10^{-7} \text{ m.s}^{-2}.\text{Hz}^{-1/2}$
Absolute atom accelerometer/gravimeter in microgravity (ICE/LP2N)	Microgravity	$6 \times 10^{-7} \text{ m.s}^{-2}.\text{Hz}^{-1/2}$
Quantum Pathfinder Mission (CARIOQA)	Satellite	$1 \times 10^{-10} \text{ m.s}^{-2}.\text{Hz}^{-1/2*}$
Quantum Space Gravimetry Missions	Satellite	$1 \times 10^{-12} \text{ m.s}^{-2}.\text{Hz}^{-1/2}$

**Targeted Quantum projection noise floor*

from Lévêque et al. 2022

An aerial photograph taken from the window of a red airplane, looking down at a vast, textured glacier in East Greenland. The glacier's surface is covered in numerous small, blue-tinged icebergs and crevasses. In the distance, a range of snow-capped mountains is visible under a clear blue sky. The red wing and tail of the airplane are visible in the upper left and right corners of the frame.

Thanks for your attention

2023 quantum gravity, East Greenland