

Atom Interferometry

An introduction

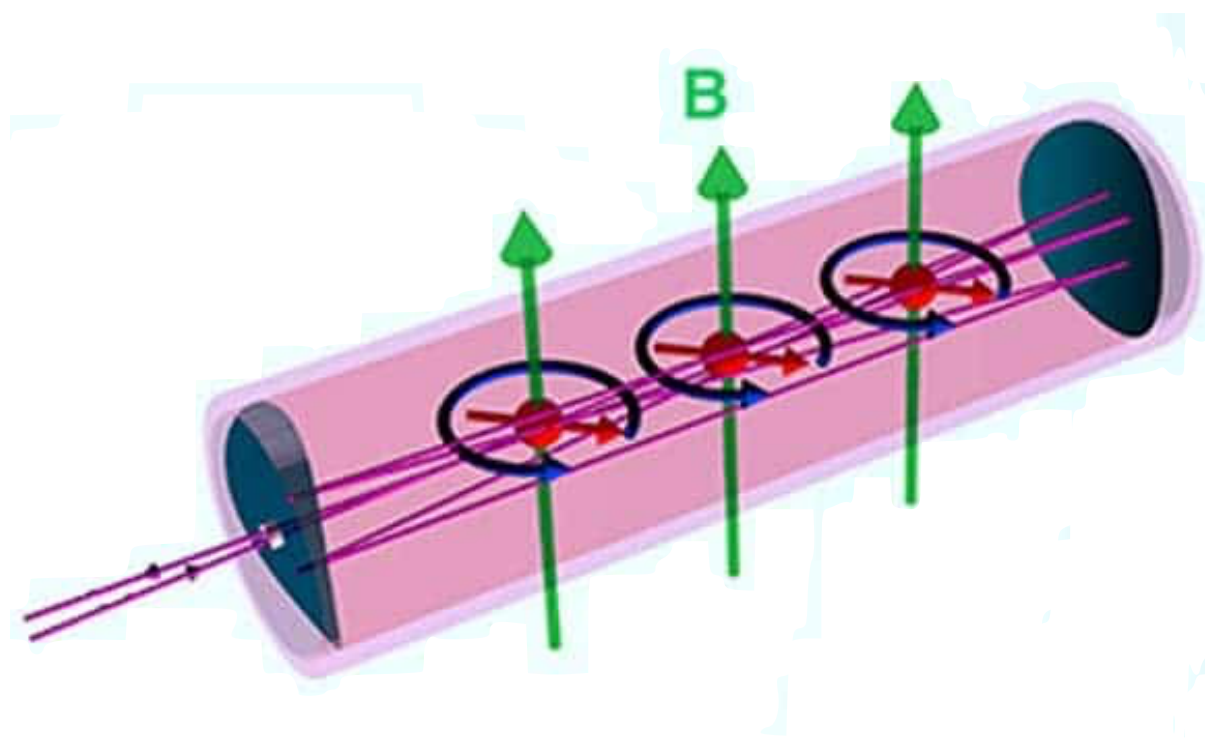
Alexandre Gauguet - FOMO Summer School 2026 - Piombino Italy

Atom Interferometer

Quantum sensors measure (sensing) things by **manipulate** and **read out** its quantum states.

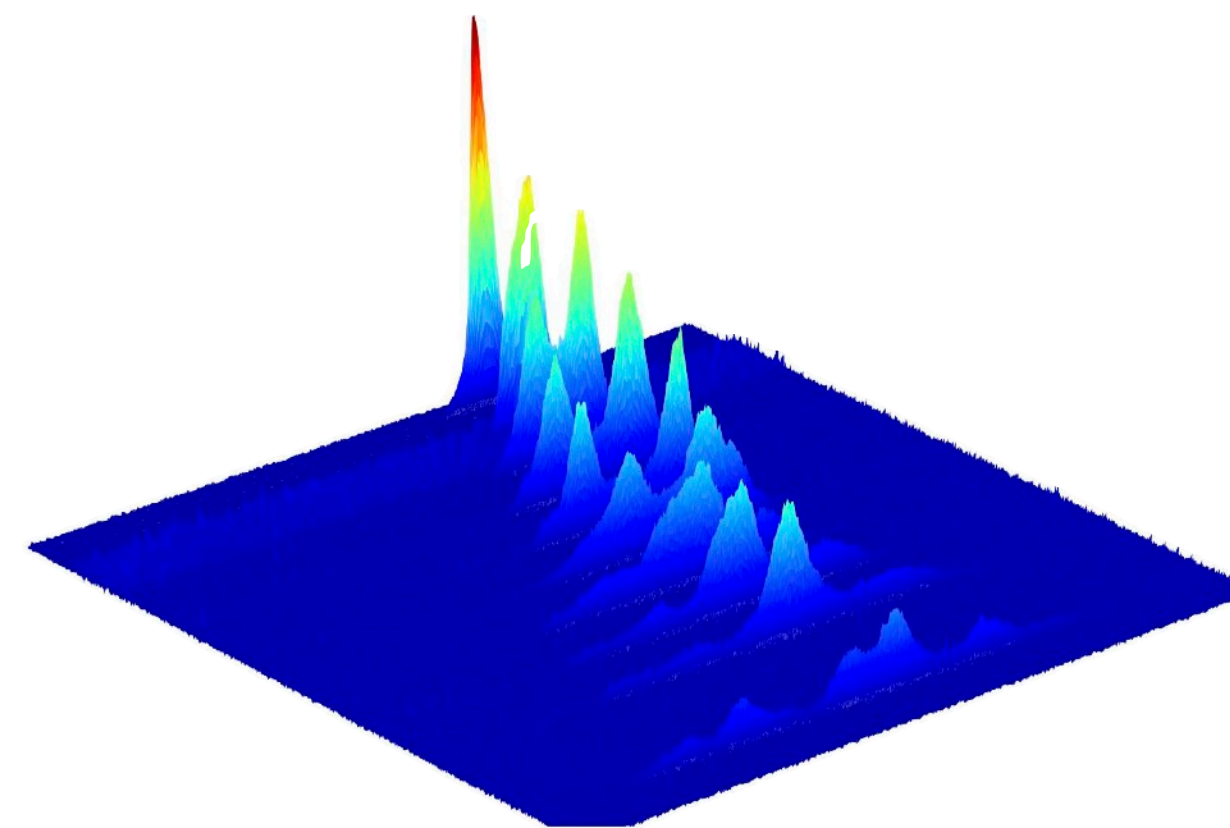
Quantum technologies and the elephants, M. S. Safronova and Dmitry Budker, Quantum Sci. Technol. 6, 040401 (2021).

$$|\Psi\rangle = \alpha_1 e^{i\phi_1} |\uparrow\rangle + \alpha_2 e^{i\phi_2} |\downarrow\rangle$$



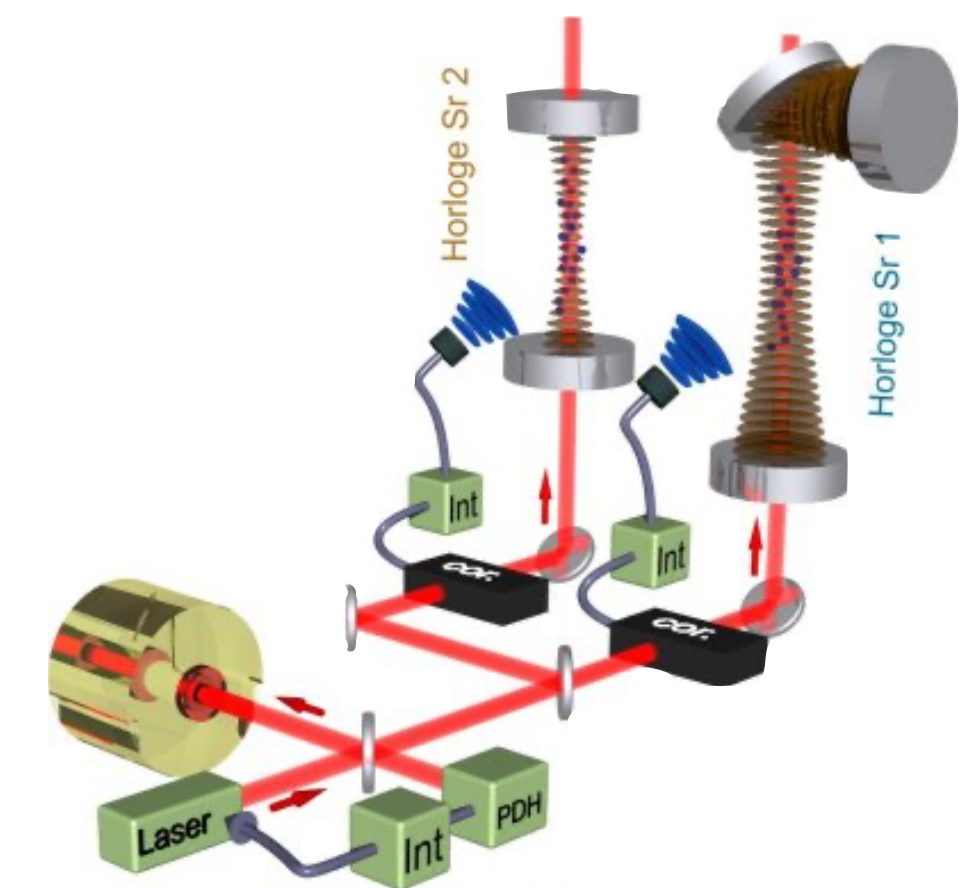
Atomic Magnetometers

$$|\Psi\rangle = \alpha_1 e^{i\phi_1} |\Psi_1\rangle + \alpha_2 e^{i\phi_2} |\Psi_2\rangle$$



Atomic inertial sensors ...

$$|\Psi\rangle = \alpha_1 e^{i\phi_1} |E_1\rangle + \alpha_2 e^{i\phi_2} |E_2\rangle$$



Atomic Clocks

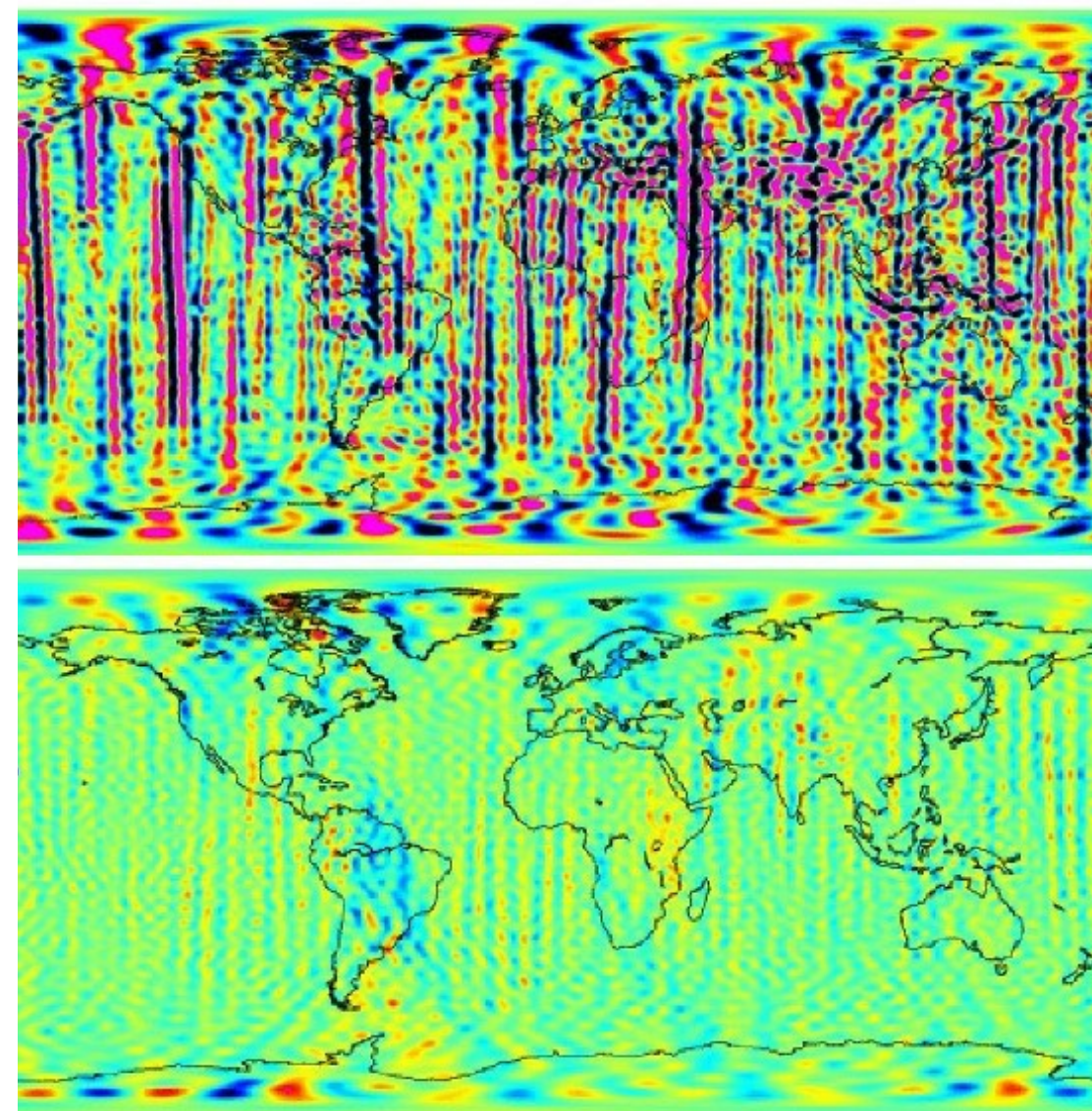
Absolute sensors + Ideal test mass + Internal structure

Atom Interferometry: inertial sensors

Geophysics

Earth observations

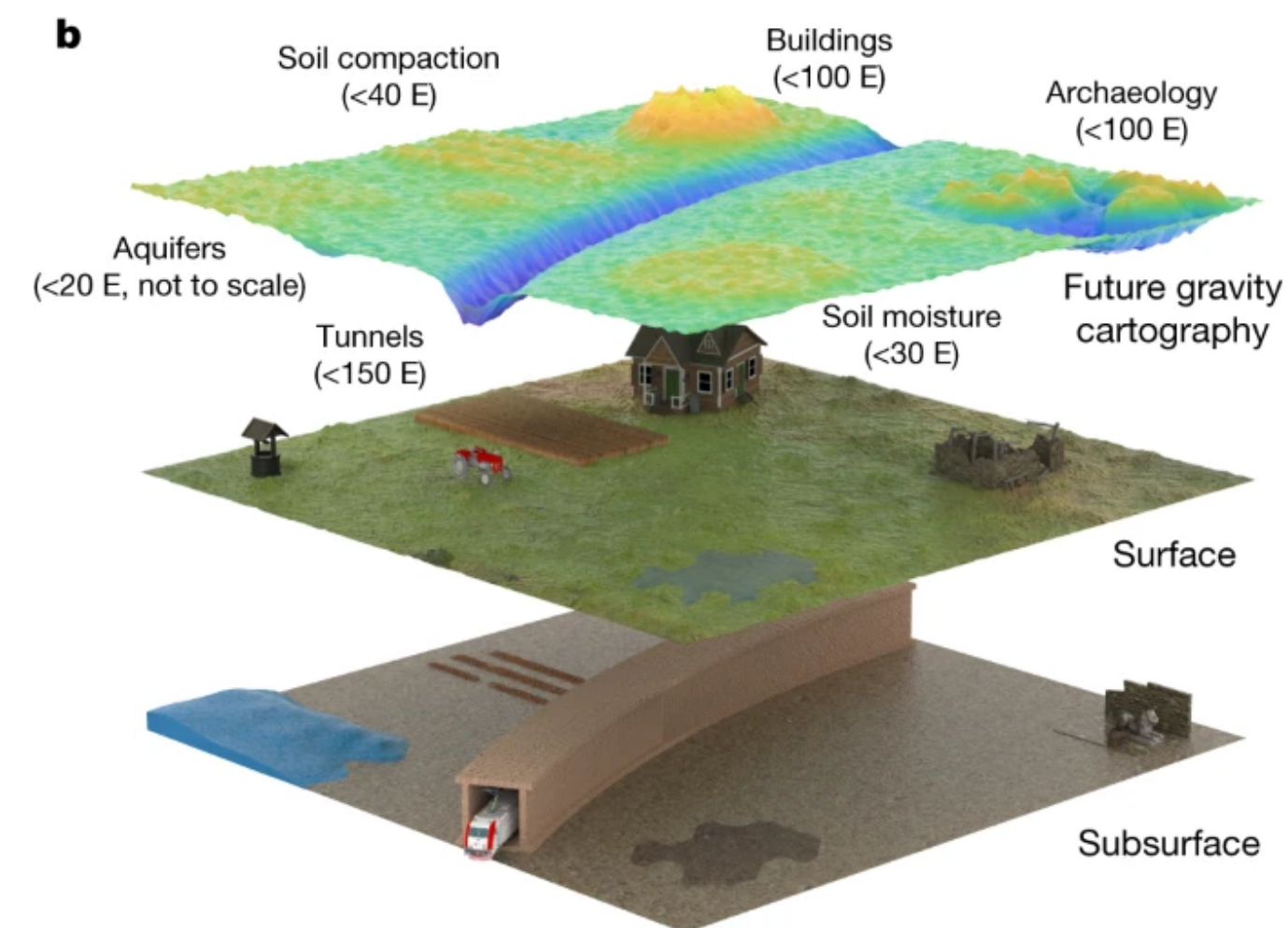
Tracking land and water masses



T. Lévêque et al. Journal of Geodesy 95:15 (2021)

Civil engineering

Gravity cartography



B. Stray et al. nature 602, 590-594 (2022)

Navigation

Navigation w/o GNSS

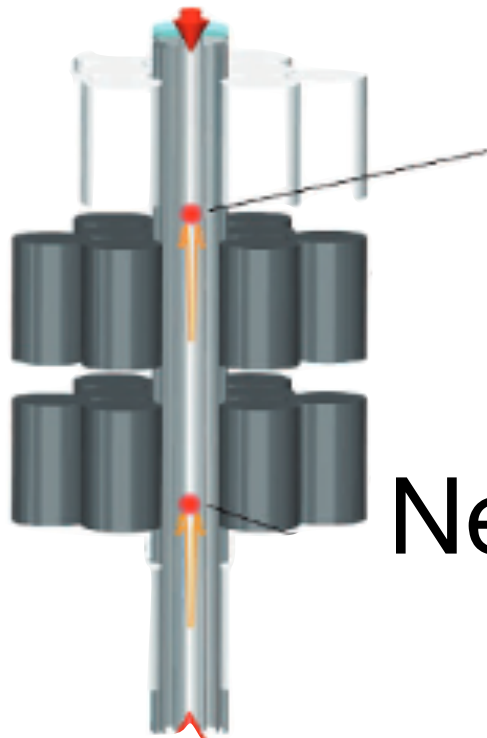


○ > 7% of EU GDP (10^{12} €) depends on GNSS

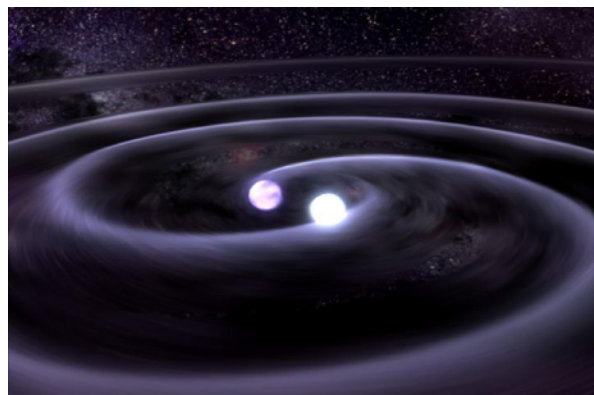
○ Strategic independence

Atom Interferometry: fundamental tests

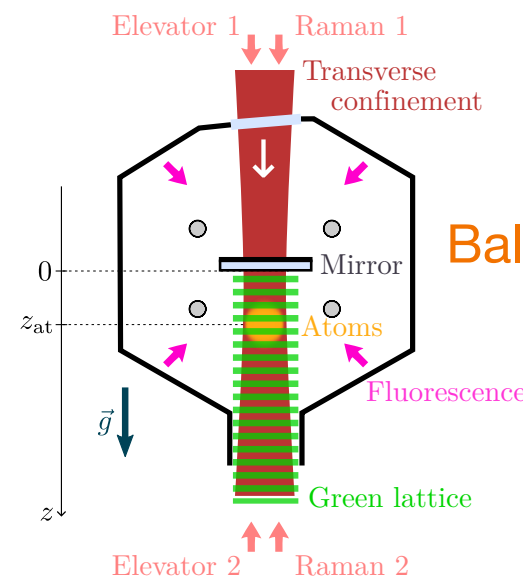
Fixler et al. science 315 5808 (2007)
Rosi et al. nature 510 7506 (2014)



Newton G



Gravitational wave



Balland et al. Phys. Rev. Lett. 133, 113403 (2024)

Short range forces

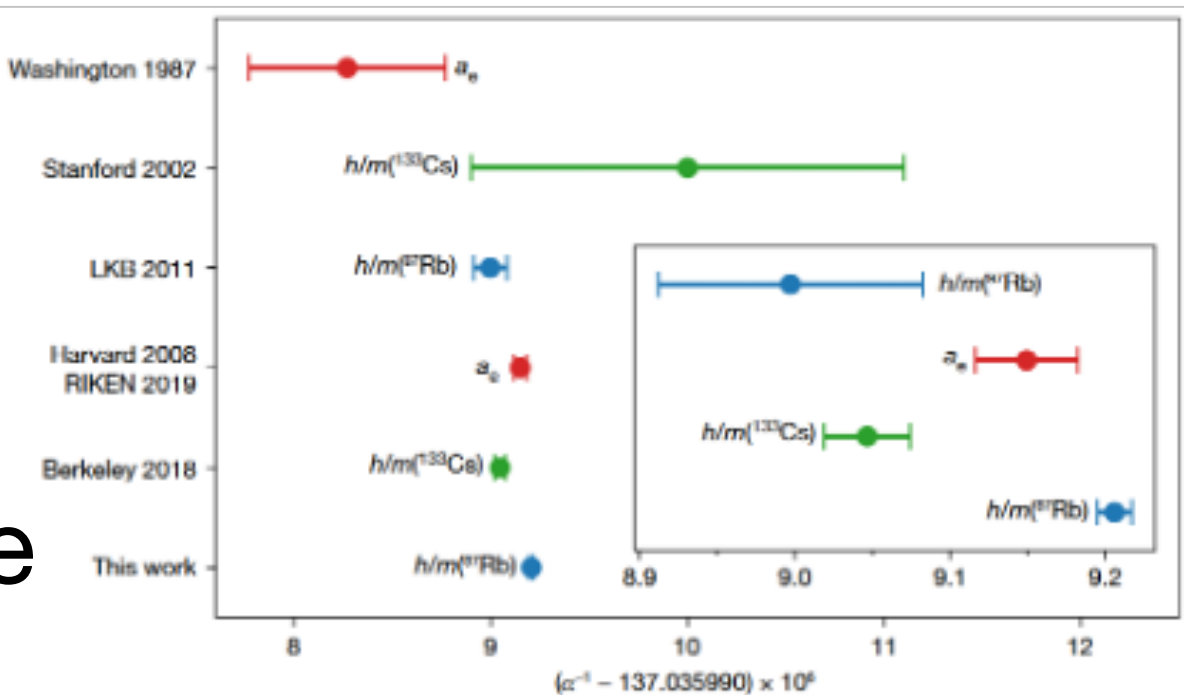


Equivalence Principle test

Asenbaum et al., PRL 125, 191101 (2020)

Atom interferometry

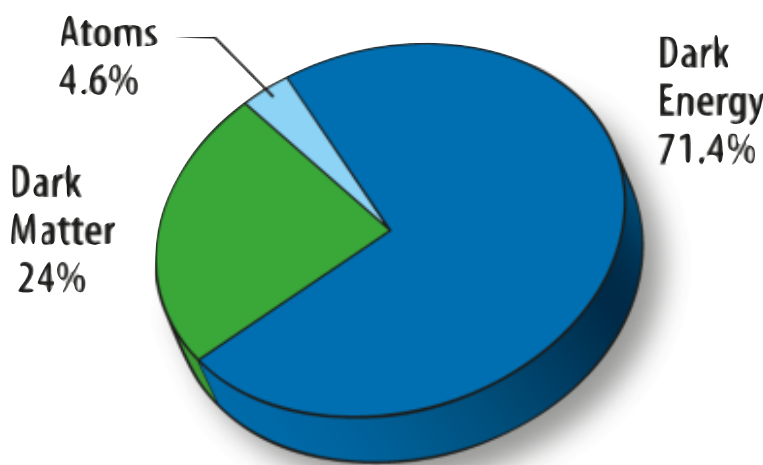
Fine structure constant



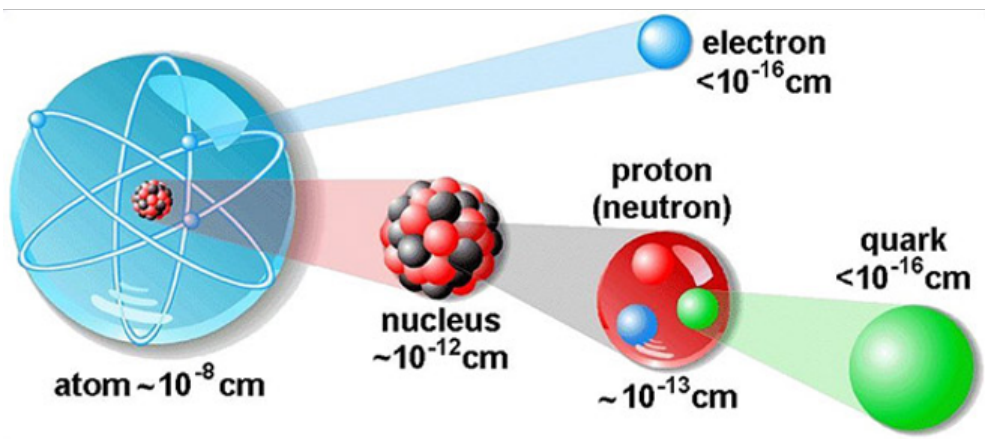
Morel et al., Nature 588, 7836 (2020)

Parker et al., Science 360(2018)

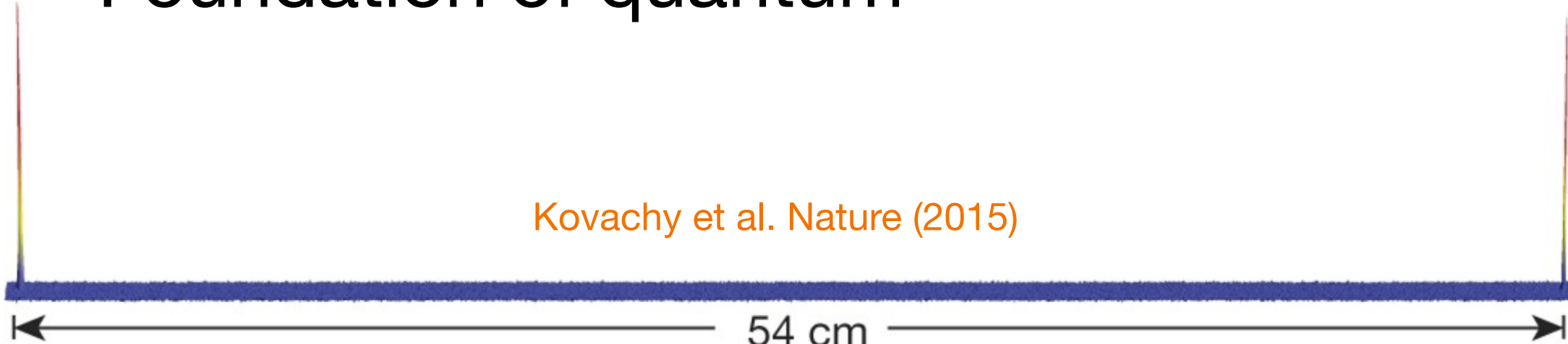
Dark Matter



Atom neutrality



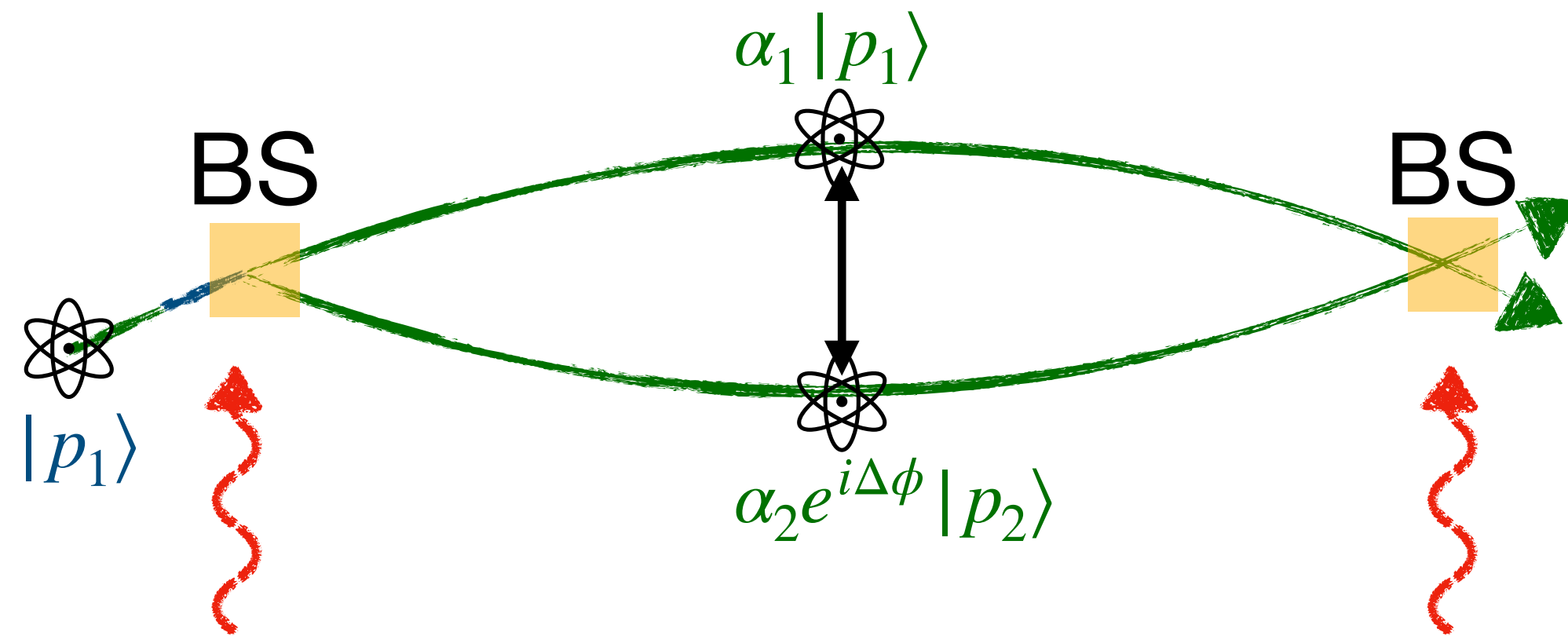
Foundation of quantum



Kovachy et al. Nature (2015)

Atom interferometry

Two wave interferometer



State preparation

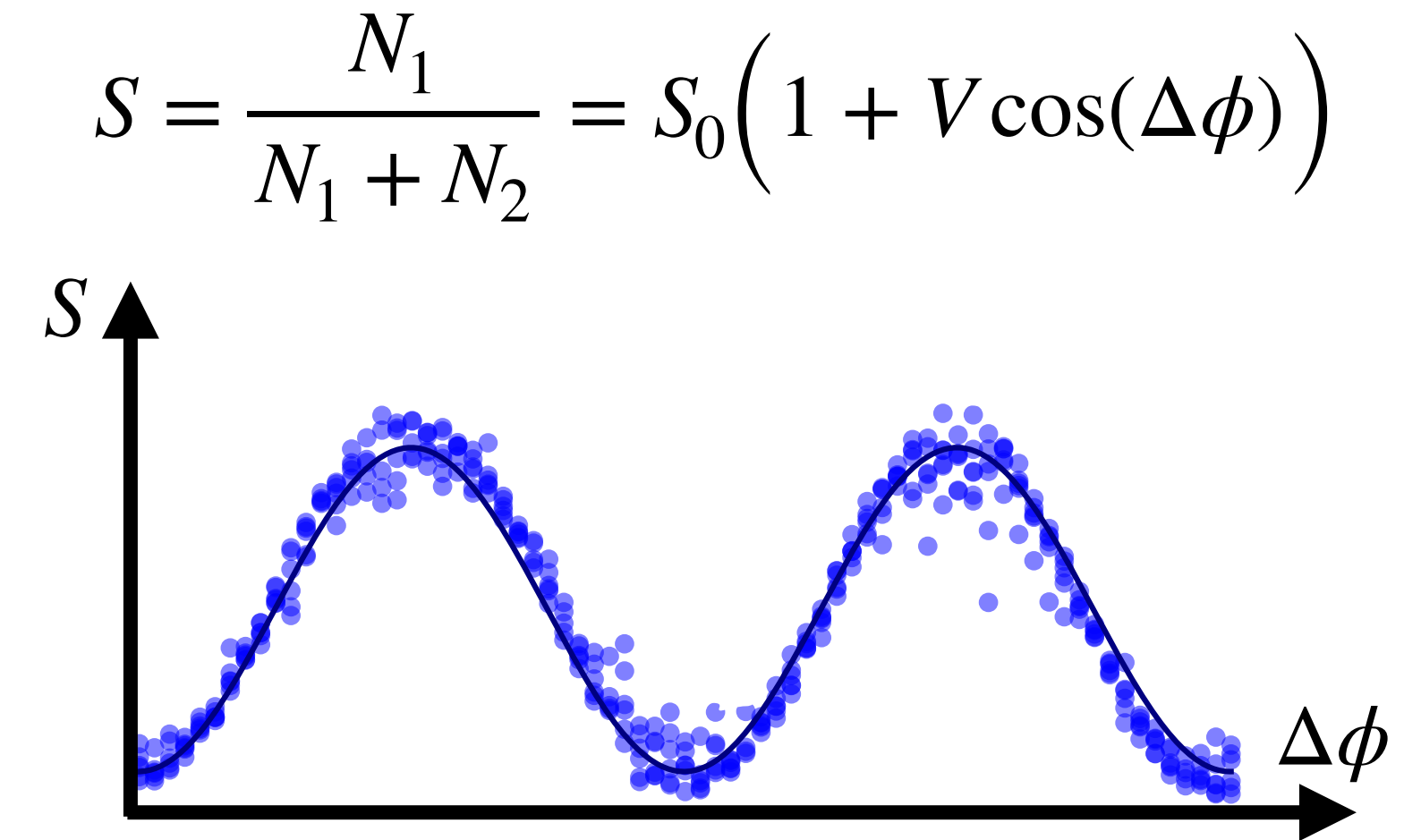
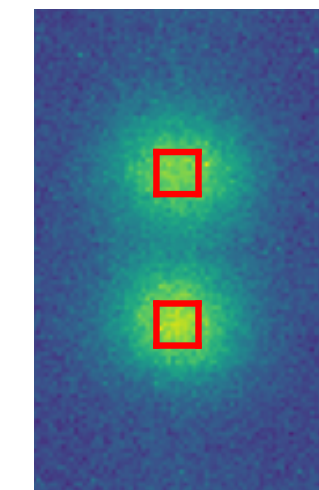
$$|p\rangle$$

Quantum superposition

$$\alpha_1 |p_1\rangle + \alpha_2 e^{i\Delta\phi} |p_2\rangle$$

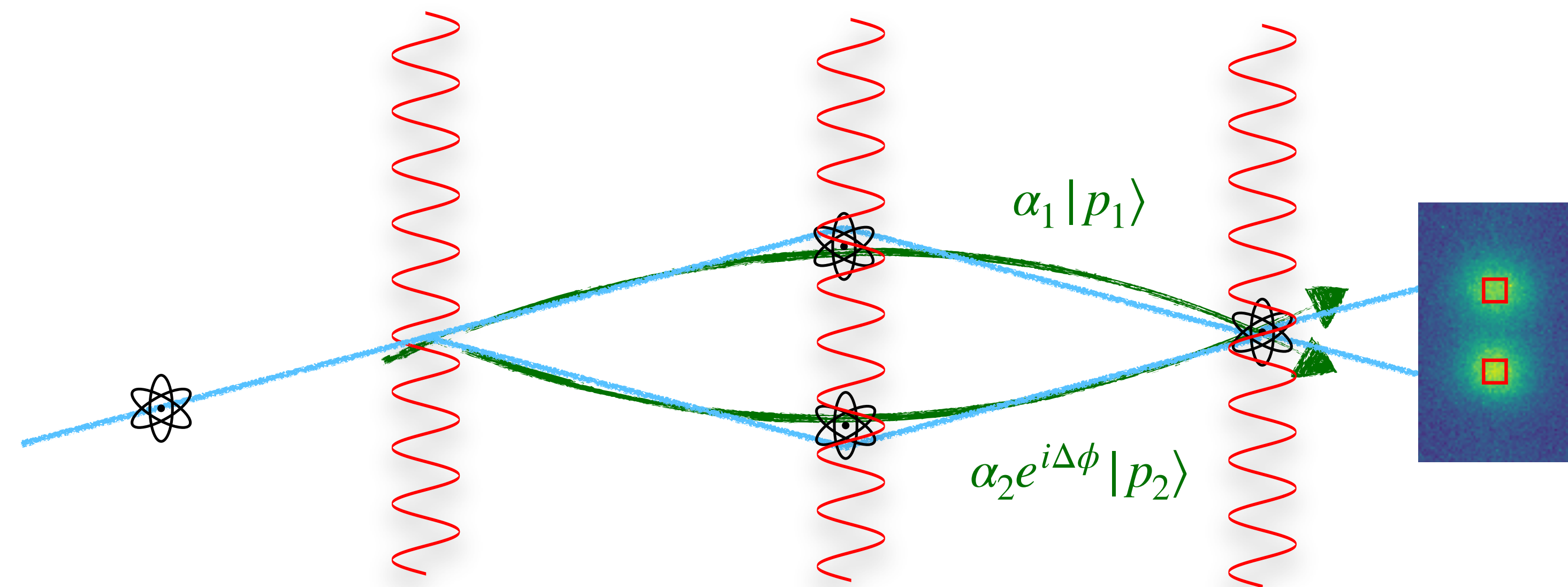
Projection

$$N_1 = |\alpha_1 + \alpha_2 e^{i\Delta\phi}|^2$$

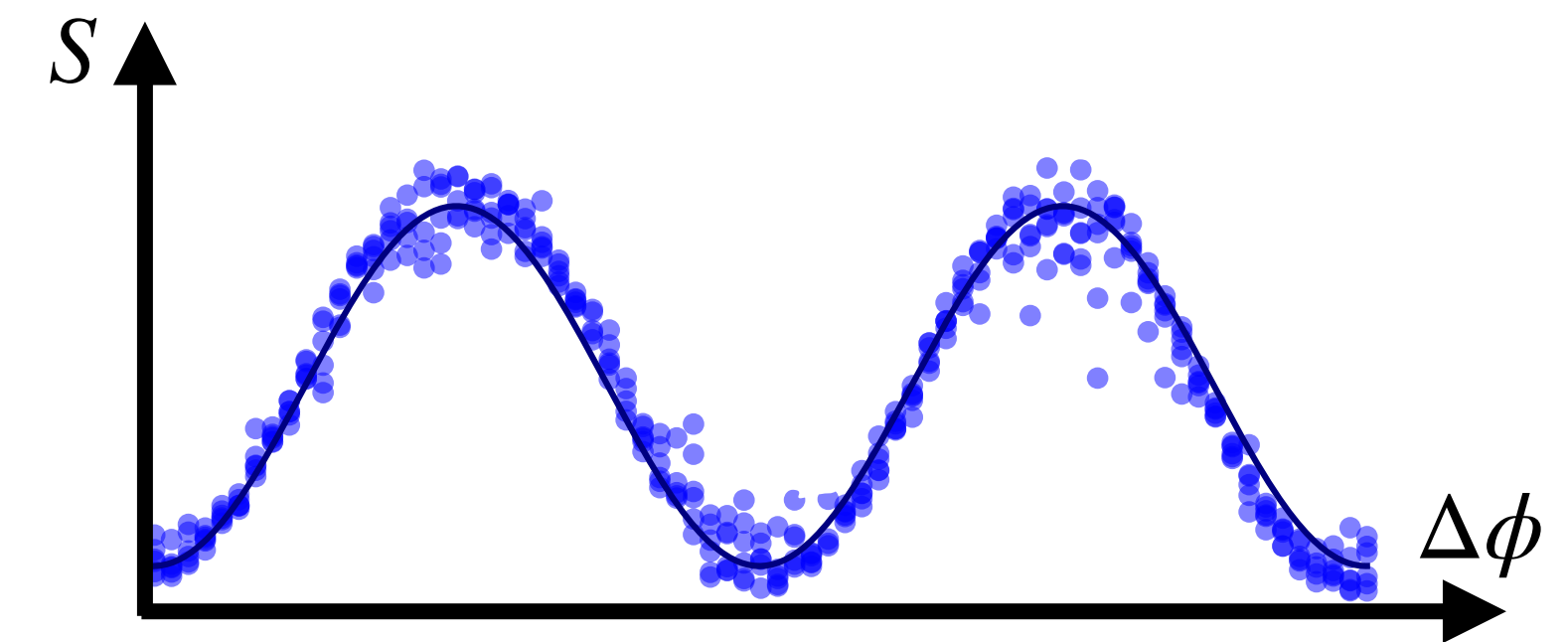


Atom interferometer

Two wave interferometer



$$S = \frac{N_1}{N_1 + N_2} = S_0 \left(1 + V \cos(\Delta\phi) \right)$$



State preparation

$$|p_1\rangle$$

Atom source

cold atoms

Quantum superposition

$$\alpha_1 |p_1\rangle + \alpha_2 e^{i\Delta\phi} |p_2\rangle$$

Atom optics:

optical lattice

Projection

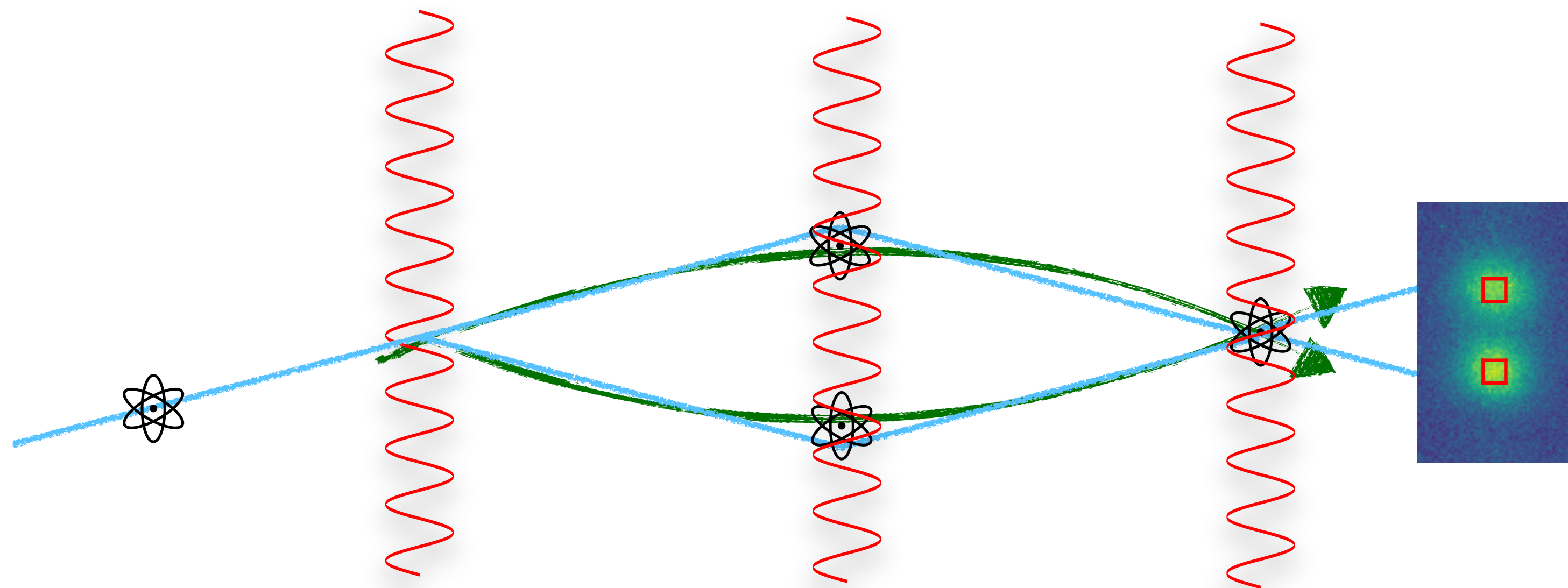
$$N_1 = |\alpha_1 + \alpha_2 e^{i\Delta\phi}|^2$$

Detection:

Fluorescence

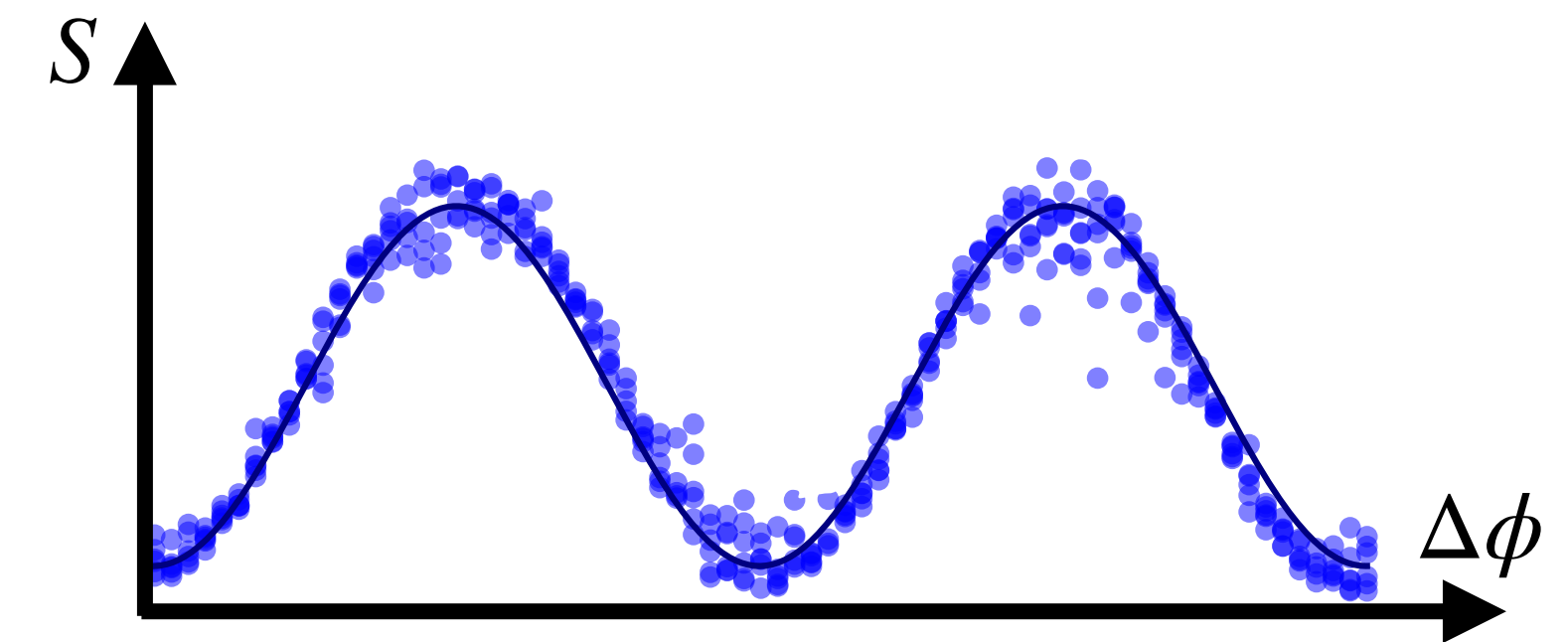
Atom interferometer

A tool for precision measurements



Two wave interferometer: single parameter $\Delta\phi$

$$S = \frac{N_1}{N_1 + N_2} = S_0 \left(1 + V \cos(\Delta\phi) \right)$$

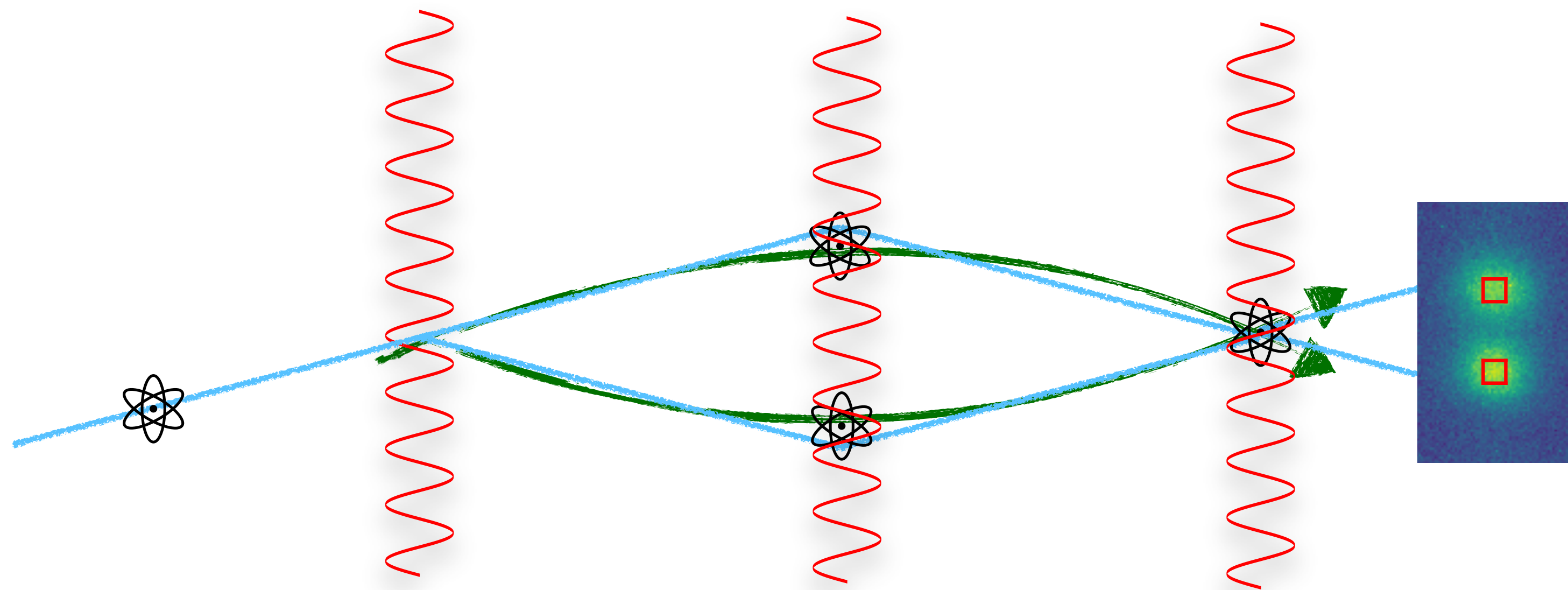


$$\Delta\phi \propto K \cdot M + \phi_{\text{syst.}}$$

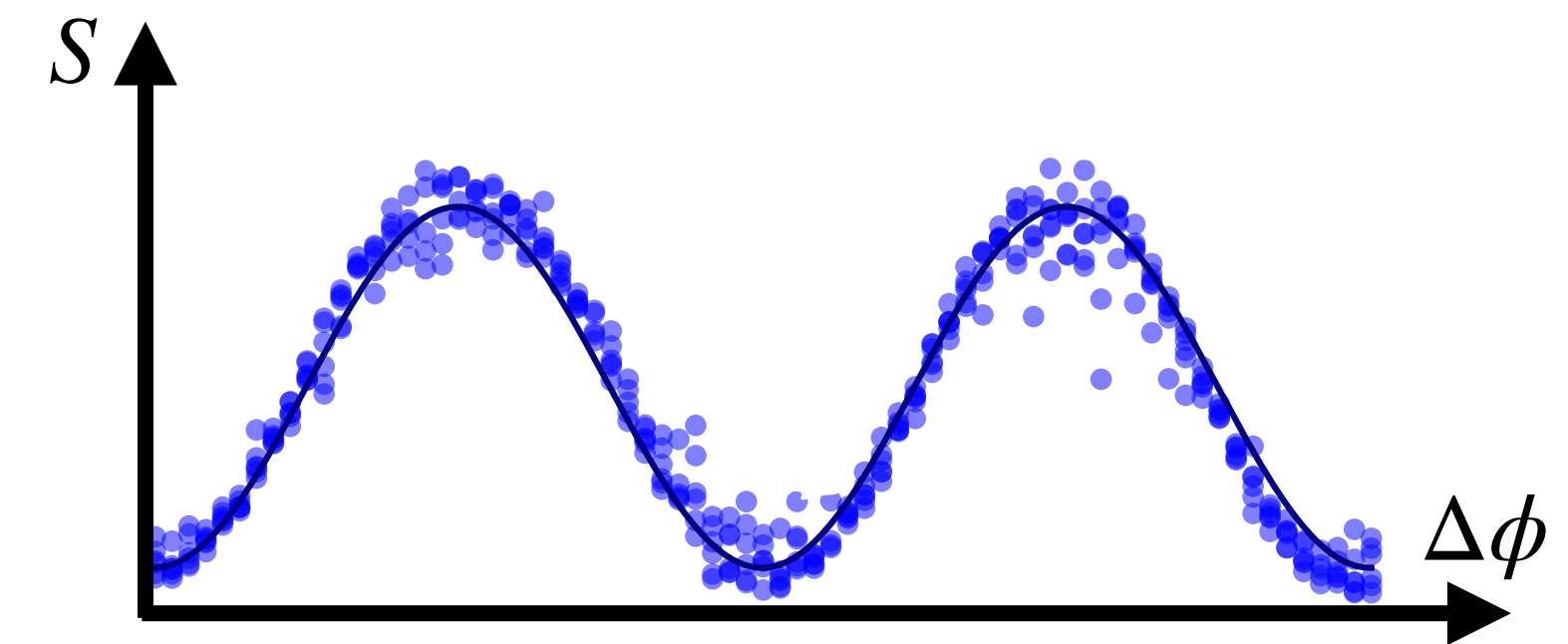
M: Measured quantity

Atom interferometer

A tool for precision measurements



$$S = \frac{N_1}{N_1 + N_2} = S_0 \left(1 + V \cos(\Delta\phi) \right)$$



$$\Delta\phi \propto K \cdot M + \phi_{\text{syst.}}$$

M: Measured quantity

Two wave interferometer: single parameter $\Delta\phi$

Sensitivity = The smallest variation of M that can be measured within a given time frame.

$$\sigma_M = \left(\frac{\partial S}{\partial \Delta\phi} \frac{\partial \Delta\phi}{\partial M} \right)^{-1} \sigma_S$$

Low noise

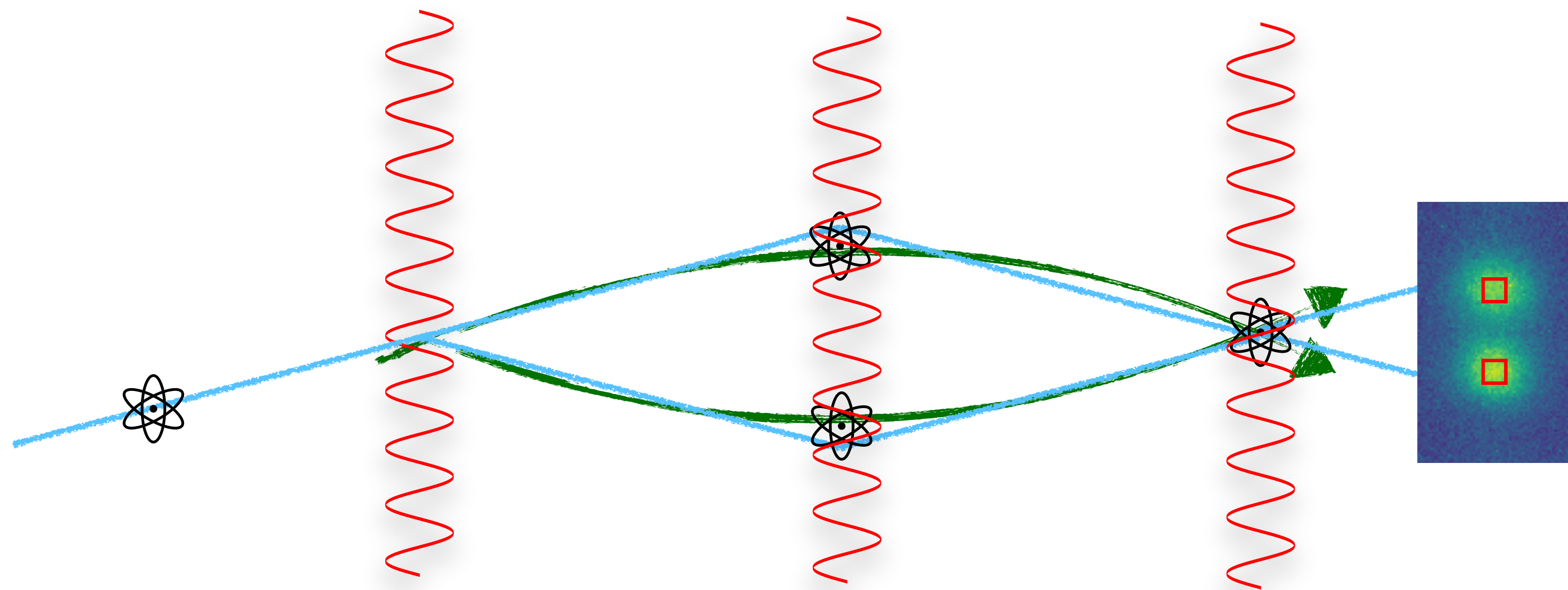
Large scale factor

Accuracy: Knowledge of $\Delta\phi$

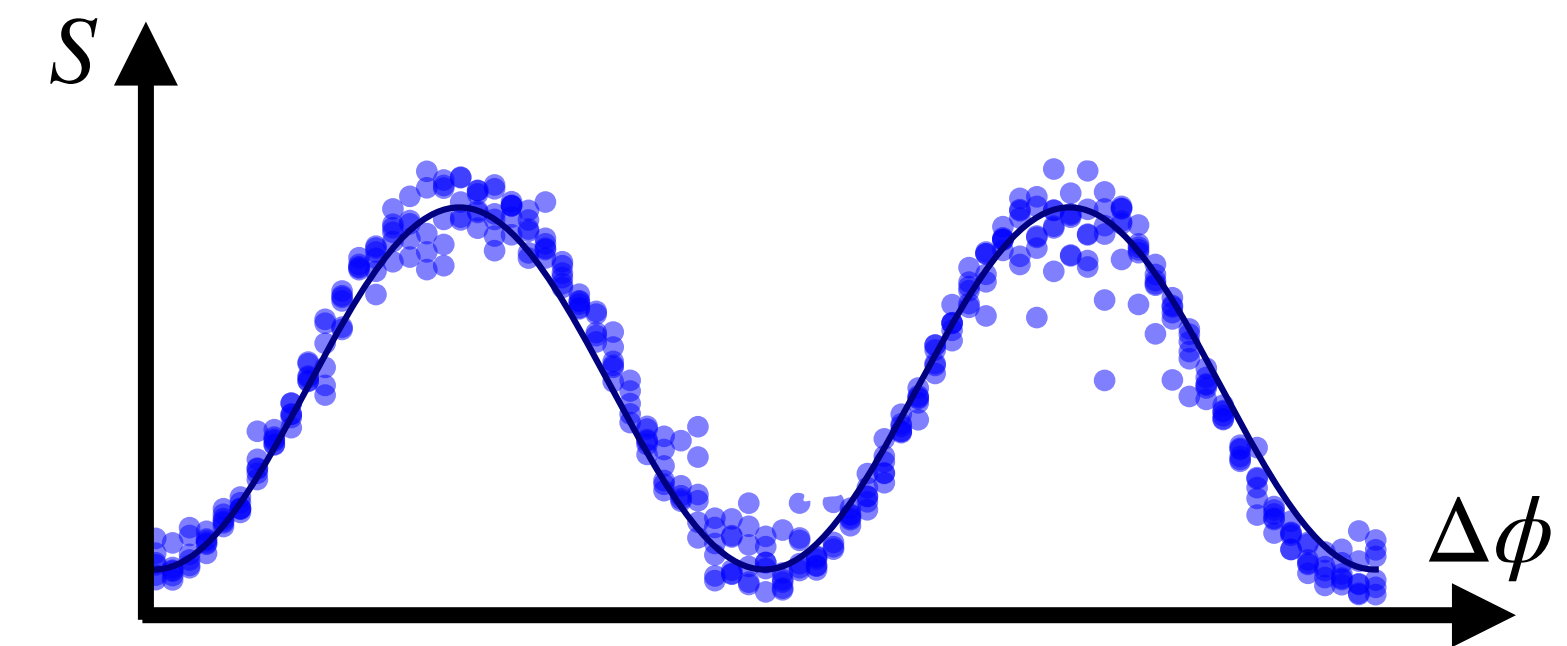
Accurate modeling of
atom interferometers

Atom interferometer

A tool for precision measurements



$$S = \frac{N_1}{N_1 + N_2} = S_0 \left(1 + V \cos(\Delta\phi) \right)$$



$$\Delta\phi \propto K \cdot M + \phi_{\text{syst.}}$$

Two wave interferometer: single parameter $\Delta\phi$

M: Measured quantity

Sensitivity = The smallest variation of M that can be measured within a given time frame.

$$\sigma_M = \left(\frac{\partial S}{\partial \Delta\phi} \frac{\partial \Delta\phi}{\partial M} \right)^{-1} \sigma_S$$

Low noise

Large scale factor

Accuracy: Known accuracy of $\Delta\phi$

QUANTUM ADVANTAGE!

Modeling of atom interferometers

Part I accuracy: Phase shift calculations

Part II sensitivity: LMT atom interferometer

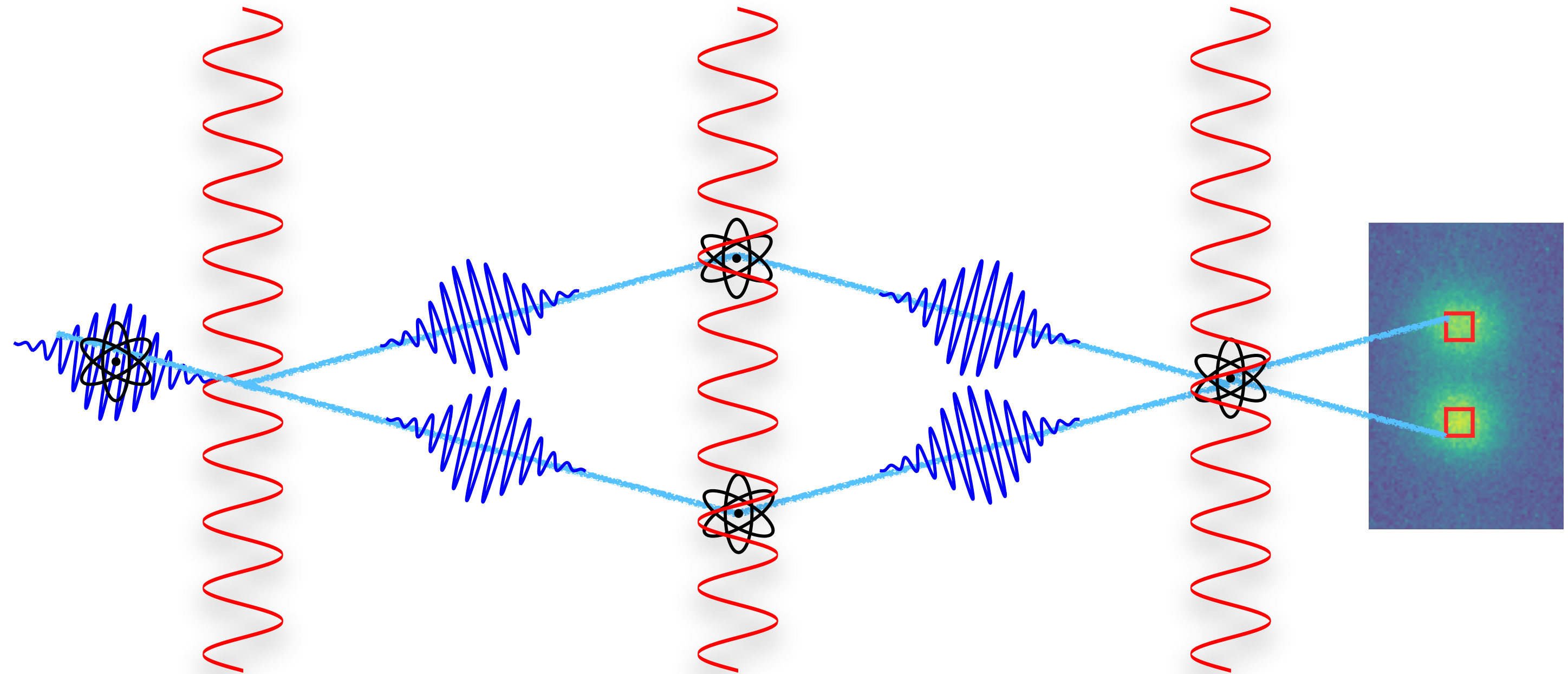
Atom Interferometry

Lecture 1: phase shift calculations

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Outline: Phase shift calculations

1. Atom Optics
2. Wave Packet propagation
3. Phase shift formulae
4. Applications



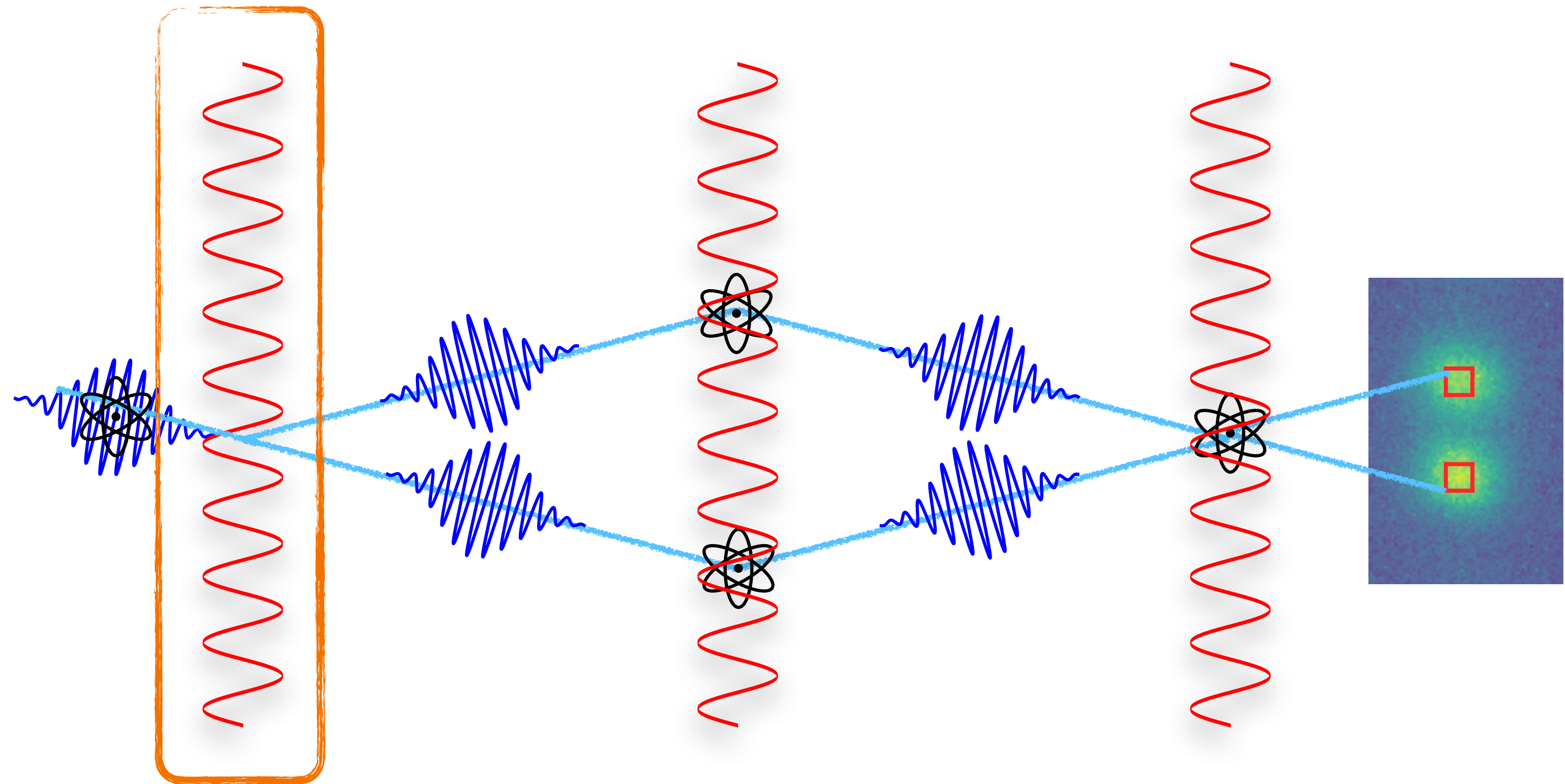
Outline: Phase shift calculations

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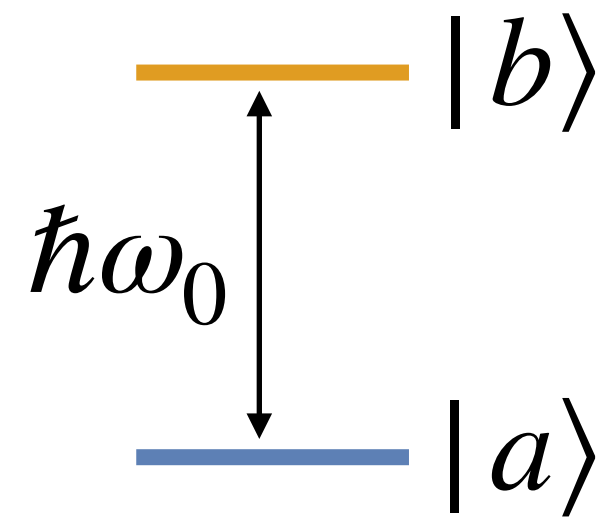
The standard model of atom optics

2-level atom in free space

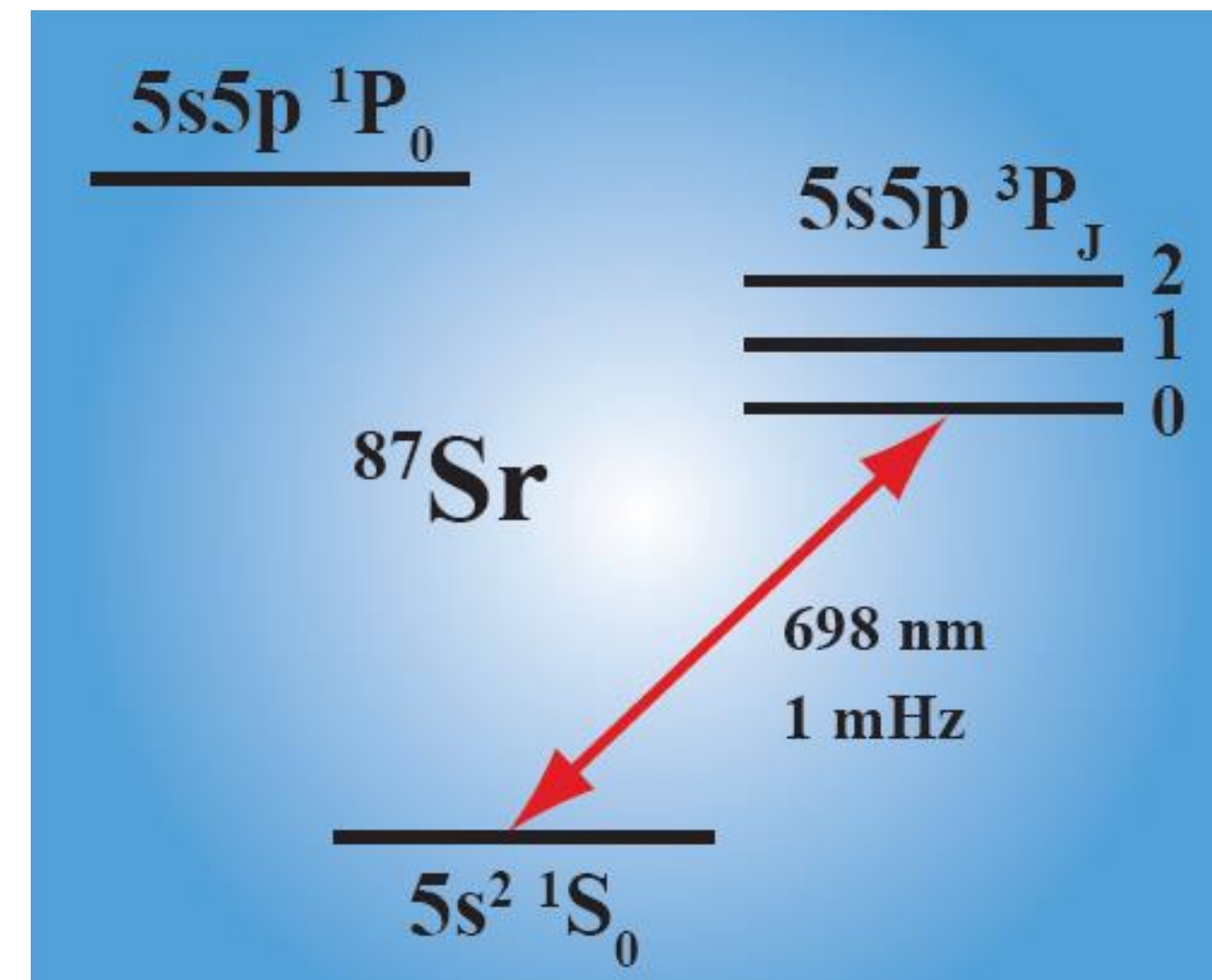
📌 **Internal variables:** (electronic, angular momentum, ...)

Two internal states: $|a\rangle$ and $|b\rangle$

Bohr frequency: $\omega_0 = (E_b - E_a)/\hbar$

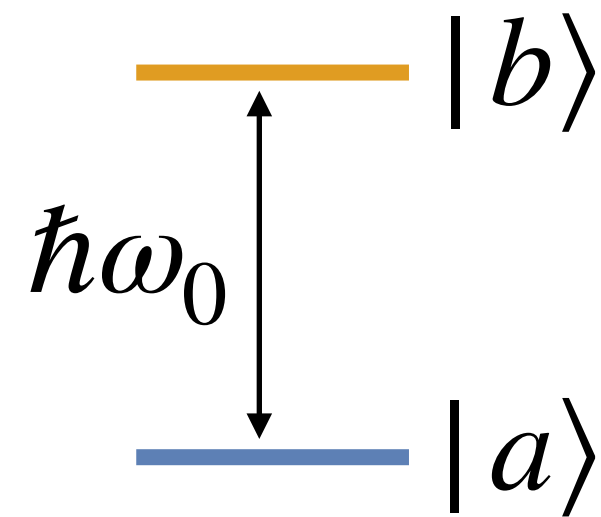


$$H_0 = \hbar\omega_b |b\rangle\langle b|$$



The standard model of atom optics

2-level atom in free space



$$H_0 = \frac{\hat{p}^2}{2M} + \hbar\omega_b |b\rangle\langle b|$$

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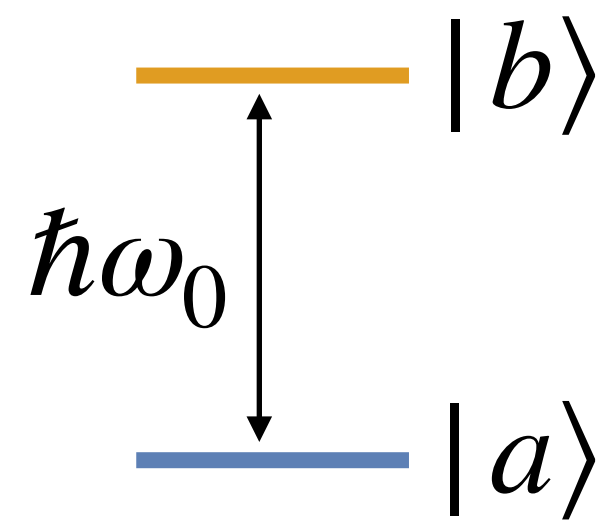
Bohr frequency: $\omega_0 = (E_b - E_a)/\hbar$

📌 **External variables:** Center of mass $\hat{r}$ and $\hat{p}$

Kinetic energy $\frac{\hat{p}^2}{2M}$

The standard model of atom optics

2-level atom in free space



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Kinetic energy $\frac{\hat{p}^2}{2M}$ $\xrightarrow{\text{eigenstates}}$ Plane wave: $|p\rangle$

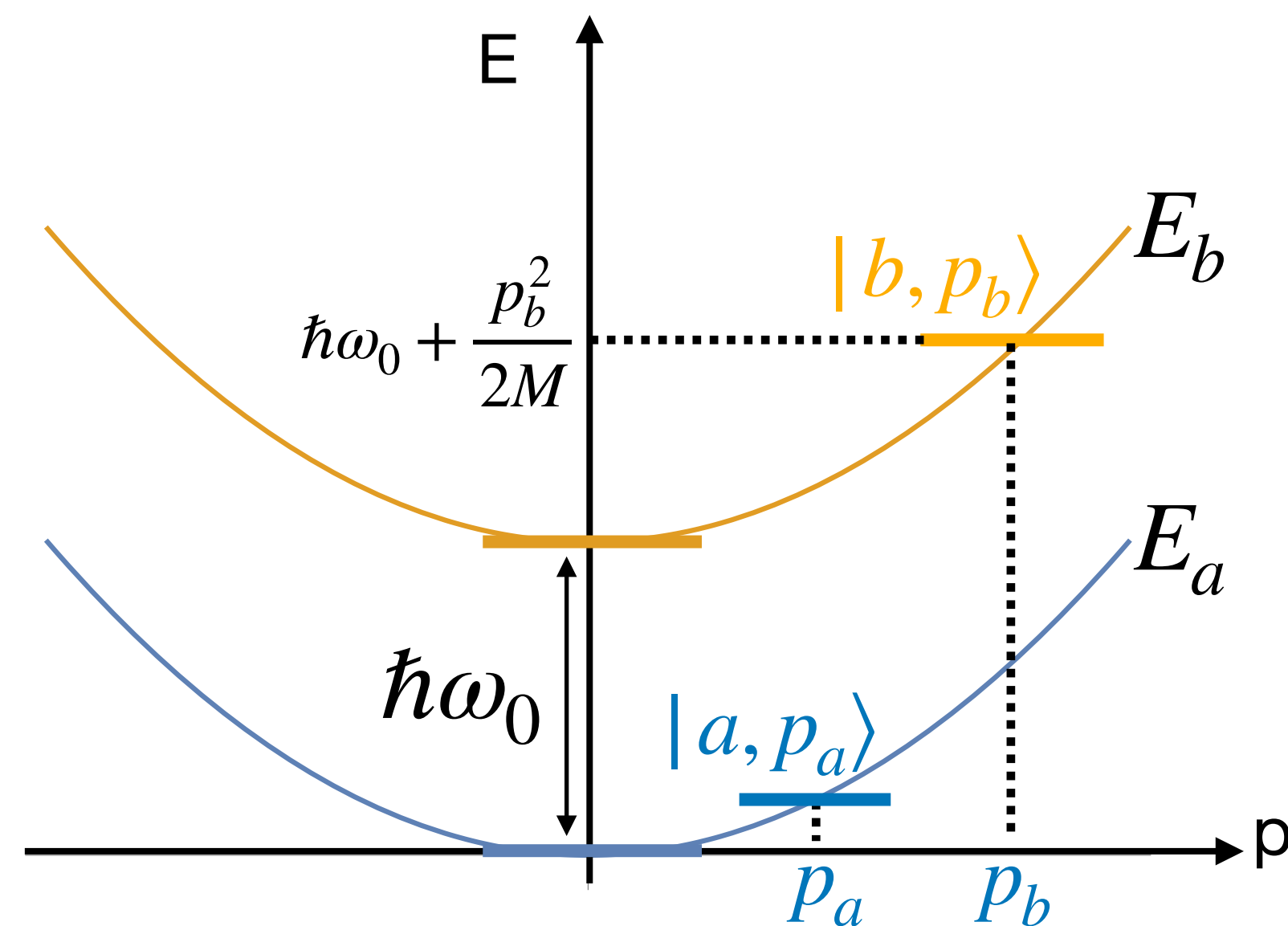
eigenvalues $\xrightarrow{\hspace{1cm}}$ $\frac{p^2}{2M}$

$$H_0 = \frac{\hat{p}^2}{2M} + \hbar\omega_b |b\rangle\langle b|$$

Uncoupled eigenstate $|i, p\rangle$

The standard model of atom optics

2-level atom in free space



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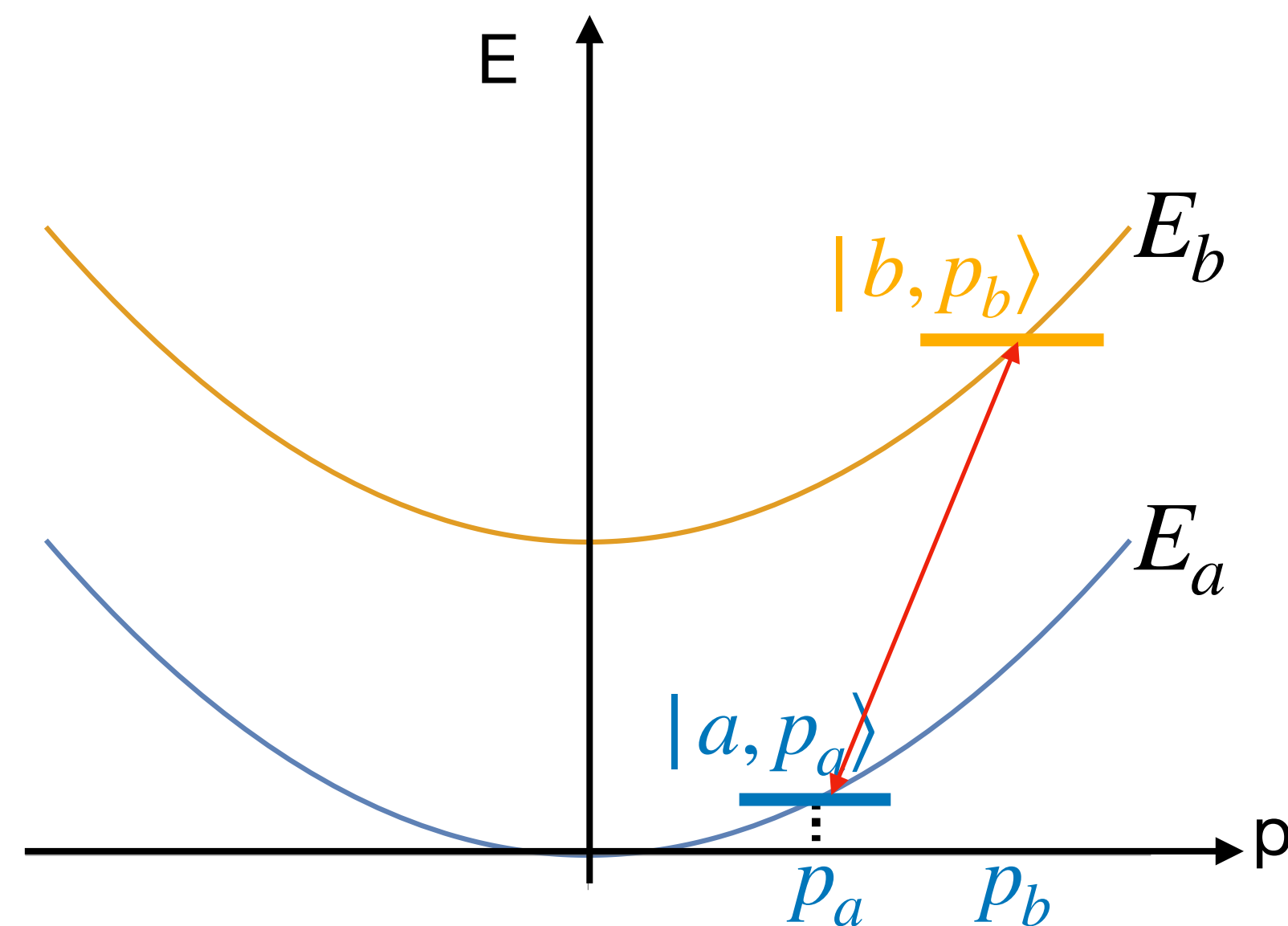
$\xrightarrow{\text{eigenvalues}}$ $\frac{p^2}{2M}$

$$H_0 = \frac{\hat{p}^2}{2M} + \hbar\omega_b |b\rangle\langle b|$$

Uncoupled eigenstate $|i, p\rangle$

The standard model of atom optics

2-level atom + classical light field



- 📌 Coupling atom - classical e.m. field

$$\mathbf{E}(z, t) = \frac{1}{2} \mathbf{e} E_0 e^{i(kz - \omega t - \phi_L)} + \text{c.c.}$$

- 📌 Atom-Laser interaction in the Rotating Wave Approximation

$$V_{AL} = -\mathbf{D} \cdot \mathbf{E} \approx \frac{\hbar \Omega}{2} \left(\underbrace{|b\rangle\langle a|}_{\text{absorption}} e^{i(k\hat{z} - \omega t - \phi_L)} + \underbrace{|a\rangle\langle b|}_{\text{emission}} e^{-i(k\hat{z} - \omega t - \phi_L)} \right)$$

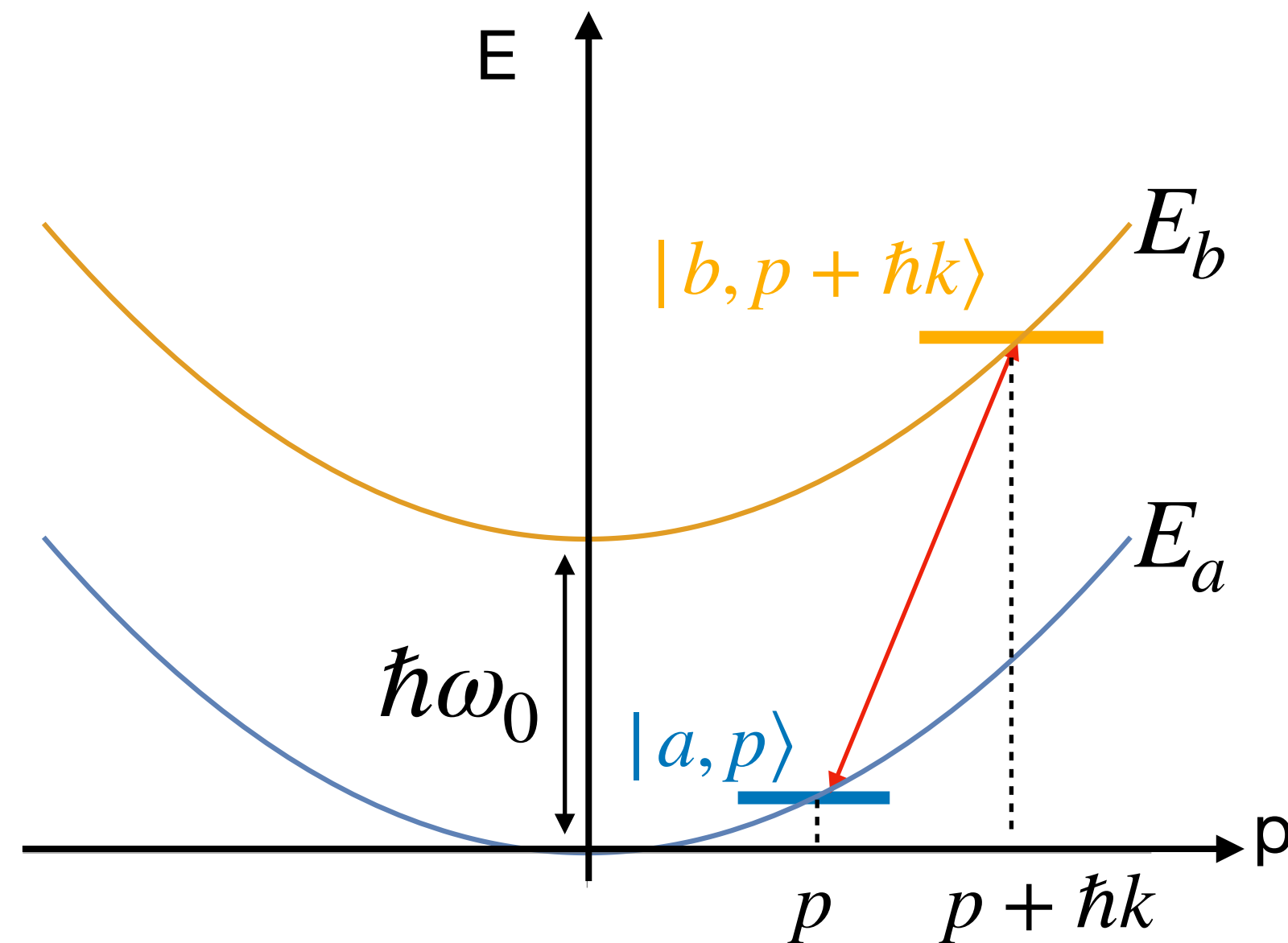
$$H = \frac{\hat{p}^2}{2m} + \hbar \omega_b |b\rangle\langle b| + V_{AL}$$

$$\text{Rabi frequency: } \Omega = \frac{-\mathbf{e} \cdot \mathbf{D} E_0}{\hbar} \text{ and } \mathbf{D} = \langle a | \mathbf{d} | b \rangle$$

🧐 homework: Show the RWA Hamiltonian. hint : use the unitary transform $U = e^{-i\hat{H}_0 t/\hbar}$

The standard model of atom optics

2-level atom + classical light field



- Initial state: $|a, p\rangle$

- Translation operator: $e^{\pm ik\hat{z}} |p\rangle = |p \pm \hbar k\rangle$

$$\langle b, p' | V_{AL} | a, p \rangle = \langle b, p' | \frac{\hbar\Omega}{2} (e^{-i(\omega t + \phi_L)} e^{ik\hat{z}} | b \rangle \langle a | + h.c.) | a, p \rangle$$

$$= \frac{\hbar\Omega}{2} e^{-i(\omega t + \phi_L)} \langle b, p' | e^{ik\hat{z}} | b, p \rangle$$

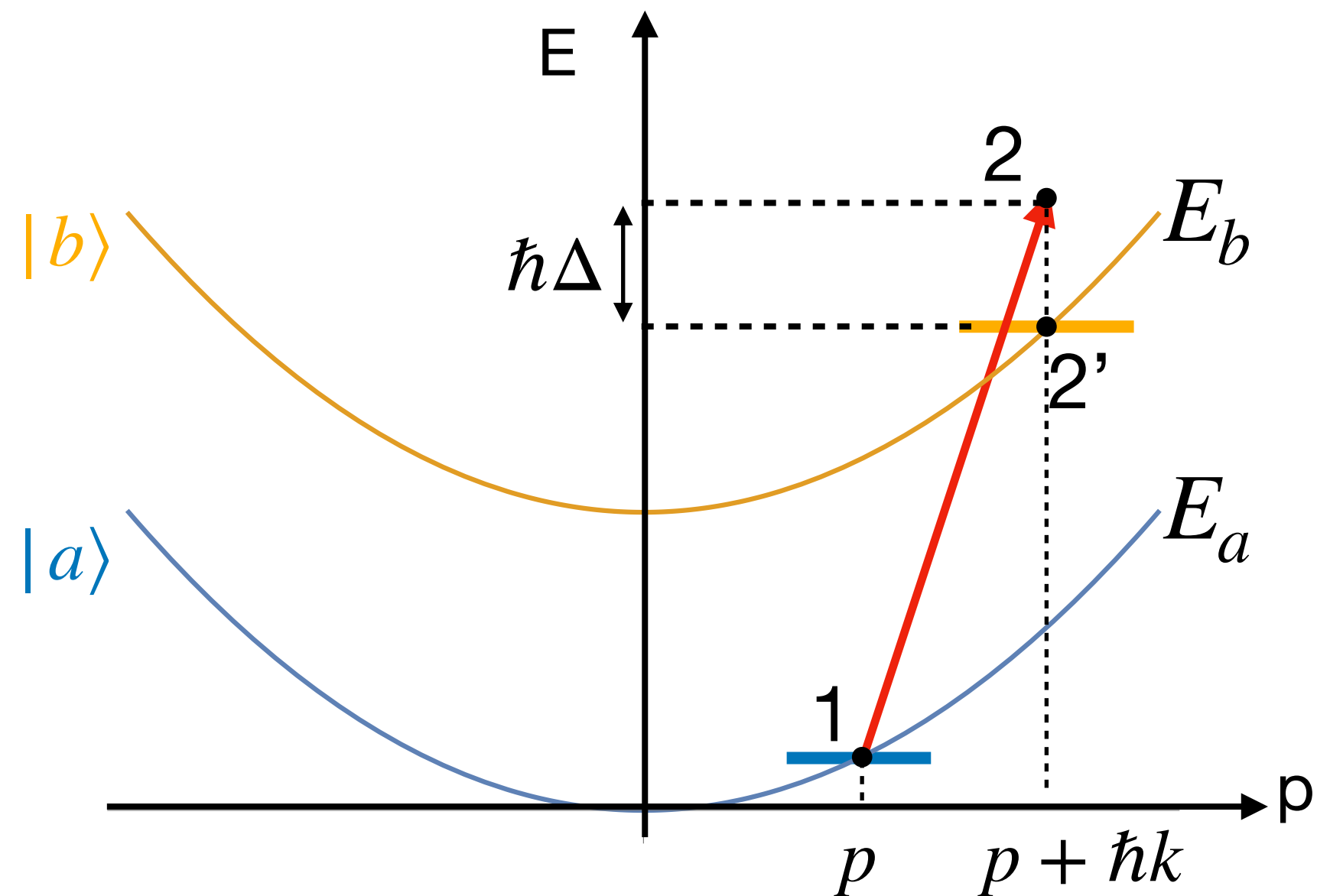
$$= \frac{\hbar\Omega}{2} e^{-i(\omega t + \phi_L)} \langle b, p' | b, p + \hbar k \rangle = \frac{\hbar\Omega}{2} e^{-i(\omega t + \phi_L)} \delta_{p', p + \hbar k}$$

$|a, p\rangle \xleftrightarrow{V_{AL}} |b, p + \hbar k\rangle$

2-level system: $|\Psi\rangle = \alpha(I, \tau) |a, p\rangle + \beta(I, \tau) |b, p + \hbar k\rangle e^{i\phi_L}$

The standard model of atom optics

Conservation laws

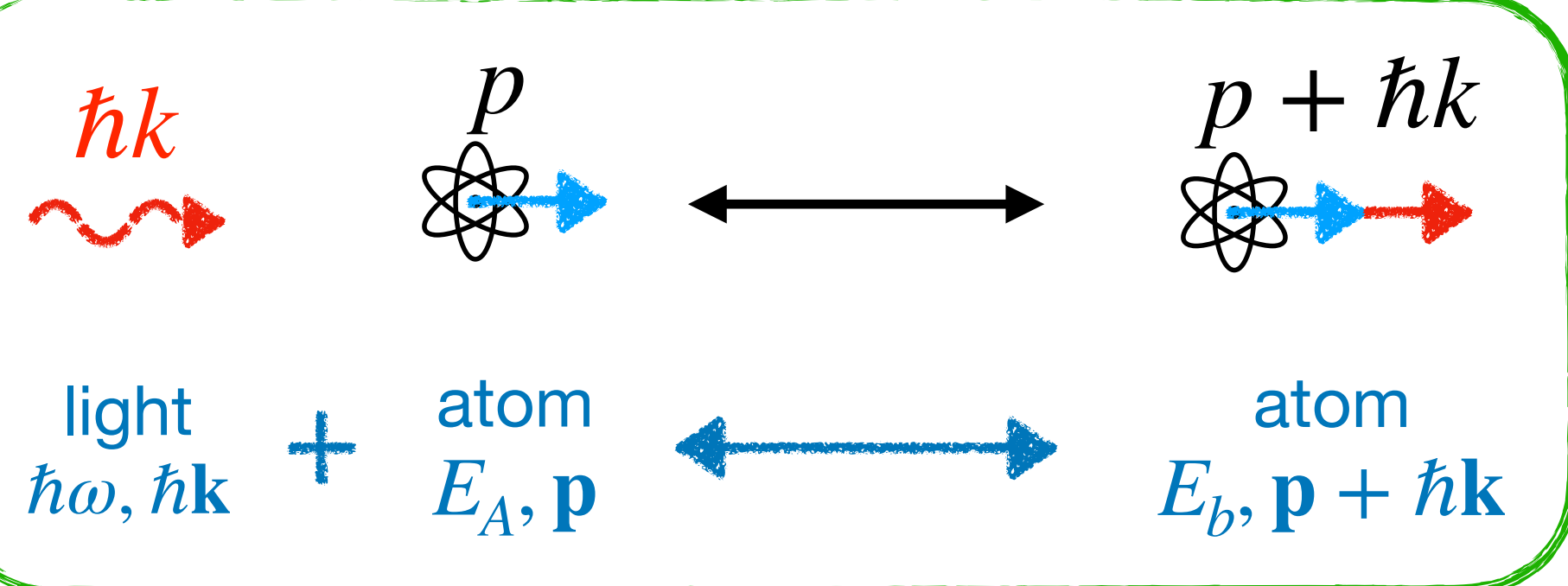


• Photon: line $E = cp$ $p_2 - p_1 = \hbar k$ $E_2 - E_1 = \hbar\omega$

• Momentum conservation : $p \rightarrow p + \hbar k$

• Energy conservation:

$$\hbar\omega + \frac{p^2}{2M} = \hbar\Delta + \hbar\omega_0 + \frac{(p + \hbar k)^2}{2M}$$



$$\Delta = \omega - \omega_0 - \mathbf{k} \frac{\mathbf{p}}{M} - \frac{\hbar k^2}{2M} = \delta - \omega_D - \omega_R$$

↙ Doppler ↘ Photon recoil

The standard model of atom optics

Interaction picture

$$H = \frac{\hat{p}^2}{2m} + \hbar\omega_b |b\rangle\langle b| + \frac{\hbar\Omega}{2} \left(|b\rangle\langle a| e^{i(k\hat{z} - \omega t - \phi_L)} + |a\rangle\langle b| e^{-i(k\hat{z} - \omega t - \phi_L)} \right)$$

Time-dependent hamiltonian:

$$H = \begin{pmatrix} \frac{p^2}{2M} & \frac{\hbar\Omega}{2} e^{i\omega t} \\ \frac{\hbar\Omega}{2} e^{-i\omega t} & \hbar\omega_0 + \frac{(p + \hbar k)^2}{2M} \end{pmatrix}$$

$$\tilde{H} = U H U^\dagger + i\hbar \dot{U} U^\dagger$$

Unitary transform:

$$U = \begin{pmatrix} e^{i\frac{p^2}{2M}t} & 0 \\ 0 & e^{i(\frac{p^2}{2M} + \omega)t} \end{pmatrix} :$$

Time-independent hamiltonian:

$$\tilde{H} = \begin{pmatrix} 0 & \frac{\hbar\Omega}{2} e^{i\phi_L} \\ \frac{\hbar\Omega}{2} e^{-i\phi_L} & \hbar\Delta \end{pmatrix}$$

The standard model of atom optics

Diagonalisation of $\tilde{H}$

Eigenvalues of $\tilde{H}$: $\lambda_{\pm} = \frac{\hbar\Delta}{2} \pm \frac{\hbar}{2}\sqrt{\Delta^2 + \Omega^2}$

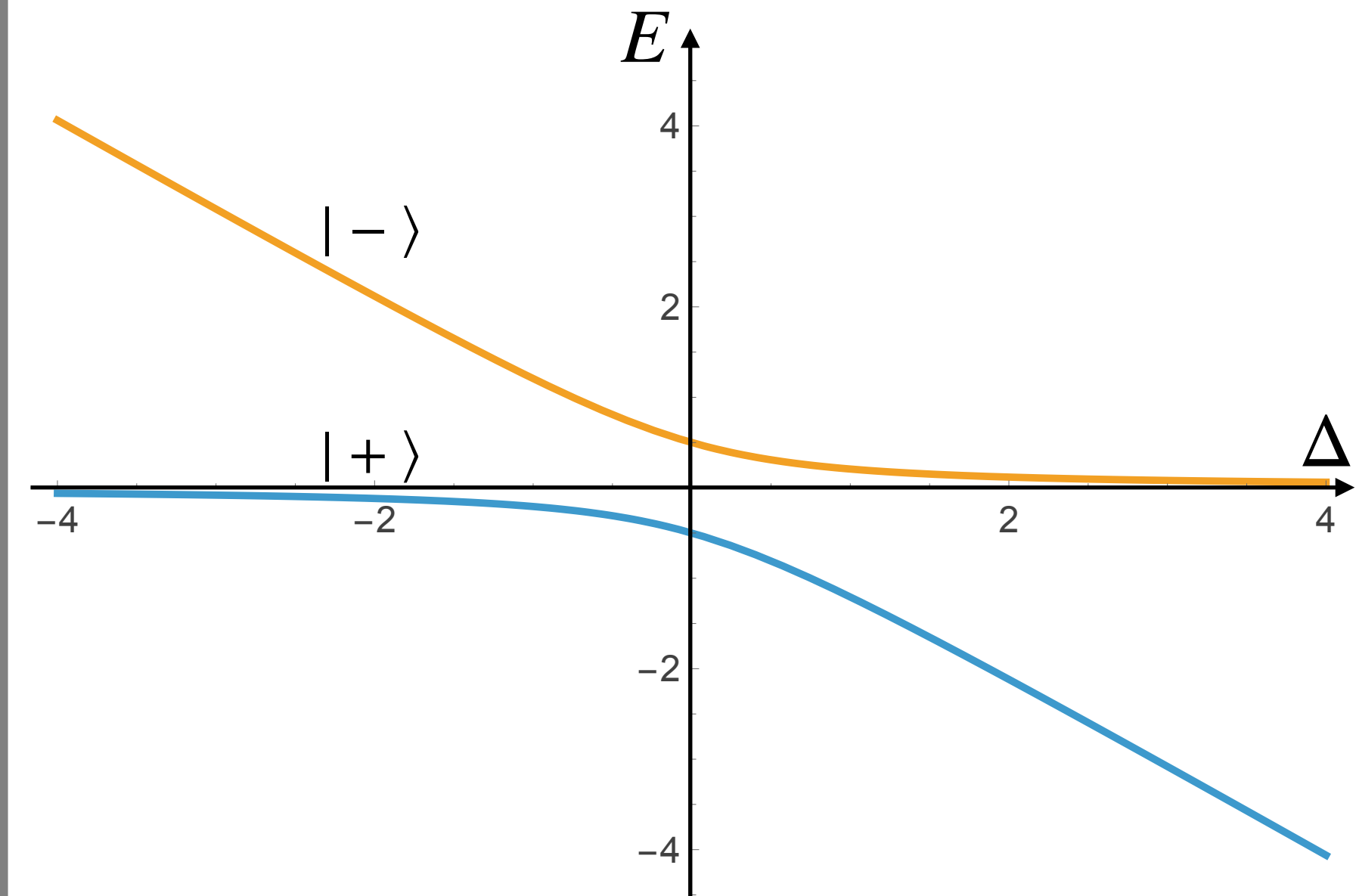
Eigenstates of $\tilde{H}$:

$$\cos 2\theta = \frac{-\Delta}{\sqrt{\Omega^2 + \Delta^2}}$$

$$\sin 2\theta = \frac{\Omega}{\sqrt{\Omega^2 + \Delta^2}}$$

$$|+\rangle = \sin\theta |\tilde{a}\rangle e^{-i\phi/2} + \cos\theta |\tilde{b}\rangle e^{i\phi/2}$$

$$|-\rangle = \cos\theta |\tilde{a}\rangle e^{-i\phi/2} - \sin\theta |\tilde{b}\rangle e^{i\phi/2}$$



The standard model of atom optics

Diagonalisation of $\tilde{H}$

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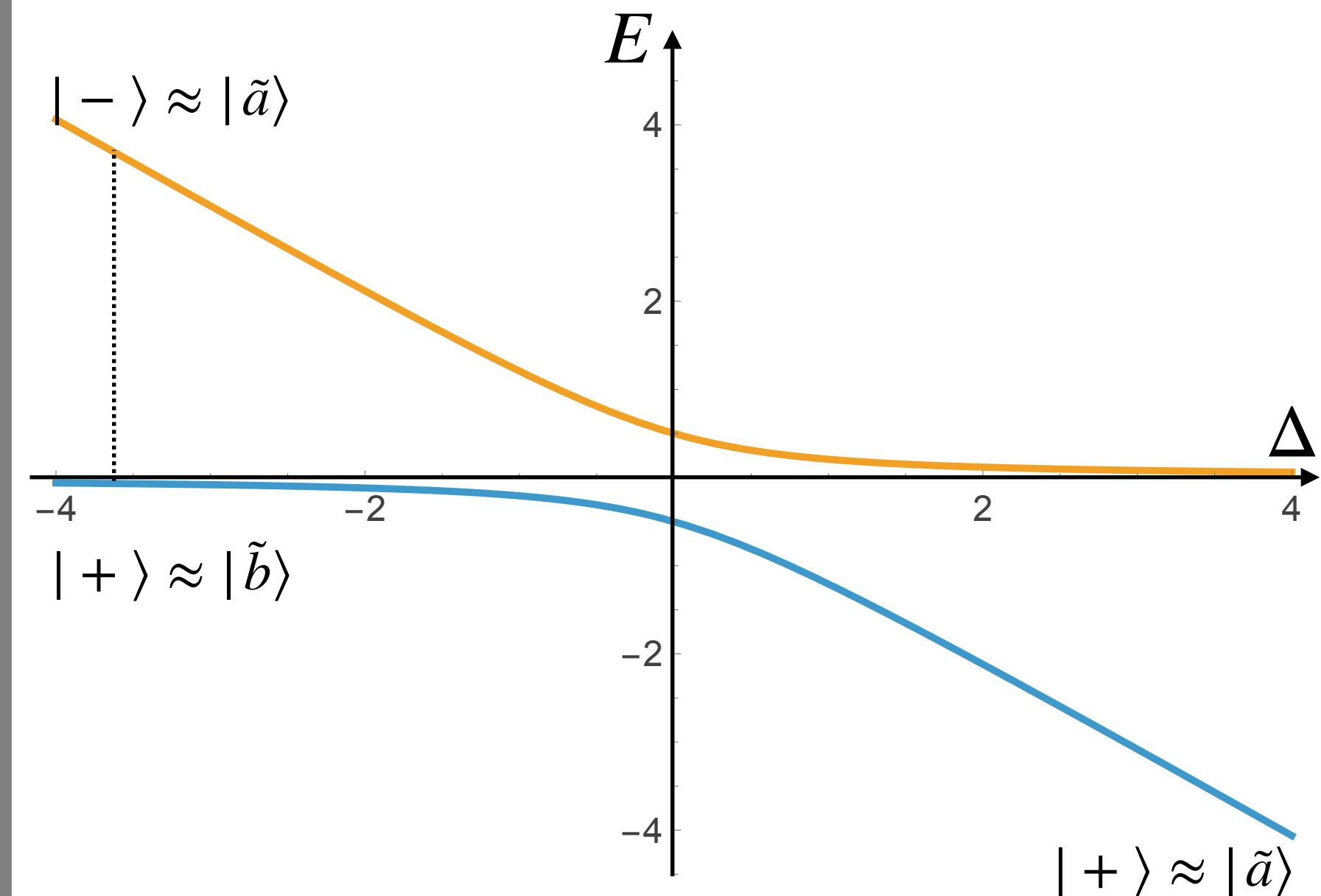
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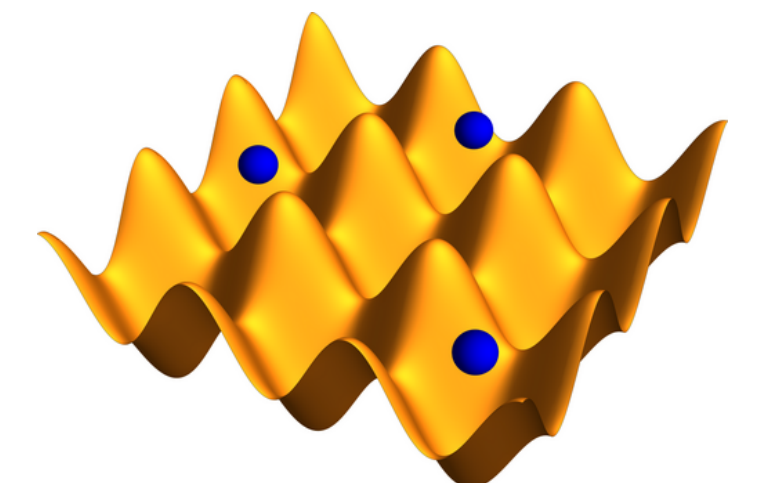
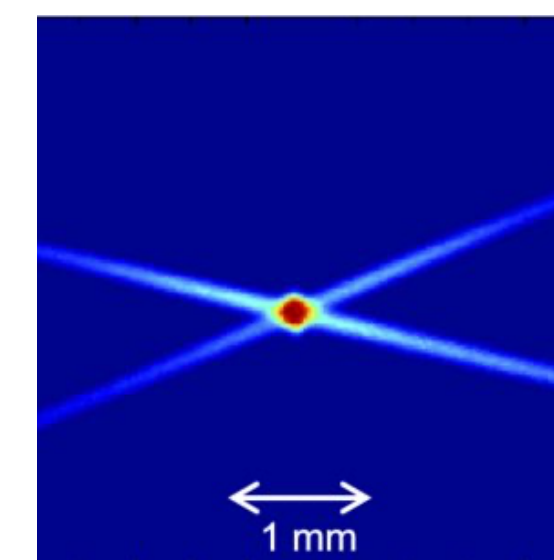
Application: Light shift

$\Delta \gg \Omega \rightarrow$

$$|+\rangle \approx |b\rangle$$

$$|-\rangle \approx |a\rangle$$

$$\approx \frac{\hbar\Omega^2}{4\Delta} \propto I$$



Engineering ground-state energy landscape ²³

The standard model of atom optics

Diagonalisation of $\tilde{H}$

Eigenvalues of $\tilde{H}$: $\lambda_{\pm} = \frac{\hbar\Delta}{2} \pm \frac{\hbar}{2}\sqrt{\Delta^2 + \Omega^2}$

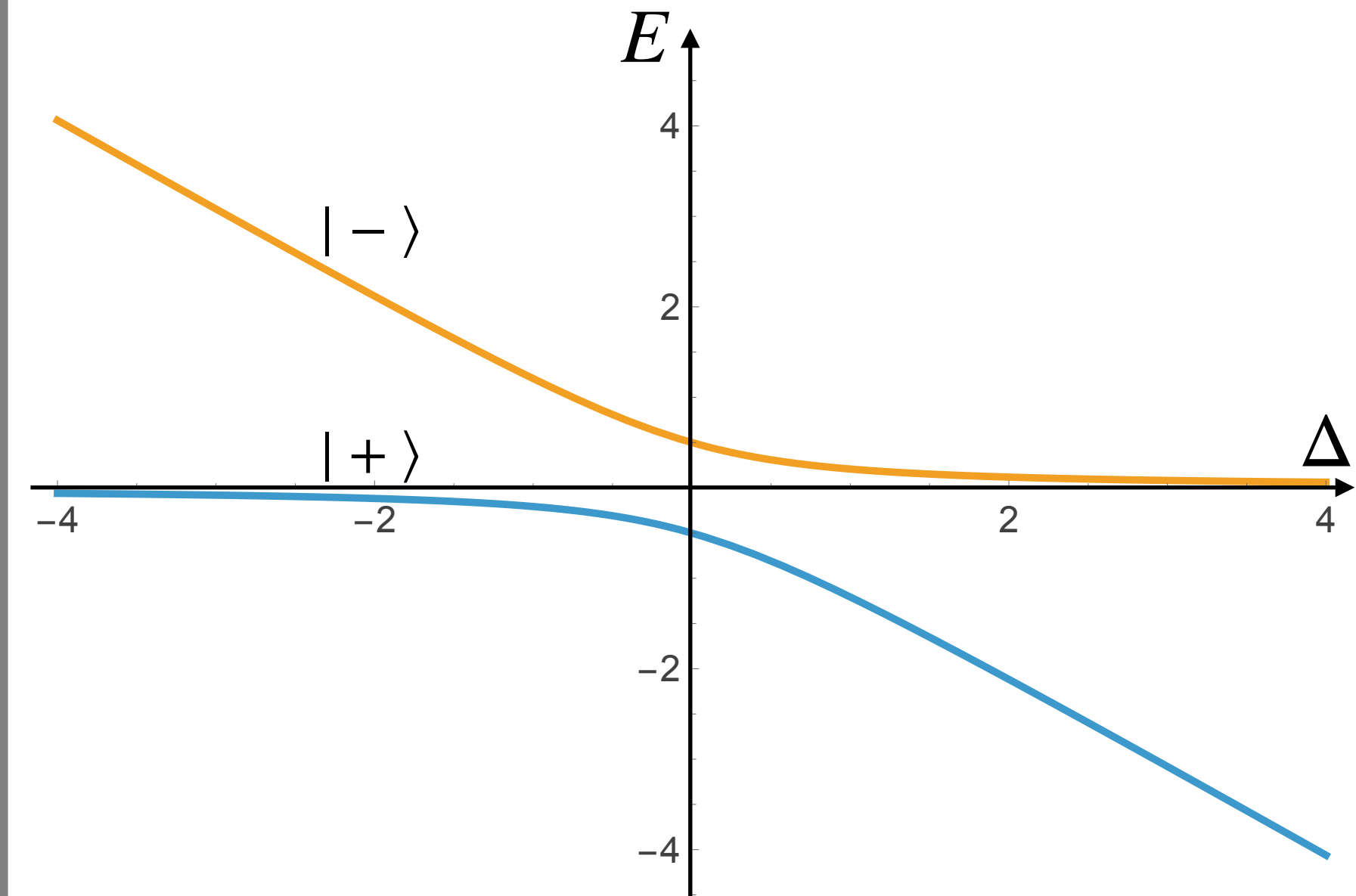
Eigenstates of $\tilde{H}$:

$$\cos 2\theta = \frac{-\Delta}{\sqrt{\Omega^2 + \Delta^2}}$$

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$$|+\rangle = \sin\theta |\tilde{a}\rangle e^{-i\phi/2} + \cos\theta |\tilde{b}\rangle e^{i\phi/2}$$

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Time evolution:

$$|+\rangle \rightarrow e^{-i\frac{\lambda_+ t}{\hbar}} |+\rangle$$

$$|-\rangle \rightarrow e^{-i\frac{\lambda_- t}{\hbar}} |-\rangle$$

The standard model of atom optics

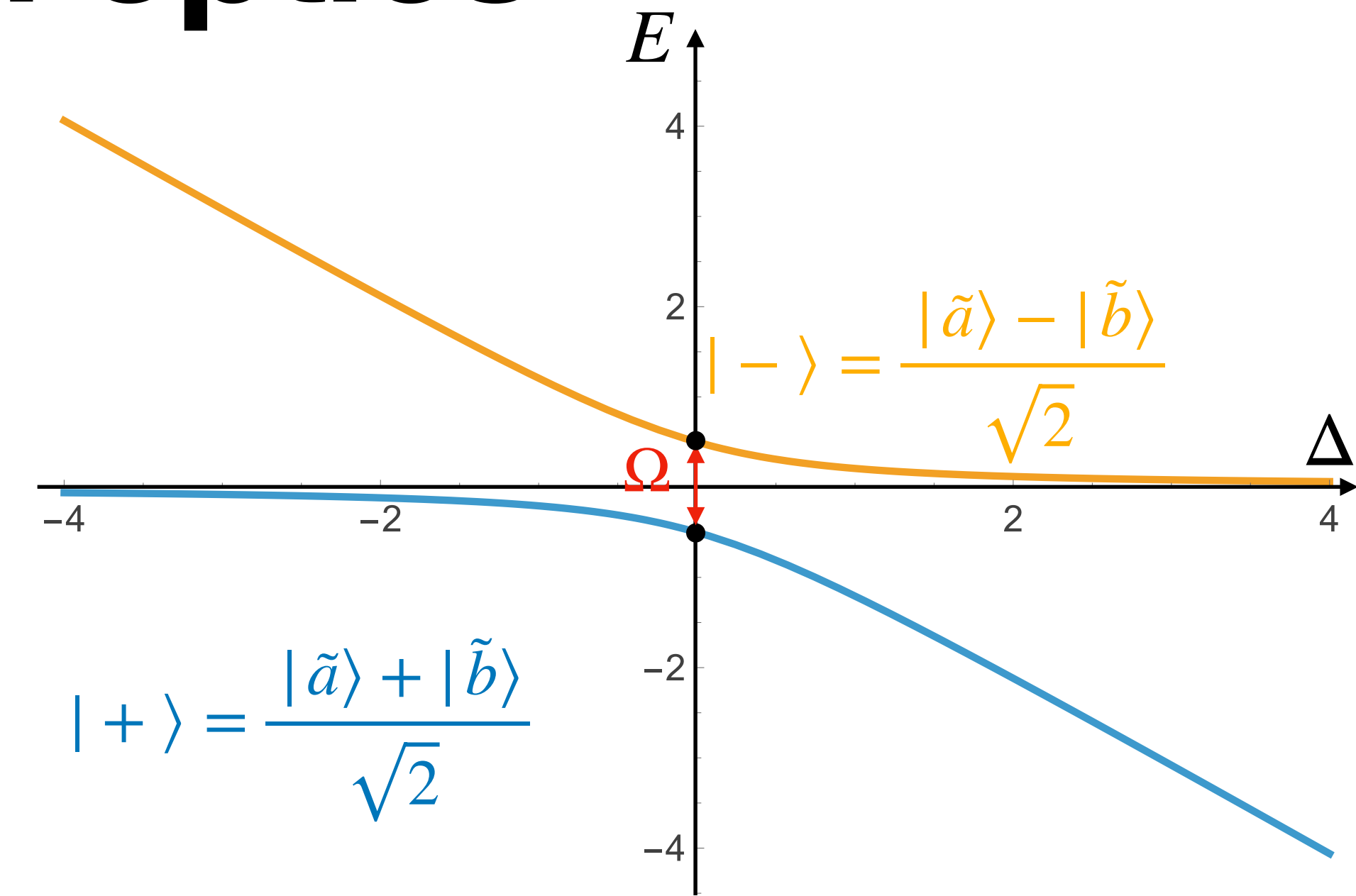
Rabi oscillations

• For $\Delta = 0$: $\lambda_{\pm} = \pm \frac{\hbar\Omega}{2}$ and $|\pm\rangle = \frac{|\tilde{a}\rangle \pm |\tilde{b}\rangle}{\sqrt{2}}$

• Initial : $|\tilde{\Psi}(0)\rangle = |\tilde{a}\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}$

• Time evolution: $|\tilde{\Psi}(t)\rangle = \frac{1}{\sqrt{2}}(|+\rangle e^{i\frac{\Omega}{2}t} + |-\rangle e^{-i\frac{\Omega}{2}t})$

$$= \frac{1}{2}(|\tilde{a}\rangle e^{i\frac{\Omega}{2}t} + |\tilde{b}\rangle e^{i\frac{\Omega}{2}t} + |\tilde{a}\rangle e^{-i\frac{\Omega}{2}t} - |\tilde{b}\rangle e^{-i\frac{\Omega}{2}t}) = \cos\left(\frac{\Omega t}{2}\right) |\tilde{a}\rangle + i \sin\left(\frac{\Omega t}{2}\right) |\tilde{b}\rangle$$



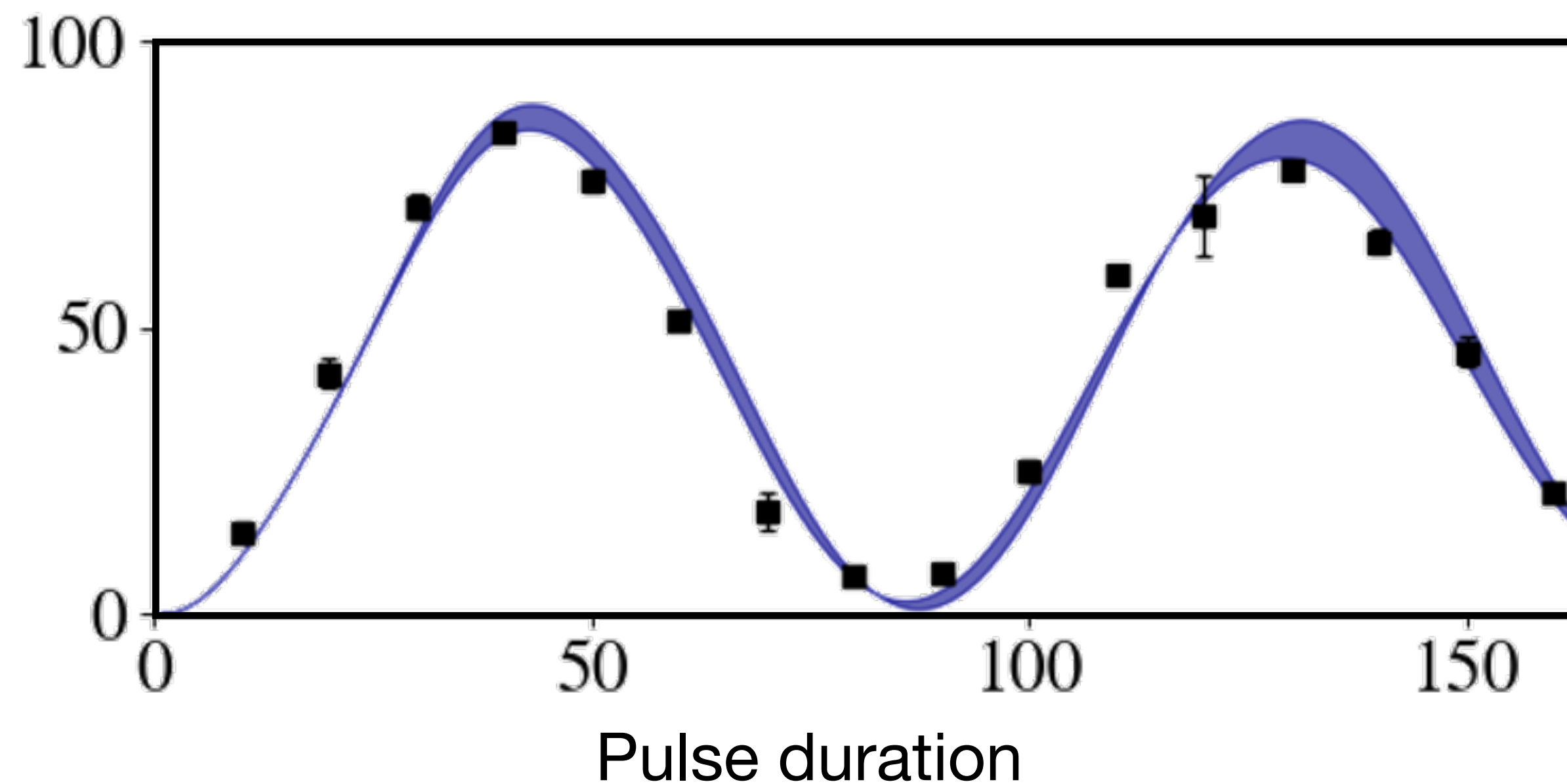
Rabi oscillations $P_{a \rightarrow b} = |\langle \tilde{b} | \tilde{\Psi}(t) \rangle|^2 = \sin^2 \frac{\Omega t}{2}$

Rabi oscillations = Interference between two eigenstates that accumulate different phases.

Atom optics

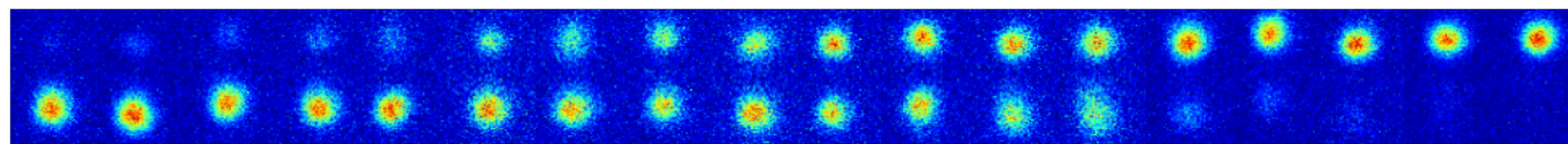
Rabi oscillations between momentum states

$$|\Psi\rangle = U^\dagger |\tilde{\Psi}\rangle = \alpha(I, \tau) |a, p\rangle + \beta(I, \tau) |b, p + \hbar k\rangle e^{i\phi_L}$$



- α and β depend on the laser intensity and duration
- Laser phase ϕ_L on the transferred state
- Transition probability:

$$P_{[p \rightarrow p + \hbar k]} = |\beta|^2 \propto \sin^2\left(\frac{\Omega\tau}{2}\right)$$

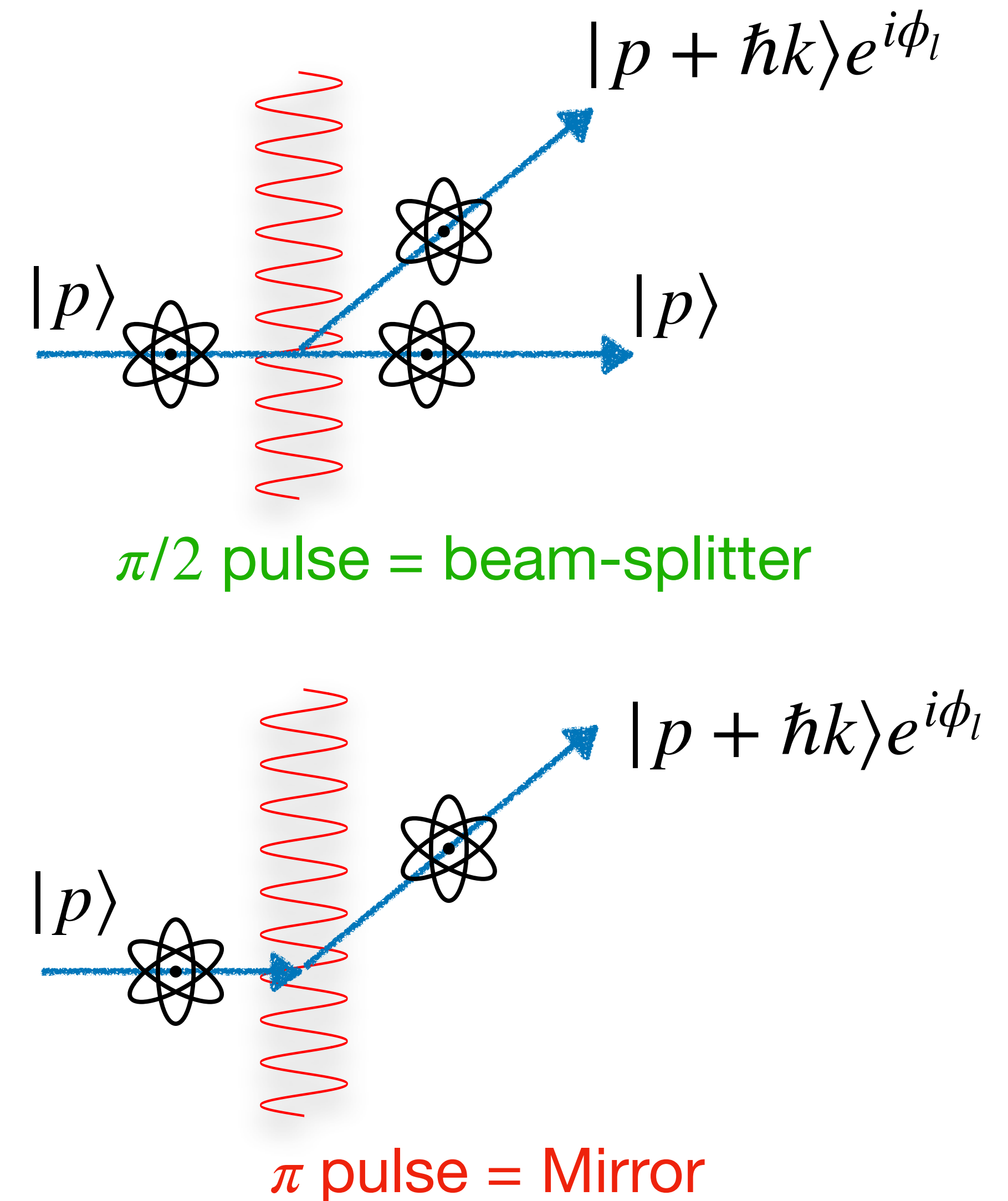
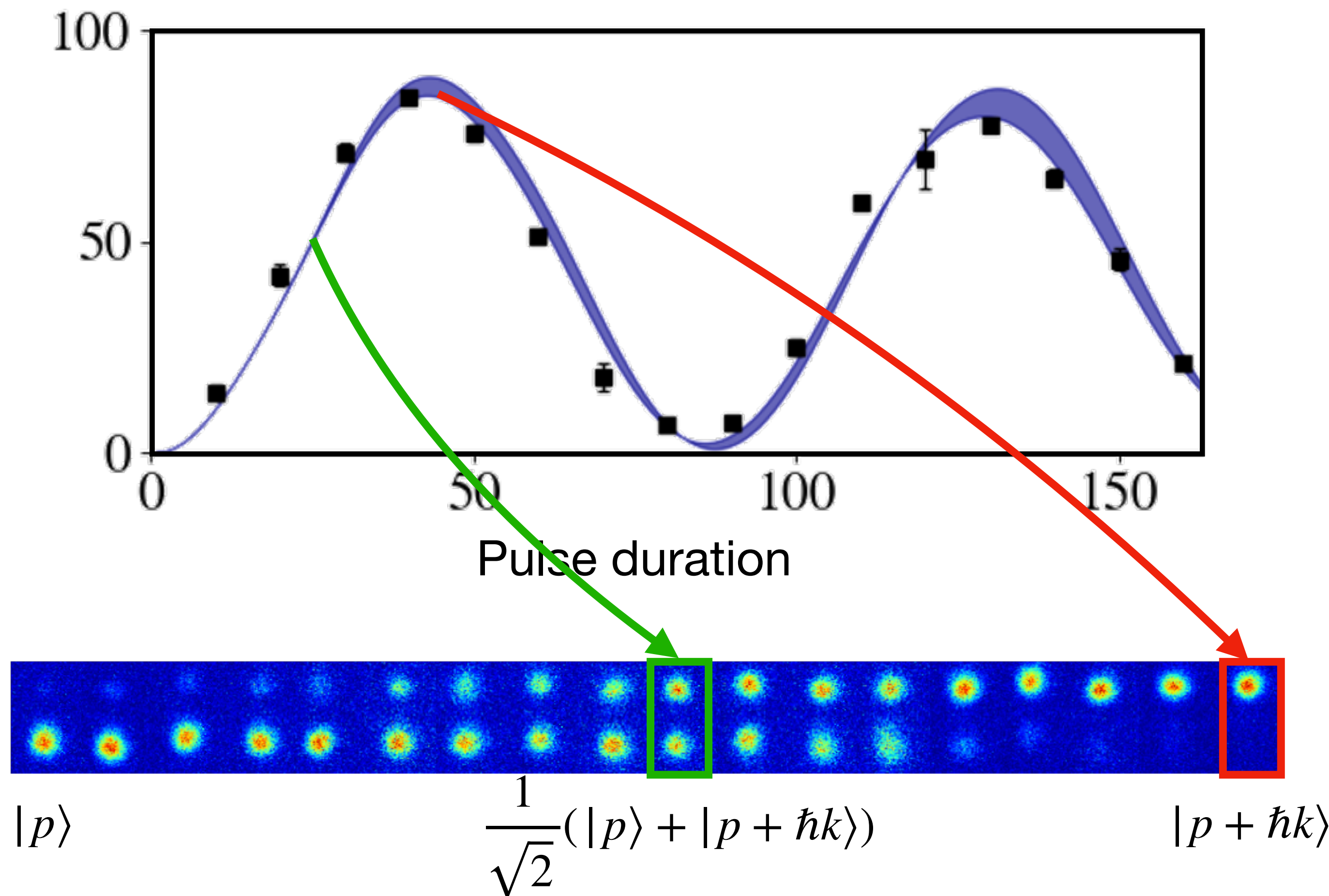


Rabi oscillations

Atom optics

Rabi oscillations between momentum states

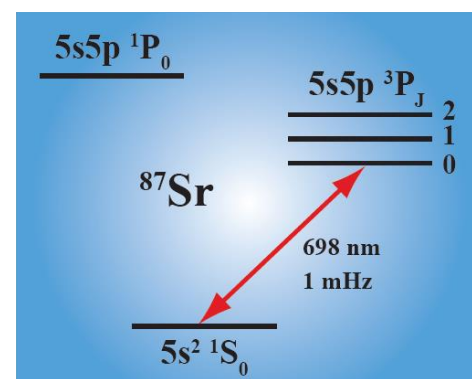
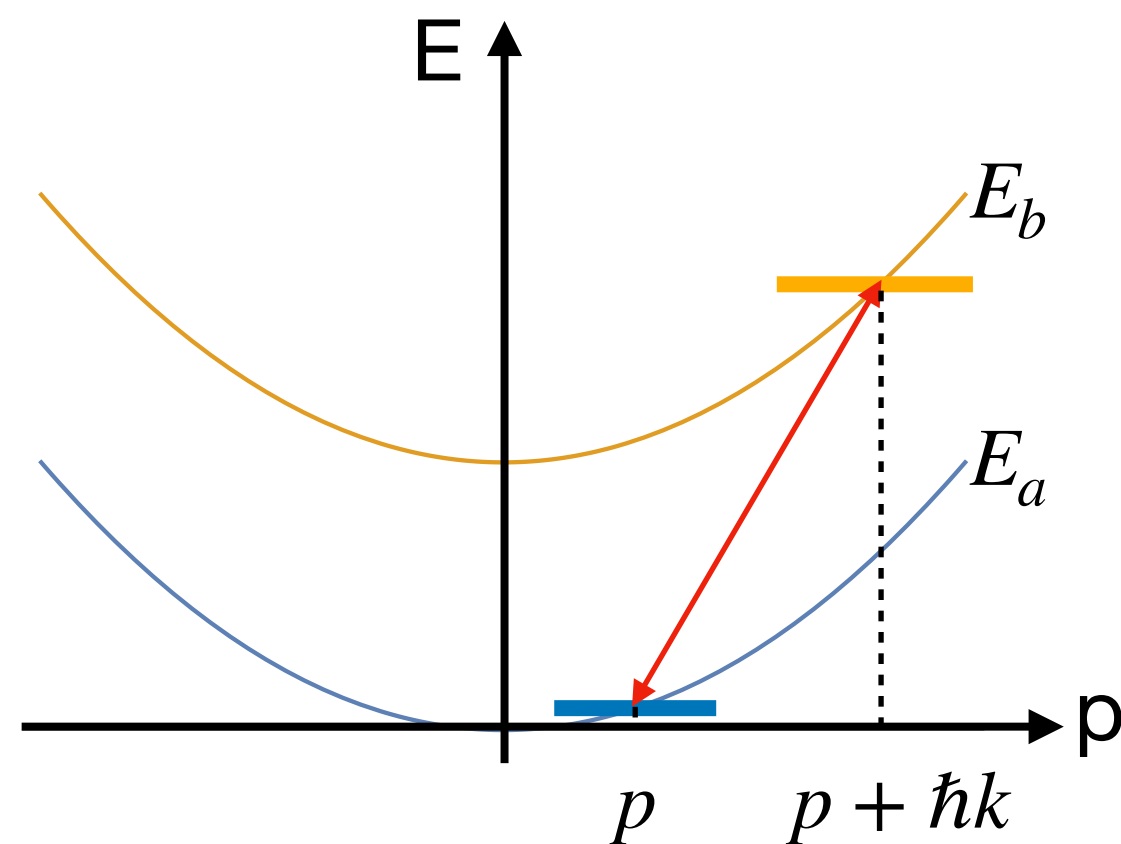
$$|\Psi\rangle = U^\dagger |\tilde{\Psi}\rangle = \alpha(I, \tau) |a, p\rangle + \beta(I, \tau) |b, p + \hbar k\rangle e^{i\phi_L}$$



Atom optics

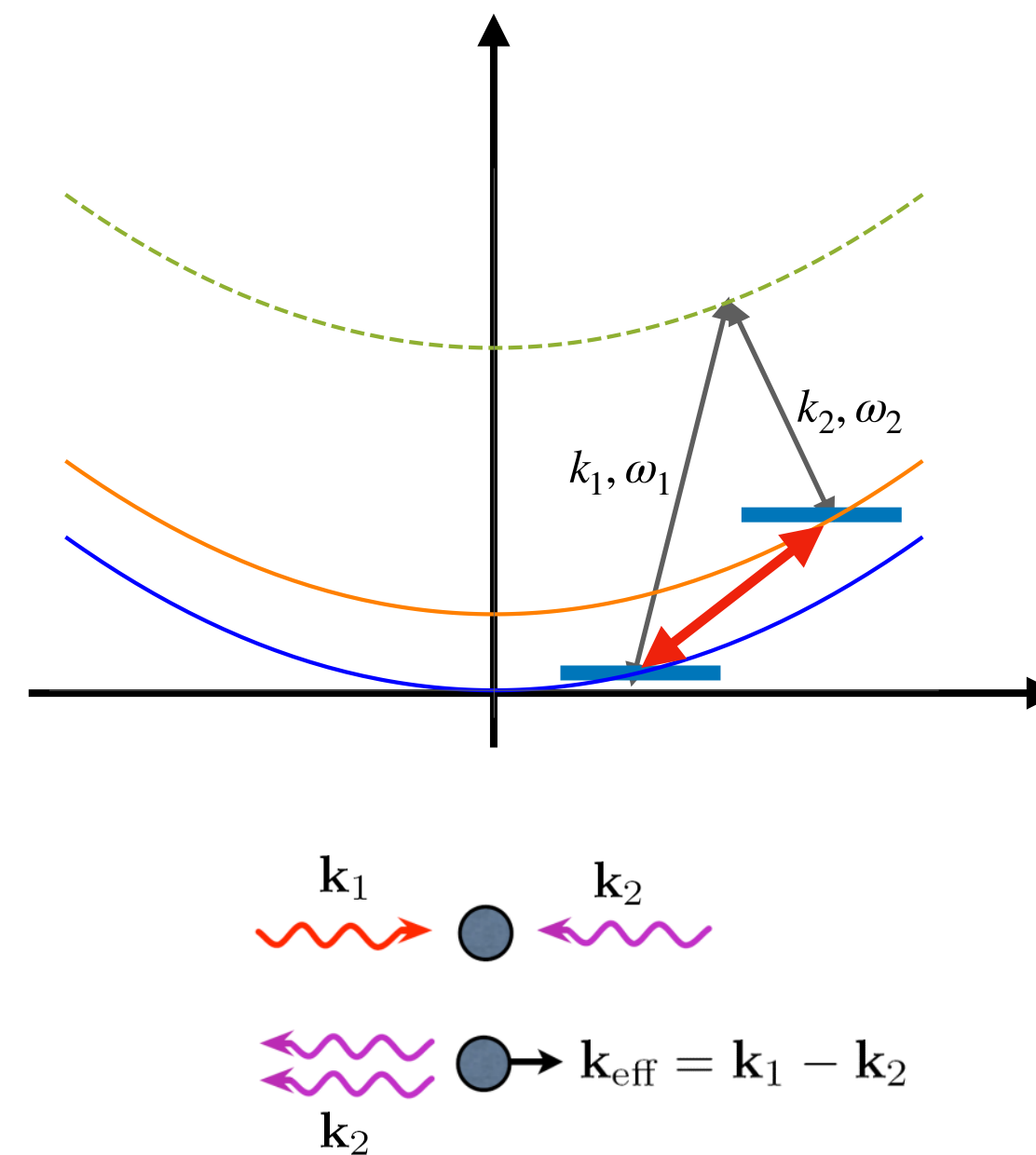
Single and multi-photon process

Single photon



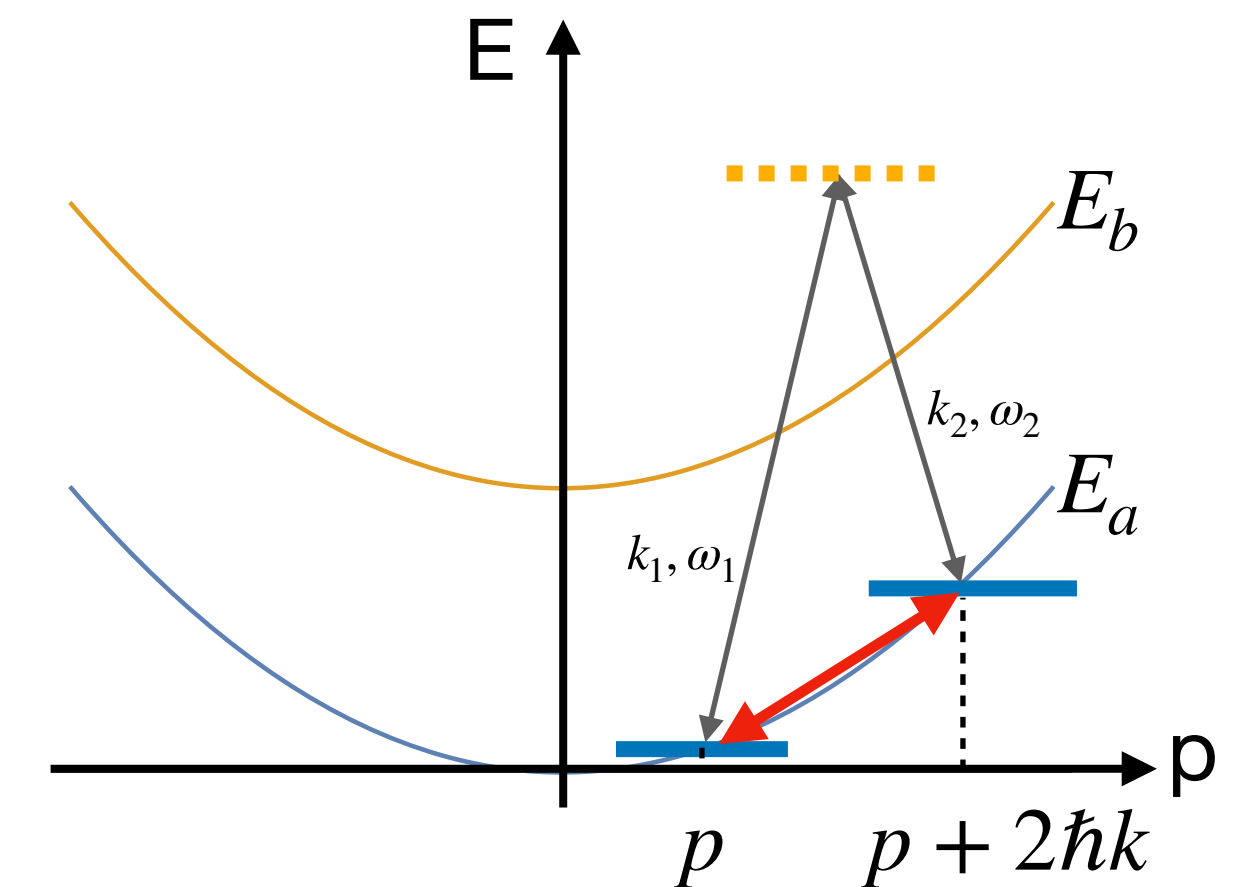
$$|\Psi\rangle = \alpha |E_a, p\rangle + \beta |E_b, p + \hbar k\rangle e^{i\varphi_{\text{opt}}(t)}$$

Two photons : Raman



$$|\Psi\rangle = \alpha |p\rangle + \beta |p + \hbar k\rangle e^{i\varphi_{\text{rf}}(t)}$$

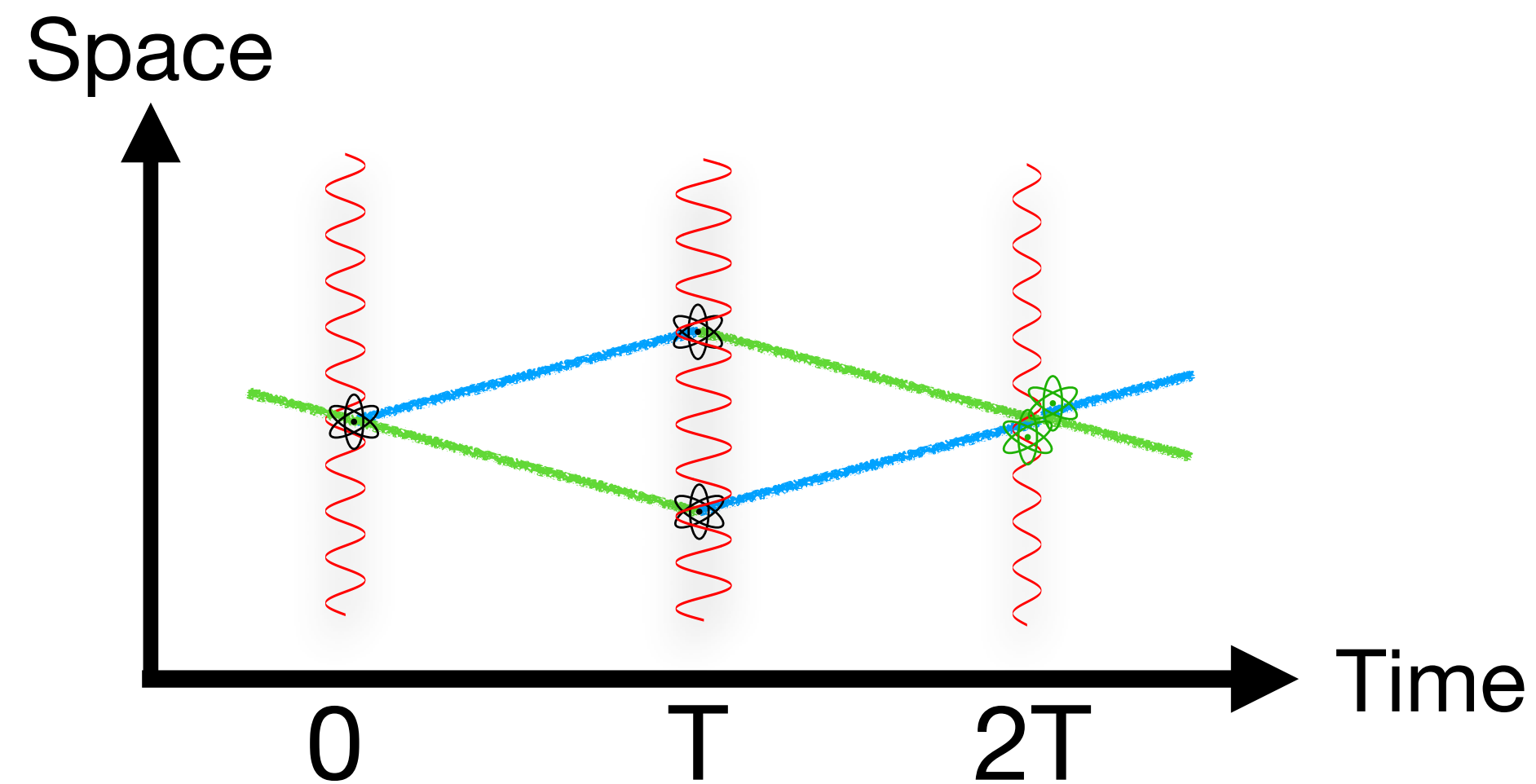
Two photons : Bragg



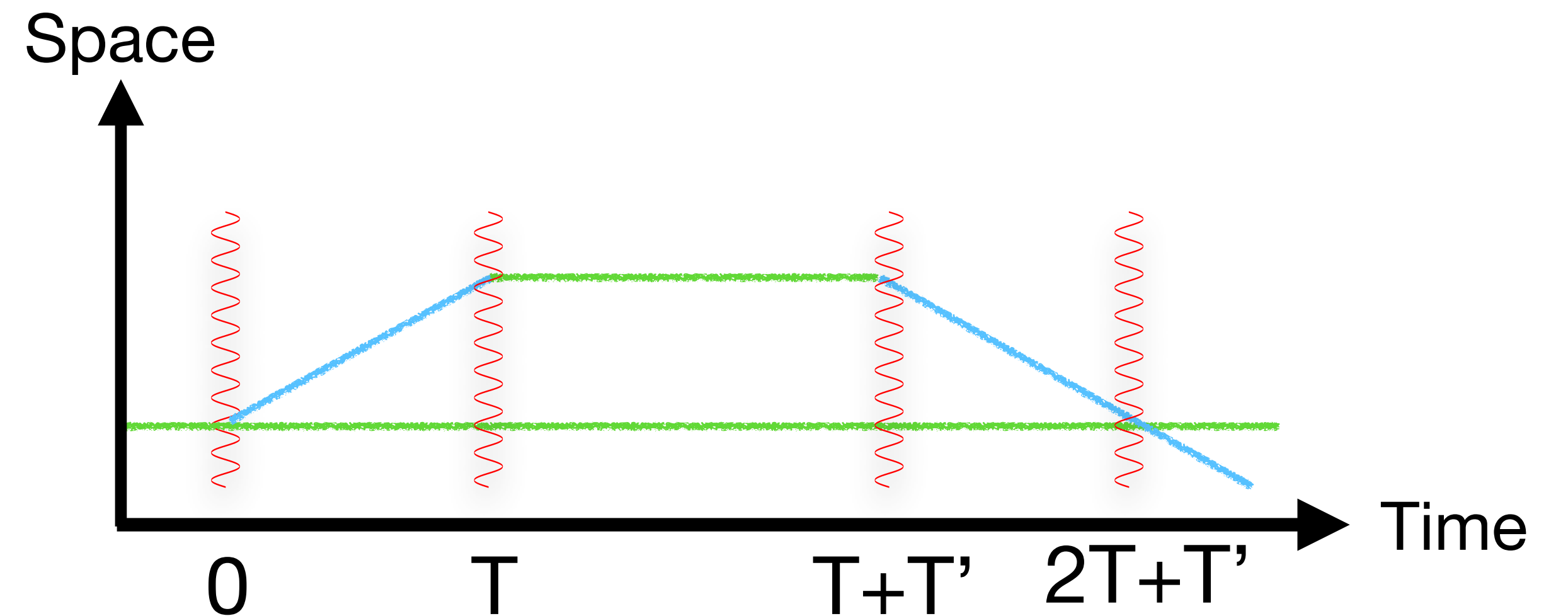
Light pulse atom interferometer

Various configurations

Various geometries, w or w/o internal states ...



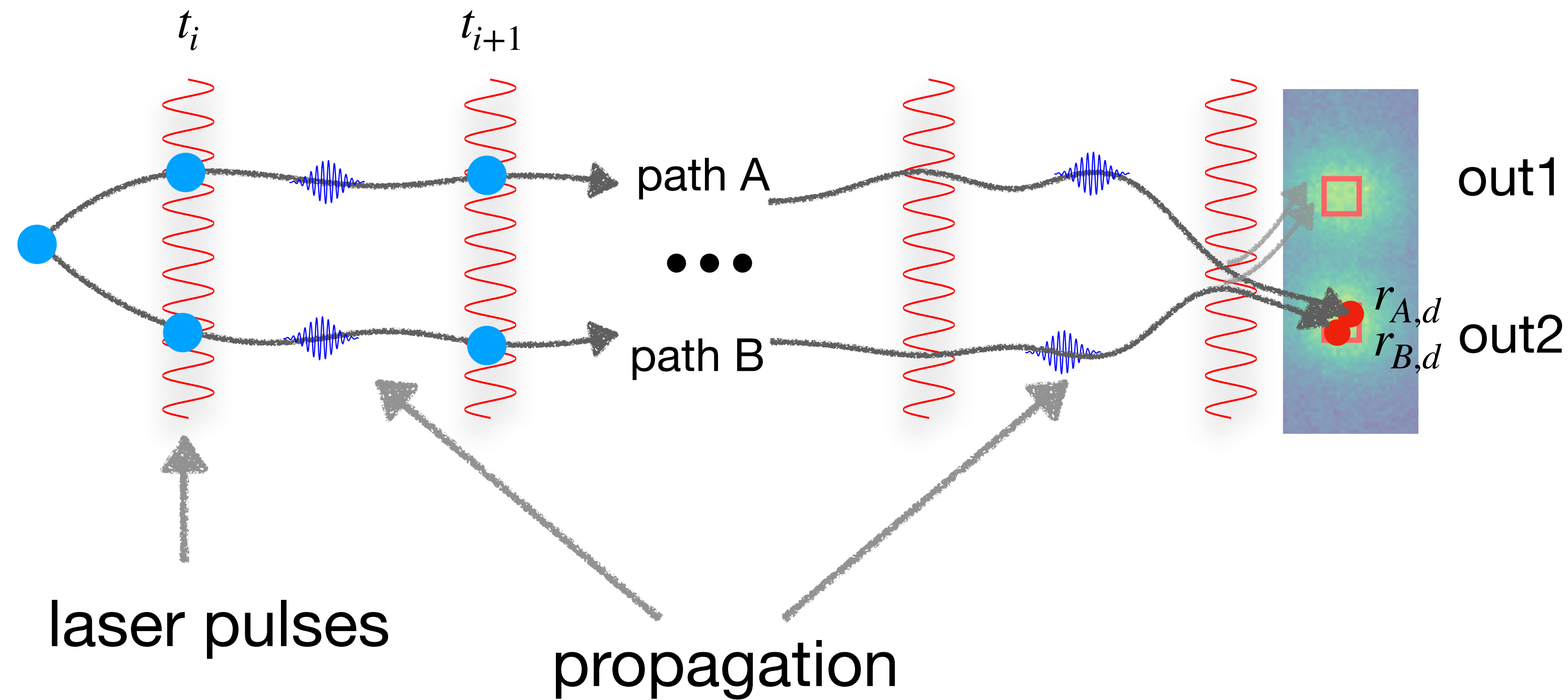
Symmetric interferometer



Ramsey-Bordé interferometer

Light pulse atom interferometer

N-pulse atom interferometer

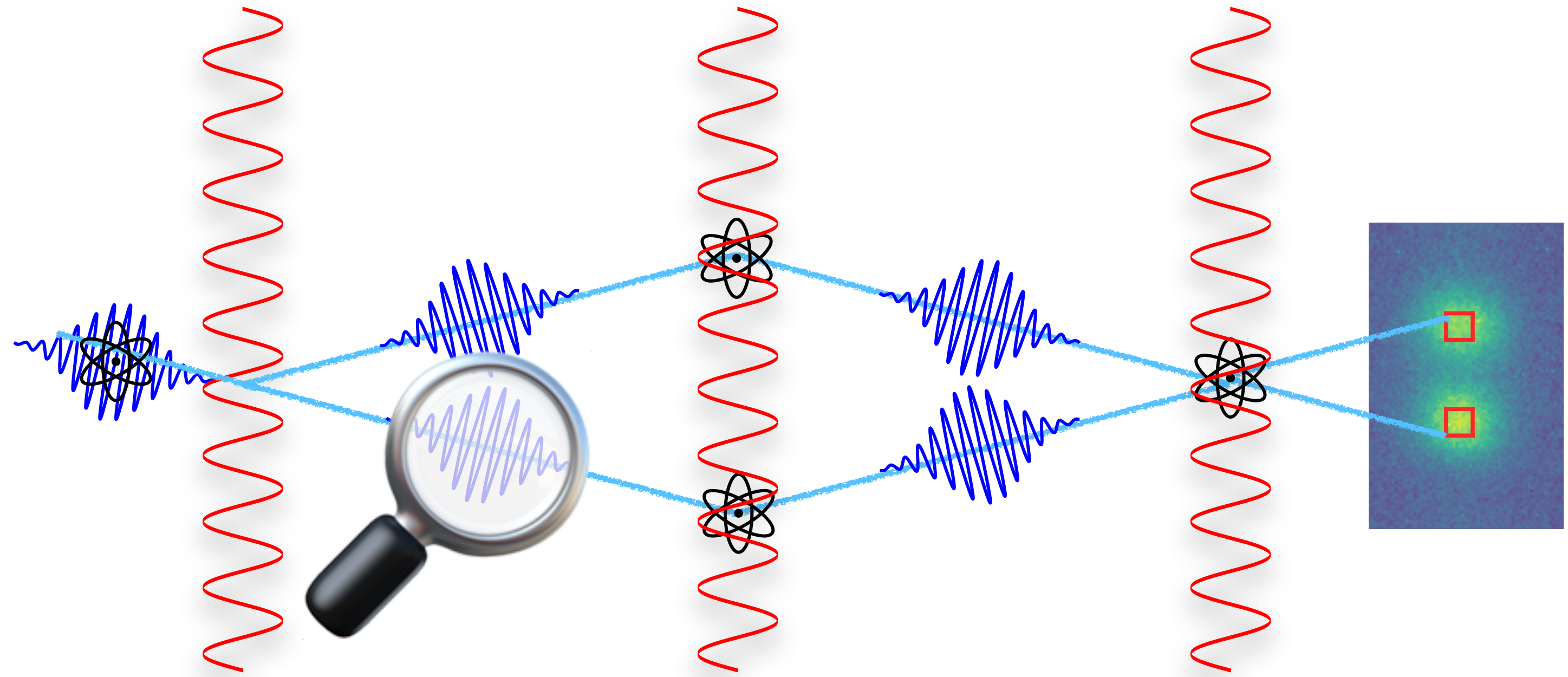


- Two paths
- Two output ports
- Integration of each ports

The total phase shift equals the sum of the phase shifts for each interferometer slice.

Outline: Phase shift calculations

1. Atom Optics
2. Wave Packet propagation
3. Phase shift formulae
4. Applications



Propagation of a wave packet

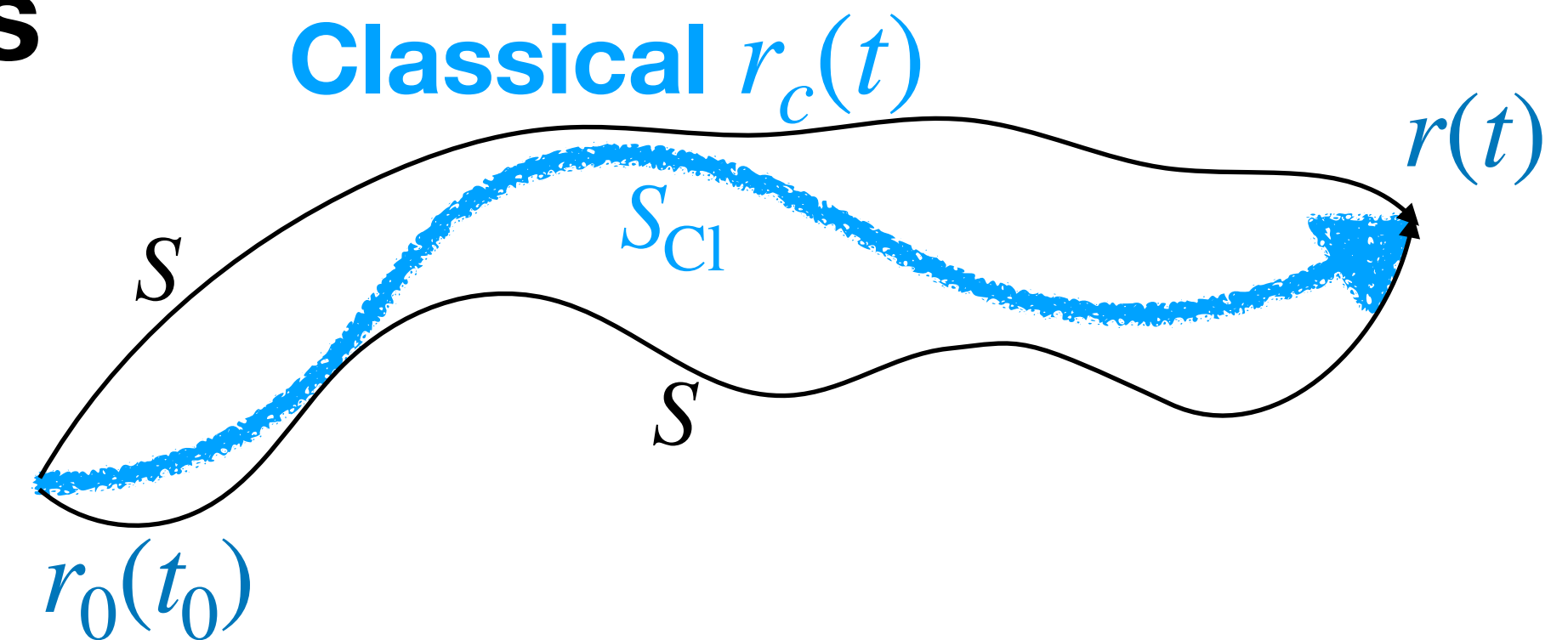
Review of classical lagrangian dynamics

Particle of mass m in a potential V

$$\text{Lagrangian : } L(r(t'), p(t')) = \frac{m}{2} \dot{r}^2 - V$$

$$\text{Canonical momentum: } p = \frac{\partial L}{\partial \dot{r}}$$

$$\text{Action : } S = \int_{t_0}^t L(r(t'), \dot{r}(t')) dt'$$



Principle of least action: particle follow a path of extremal action ($\delta S = 0$).

↓

$$S_{Cl}(r, t; r_0, t_0)$$

Alternative expression for the classical action:

$$S_{Cl} = \int_{r(t)} (p dr - H dt)$$

🤔 think phase of plane wave: $\frac{1}{\hbar}(p \cdot r - Et)$

Propagation of a wave packet

Quantum Propagator

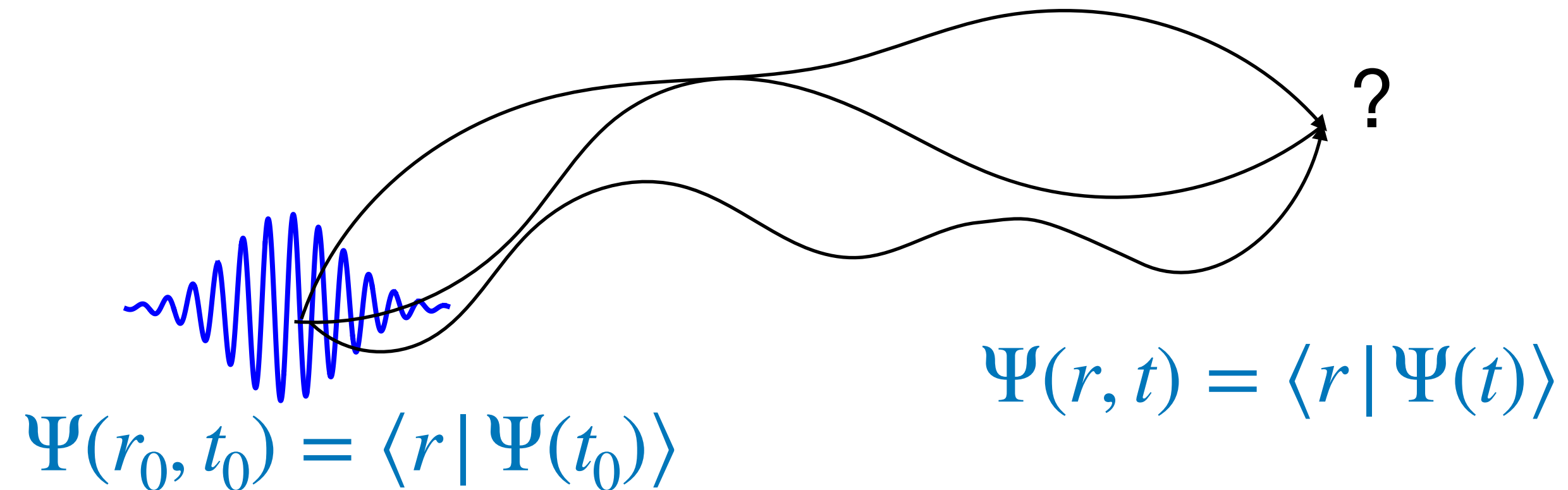
Schrödinger equation: $i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = H(t) |\Psi(t)\rangle$

$$|\Psi(t)\rangle = U(t, t_0) |\Psi(t_0)\rangle$$

$$\begin{aligned} \Psi(r, t) &= \langle r | \Psi(t) \rangle = \langle r | U(t, t_0) | \Psi(t_0) \rangle \\ &= \int dr_0 \langle r | U(t, t_0) | r_0 \rangle \langle r_0 | \Psi(t_0) \rangle \end{aligned}$$

$$\Psi(r, t) = \int dr_0 K(r_0, t_0; r, t) \Psi(r_0, t_0)$$

Propagator: $K(r_0, t_0; r, t)$



- $K(r, t)$ is solution of the Schrödinger equation with initial wave packet localized in (r_0, t_0) .
- Green's function of Schrödinger equation.
- $K(r_0, t_0; r, t)$ = amplitude from a space-time (r_0, t_0) to (r, t)

Your favorite book in quantum e.g. :

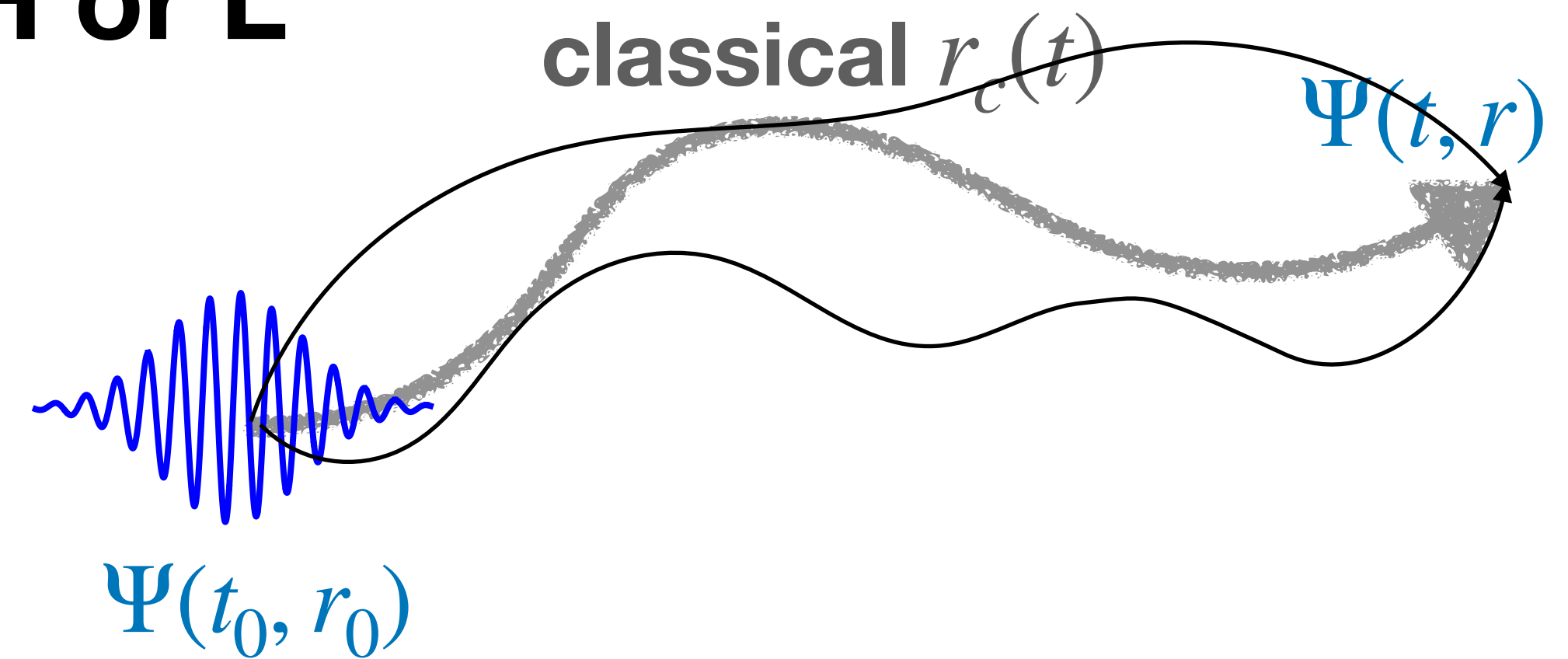
Cohen-Tannoudji et al. Quantum Mechanics, Tome I (Wiley, New-York, 1977)

Propagation of a wave packet

Quantum propagator for quadratic H or L

Feynman path integral approach

$$K(r, t; r_0, t_0) = \mathcal{N} \sum_{\gamma} e^{iS_{\gamma}/\hbar} = \int \mathcal{D}[r(t)] e^{iS/\hbar}$$



At most quadratic hamiltonian in r and p

$$H = \frac{\mathbf{p}\hat{\beta}\mathbf{p}}{2m} - m\mathbf{g} \cdot \mathbf{r} - \frac{m}{2}\mathbf{r}\hat{\gamma}\mathbf{r} - \mathbf{r}\hat{\alpha}\mathbf{p} + \mathbf{f} \cdot \mathbf{p}$$

kinetic gravity gravity grad Rotation
GW acc. harm. pot.

Simplified expression
for the propagator



van Vleck Propagator

$$K(r, t; r_0, t_0) = \frac{1}{C} \exp\left[\frac{i}{\hbar} S_{\text{Cl}}(r, t; r_0, t_0)\right]$$

J.H. van Vleck PNAS (1928)

C.J. Bordé Les Houches Lectures (1991)

Still a good approximation in the semi-classical limit: $S_{\text{Cl}} \gg \hbar$

● Wave packet width smaller than typical variations of H: $\Delta_r^n \ll \partial_r^n H$ & $\Delta_p^n \ll \partial_p^n H$

Propagation of a wave packet

Propagator for at most quadratic hamiltonian

quadratic hamiltonian - lagrangian



$S_{\text{Cl}}(r, t)$ is quadratic



$K(r, t)$ is a gaussian



Propagation of a wave packet

Gaussian wave packet

📌 Quadratic hamiltonian - lagrangian



$S_{Cl}(r, t)$ is quadratic

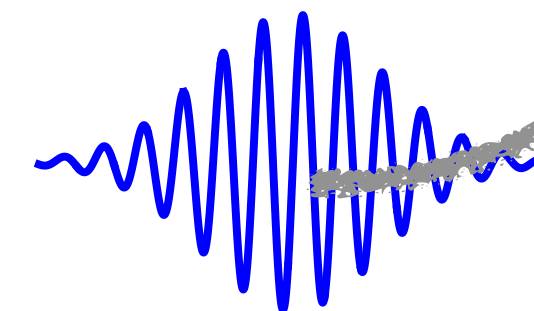


$K(r, t)$ is a gaussian

📌 Initial gaussian wave packet

C.J. Bordé Les Houches Lectures (1991)

Gaussian



$\Psi(t_0, r_0)$

$\Psi(t, r)$

$$\Psi(r, t_0) = \frac{1}{\sqrt{|\det(X(t_0))|}} \exp \left[\frac{im}{2\hbar} (r - r_c) Y_0 X_0^{-1} (r - r_c) + \frac{i}{\hbar} (r - r_c) \cdot p_c \right]$$

Centers of the wave packet

Center of the w.p. r_c

Velocity $v_c = p_c/m$

Widths of the wave packet

Y Width in velocity space

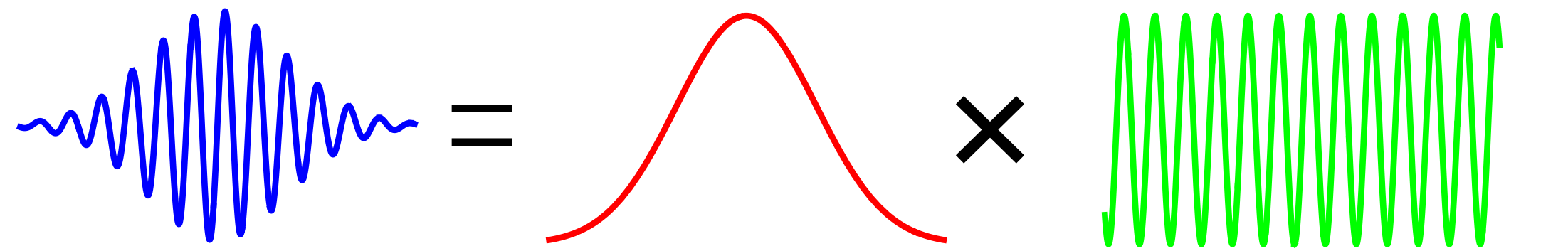
X Complex width in physical space

- Analogous to the complex radius of curvature of Gaussian beams.

- Tricky to move from X and Y to the usual widths in phase space.

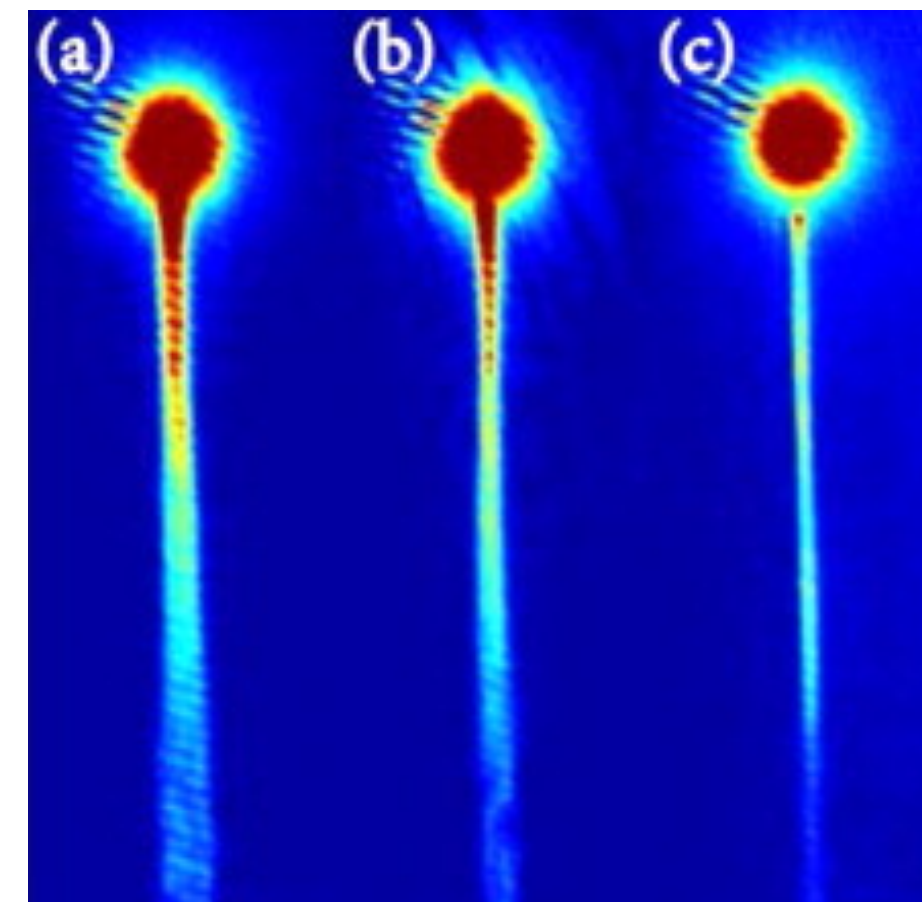
Propagation of a wave packet

Example: free propagation of a 1D wave packet



$$\Psi(z, t_0) \propto \exp\left[\frac{im}{2\hbar} \frac{Y}{X_0} (r - r_c)^2\right] \times \exp\left[\frac{i}{\hbar} (r - r_c) \cdot p_c\right]$$

Initial conditions: $t_0 = 0$ & $r_c(0) = 0$



M.Jeppesen et al, PRA (2008)

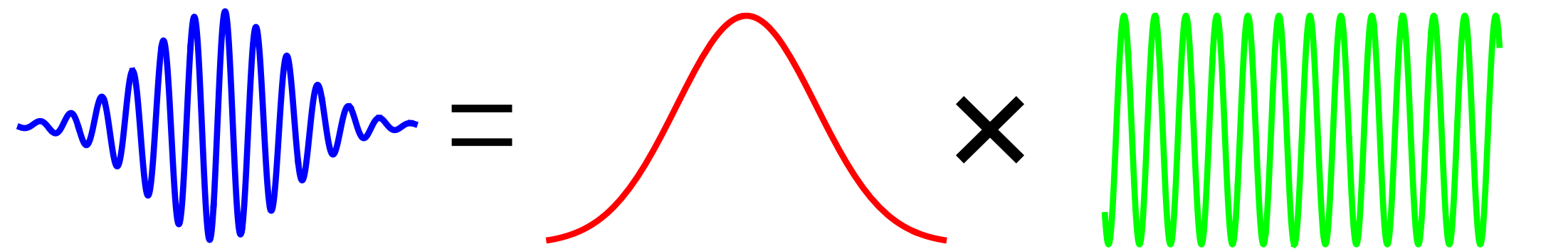
minimal wave packet

$$X(0) = -i\sigma \quad Y(0) = \frac{2\hbar}{m\sigma}$$

$$\Psi(z, 0) \propto \exp\left[i\frac{p_c}{\hbar} r\right] \exp\left[-\frac{r^2}{\sigma^2}\right]$$

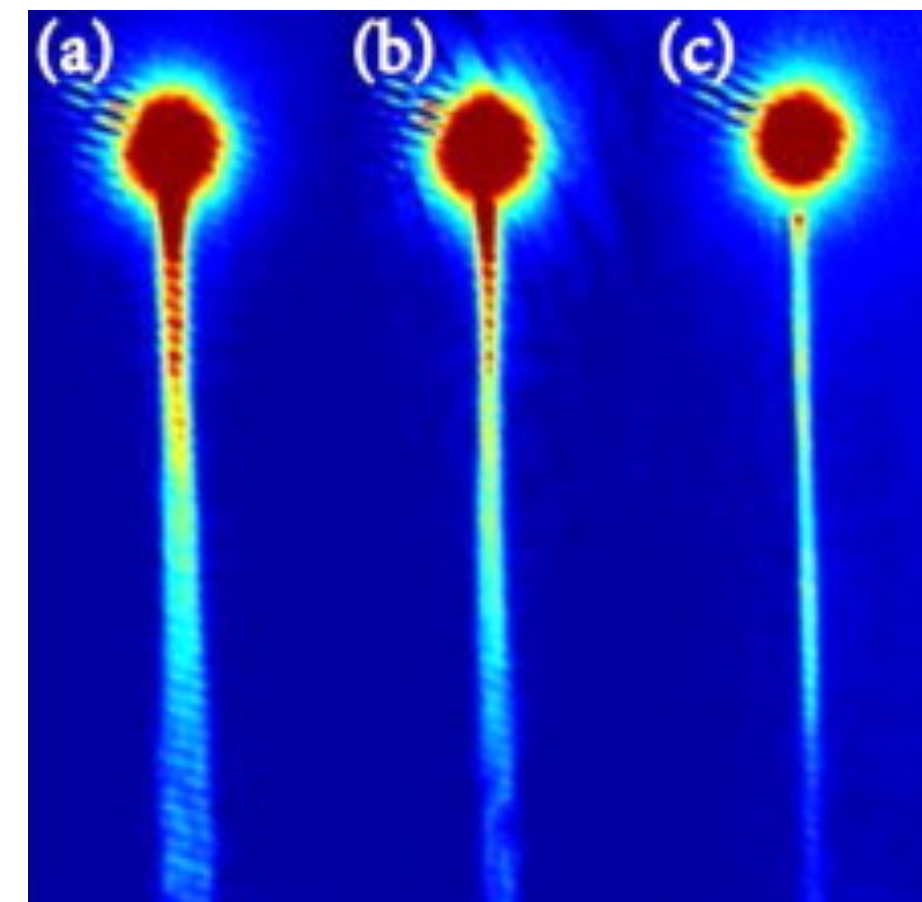
Propagation of a wave packet

Example: free propagation of a 1D wave packet



$$\Psi(z, t_0) \propto \exp\left[\frac{im}{2\hbar} \frac{Y_0}{X_0} (r - r_c)^2\right] \times \exp\left[\frac{i}{\hbar} (r - r_c) \cdot p_c\right]$$

Initial conditions: $t_0 = 0$ & $r_c(0) = 0$



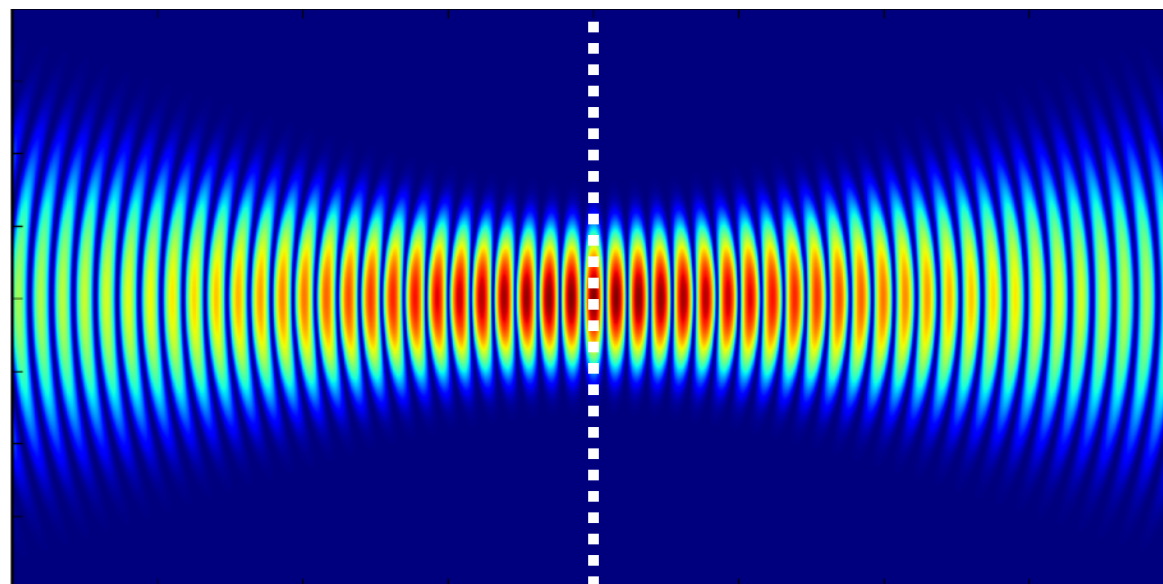
M. Jeppesen et al, PRA (2008)

minimal wave packet

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$$\Psi(z, 0) \propto \exp\left[i\frac{p_c}{\hbar} r\right] \exp\left[-\frac{r^2}{\sigma^2}\right]$$

Comparison with Laser Gaussian beam (2 + 1)D :



At the waist position $z = 0$



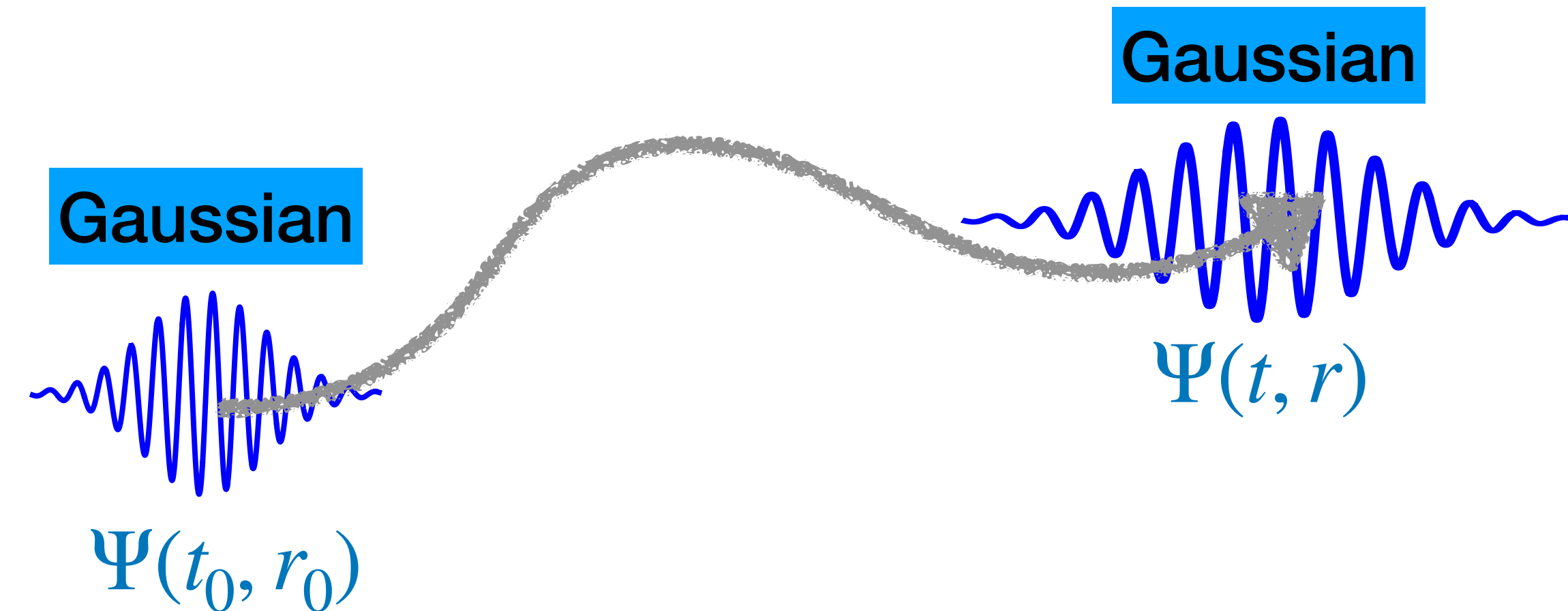
$$E \propto \exp\left[ikz\right] \exp\left[-\frac{r^2}{w_0^2}\right]$$

Propagation of a wave packet

Gaussian wave packet

$$\Psi(r, t) = \frac{1}{C} \int e^{\frac{i}{\hbar} S_{\text{Cl}}(r, t; r_0, t_0)} \Psi(r_0, t_0) dr_0$$

Convolution product between
2 gaussians = gaussian

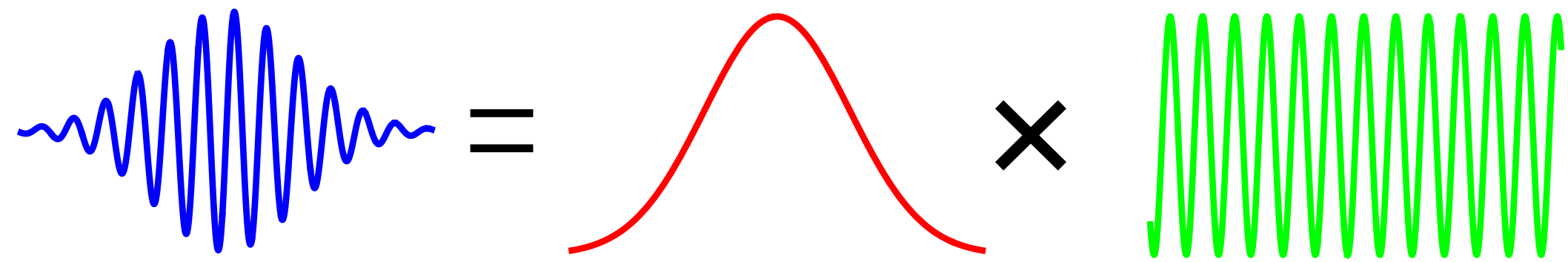


$$\Psi(r, t) = \frac{e^{\frac{i}{\hbar} S_{\text{Cl}}(r, t; r_0, t_0)}}{C \sqrt{|\det(X)|}} \exp \left[\frac{im}{2\hbar} (r - r_c) Y X^{-1} (r - r_c) \right] \exp \left[i \frac{p_c}{\hbar} (r - r_c) \right]$$

- Shape of the wave packet is not changed : gaussian stay gaussian
- Propagation : **classical propagation** of r_c, p_c, X, Y

Propagation of a wave packet

Example: free propagation of a 1D wave packet



$$\Psi(z, t) \propto \exp\left[\frac{i}{\hbar}S_{Cl}\right] \exp\left[\frac{im}{2\hbar X}(r - r_c)^2\right] \times \exp\left[\frac{i}{\hbar}(r - r_c) \cdot p_c\right]$$

$$X(t) = -i\sigma + \frac{2\hbar}{m\sigma}t \quad Y(t) = \frac{2\hbar}{m\sigma}$$

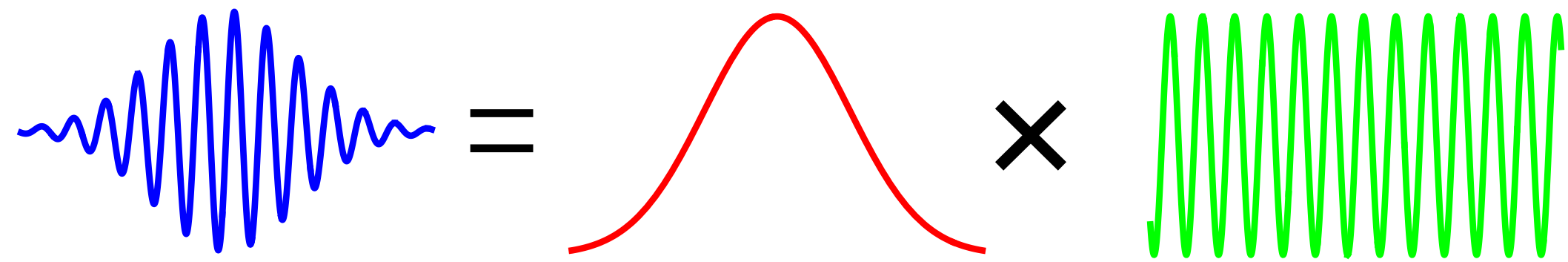
$$\Psi(z, t) \propto \exp\left[i\frac{p_c}{\hbar}(r - r_c)\right] \exp\left[\frac{-(r - r_c)^2}{\sigma^2 + (\frac{2\hbar}{m\sigma}t)^2}\right] \exp\left[i\frac{2\hbar t/m}{\sigma^4 + (2\hbar t/m)^2}(r - r_c)^2\right]$$

Conservation of phase-space volume: $Im(YX^*) = \frac{2\hbar}{m}$

Free particle $S_{Cl} = \frac{mr_c^2}{2t}$

Propagation of a wave packet

Example: free propagation of a 1D wave packet



$$\Psi(z, t) \propto \exp\left[\frac{i}{\hbar}S_{Cl}\right] \exp\left[\frac{im}{2\hbar X}(r - r_c)^2\right] \times \exp\left[\frac{i}{\hbar}(r - r_c) \cdot p_c\right]$$

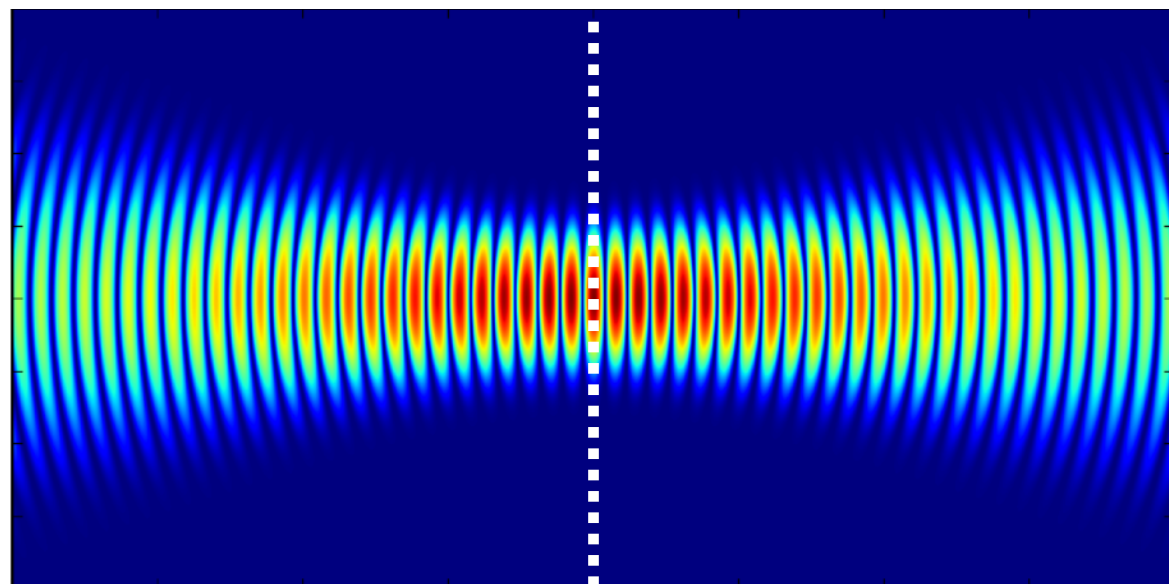
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$$X(t) = -i\sigma + \frac{2\hbar}{m\sigma}t \quad Y(t) = \frac{2\hbar}{m\sigma}$$

$$\Psi(z, t) \propto \exp\left[i\frac{p_c}{\hbar}(r - r_c)\right] \exp\left[\frac{-(r - r_c)^2}{\sigma^2 + (\frac{2\hbar}{m\sigma}t)^2}\right] \exp\left[i\frac{2\hbar t/m}{\sigma^4 + (2\hbar t/m)^2}(r - r_c)^2\right]$$

Comparison with Laser Gaussian beam: (2 + 1)D :



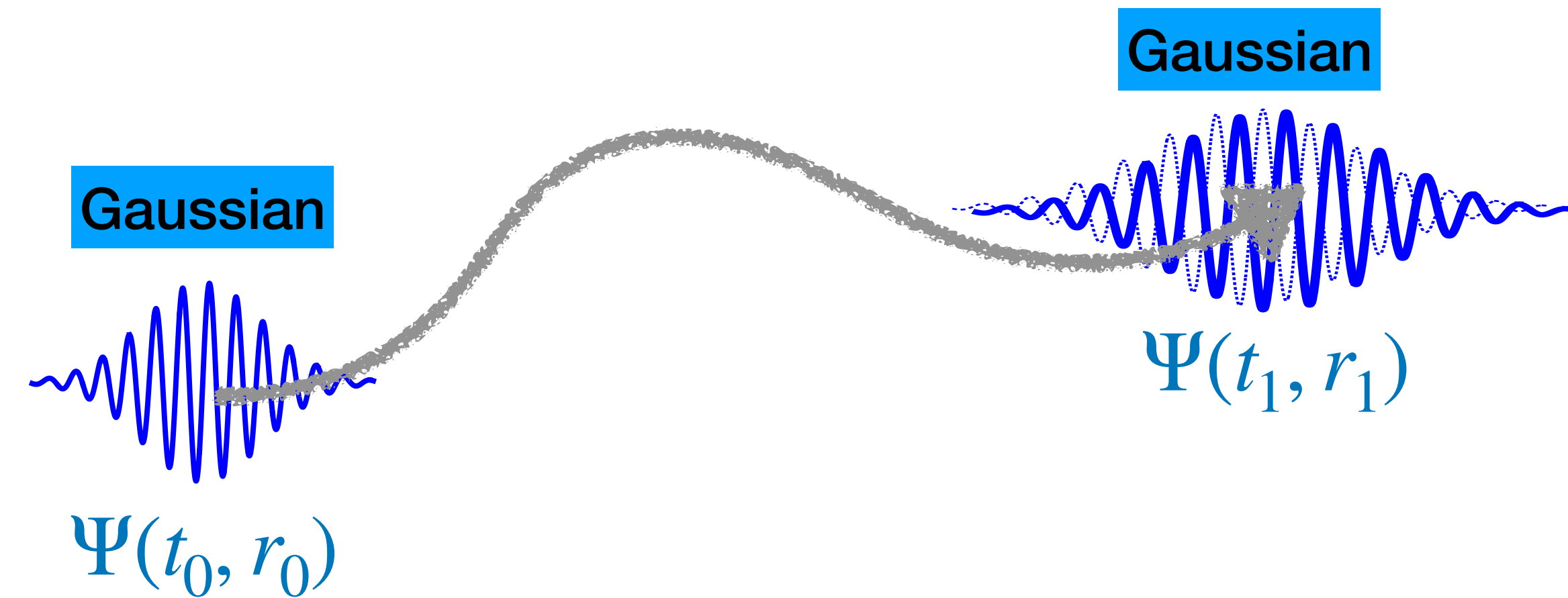
$$E \propto \exp\left[i\phi(z)\right] \exp\left[ikz\right] \exp\left[\frac{-r^2}{w(z)^2}\right] \exp\left[-ik\frac{r^2}{2R(z)}\right] = \exp\left[i\phi(z)\right] \exp\left[ikz\right] \exp\left[\frac{ik}{2Q}r^2\right]$$

Propagation of a wave packet

Gaussian wave packet

$$\Psi(r, t) = \frac{1}{C} \int e^{\frac{i}{\hbar} S_{\text{Cl}}(r, t; r_0, t_0)} \Psi(r_0, t_0) dr_0$$

Convolution product between
2 gaussians = gaussian



$$\Psi(r, t) = \frac{e^{\frac{i}{\hbar} S_{\text{Cl}}(r, t; r_0, t_0)}}{C \sqrt{|\det(X)|}} \exp \left[\frac{im}{2\hbar} (r - r_c) Y X^{-1} (r - r_c) \right] \exp \left[i \frac{p_c}{\hbar} (r - r_c) \right]$$

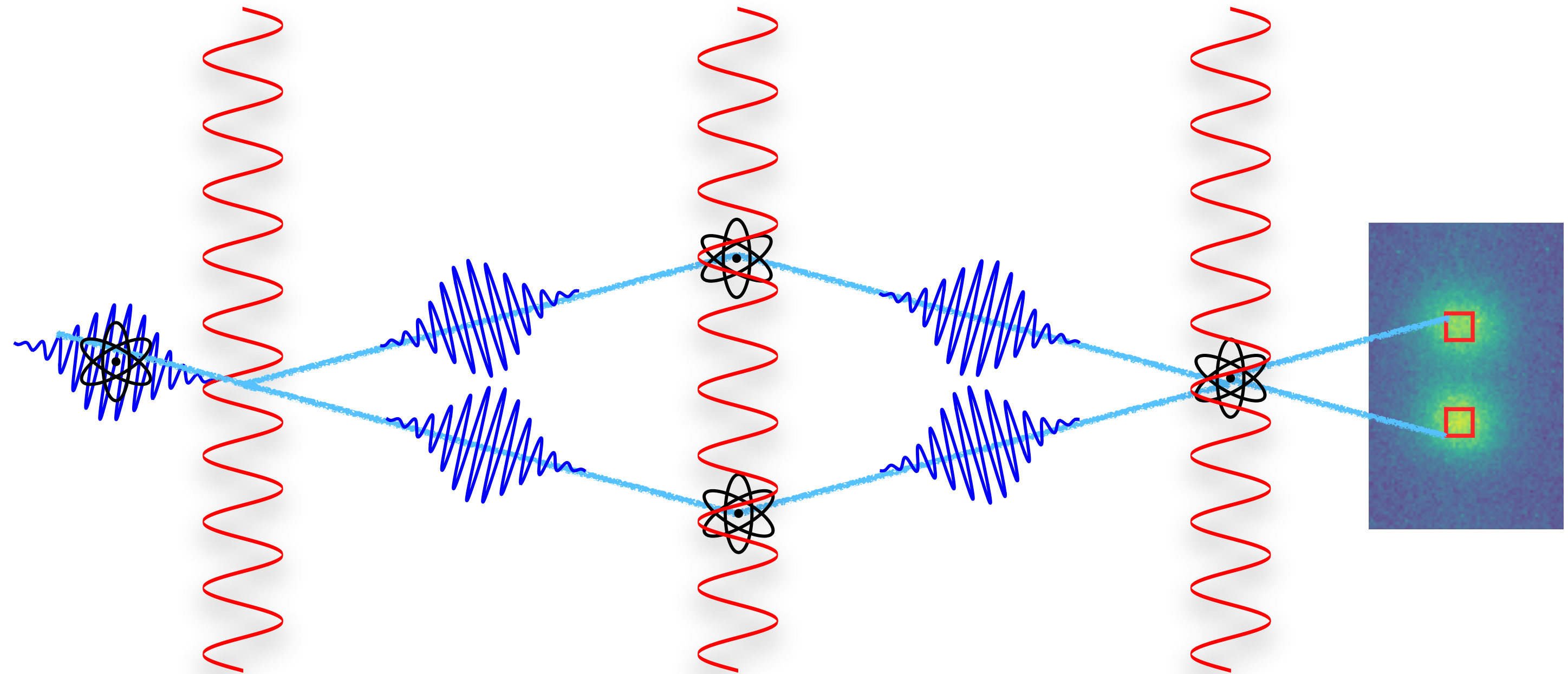
Propagation phase $\phi_p = \frac{1}{\hbar} \left(\underbrace{S_{\text{Cl}}(t_0, t_1) + p_{c,1} (r - r_{c,1}) - p_{c,0} (r - r_{c,0})}_{\text{center of the wp at } t_i \text{ and } t_{i+1}} \right) + \underbrace{\vartheta(X_0, Y_0, X_1, Y_1)}_{\text{width of the wp at } t_i \text{ and } t_{i+1}}$

center of the wp at t_i and t_{i+1}

width of the wp at
 t_i and t_{i+1}

Outline: Phase shift calculations

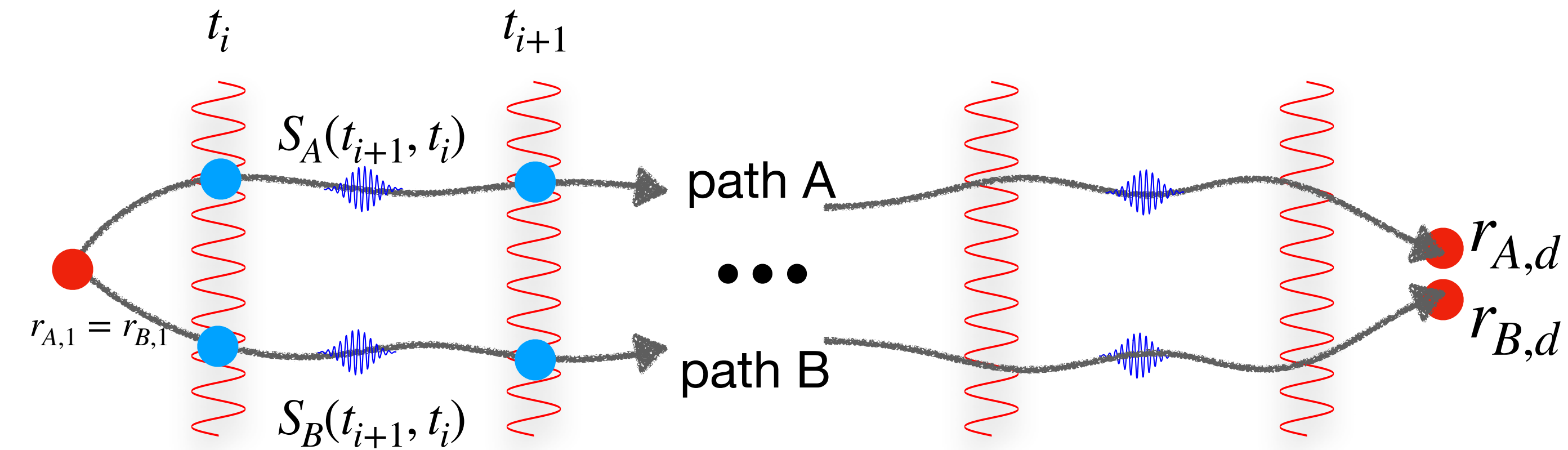
1. Atom Optics
2. Wave Packet propagation
- 3. Phase shift formulae**
4. Applications



Propagation phase

$$\phi_p^{(A,B)} = \frac{1}{\hbar} \left(S_{\text{Cl}}(t_i, t_{i+1}) + p_{c,i+1} (r - r_{c,i+1}) - p_{c,i} (r - r_{c,i}) \right) + \varphi(X_i, Y_i, X_{i+1}, Y_{i+1})$$

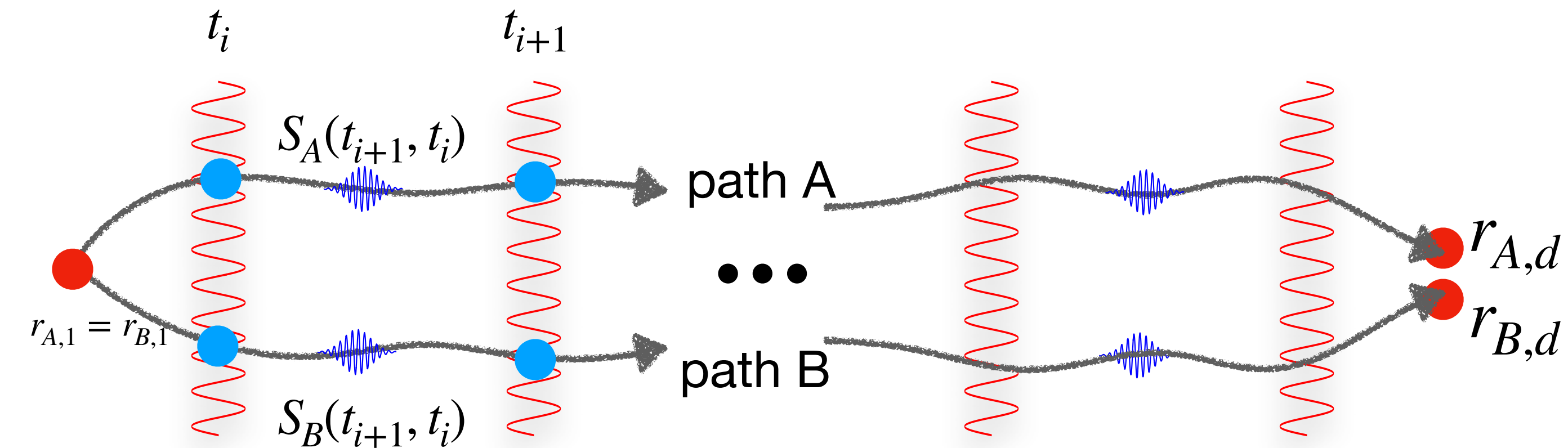
the last term of the i-th slice cancels out
with the first term of the (i+1)th slice.



Propagation phase

$$\phi_p^{(A,B)} = \frac{1}{\hbar} \left(S_{Cl}(t_i, t_{i+1}) + p_{c,i+1} (r - r_{c,i+1}) - p_{c,i} (r - r_{c,i}) \right) + \varphi(X_i, Y_i, X_{i+1}, Y_{i+1})$$

the last term of the i-th slice cancels out
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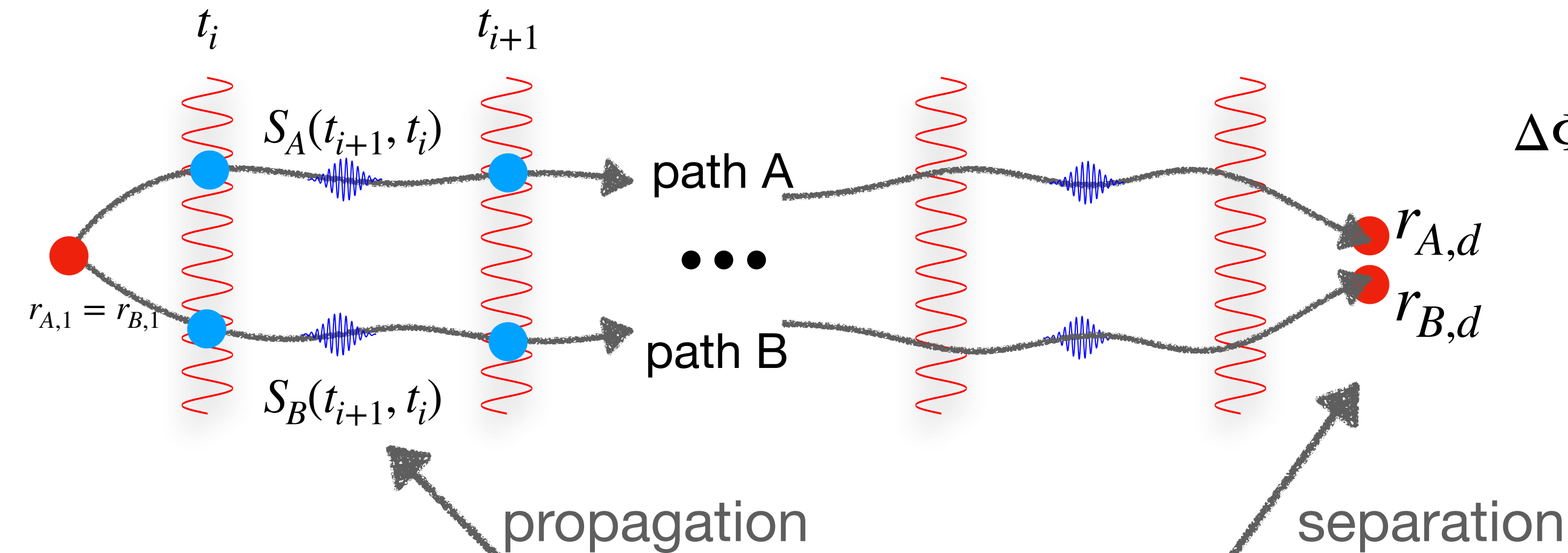


$$\Delta\Phi_p = \Phi_{p,B} - \Phi_{p,A} = \frac{1}{\hbar} \sum_i \left(S_B(t_{i+1}, t_i) - S_A(t_{i+1}, t_i) \right) + \frac{p_{B,d}}{\hbar} (r - r_{B,d}) - \frac{p_{A,d}}{\hbar} (r - r_{A,d}) + \varphi_B - \varphi_A$$

Propagation phase

$$\phi_p^{(A,B)} = \frac{1}{\hbar} \left(S_{Cl}(t_i, t_{i+1}) + p_{c,i+1} (r - r_{c,i+1}) - p_{c,i} (r - r_{c,i}) \right) + \varphi(X_i, Y_i, X_{i+1}, Y_{i+1})$$

the last term of the i-th slice cancels out with the first term of the (i+1)th slice.



$$\Delta\Phi_p = \Phi_{p,B} - \Phi_{p,A} = \frac{1}{\hbar} \sum_i \left(S_B(t_{i+1}, t_i) - S_A(t_{i+1}, t_i) \right) + \frac{p_{B,d}}{\hbar} (r - r_{B,d}) - \frac{p_{A,d}}{\hbar} (r - r_{A,d}) + \cancel{\varphi_B - \varphi_A}$$

detection = integration

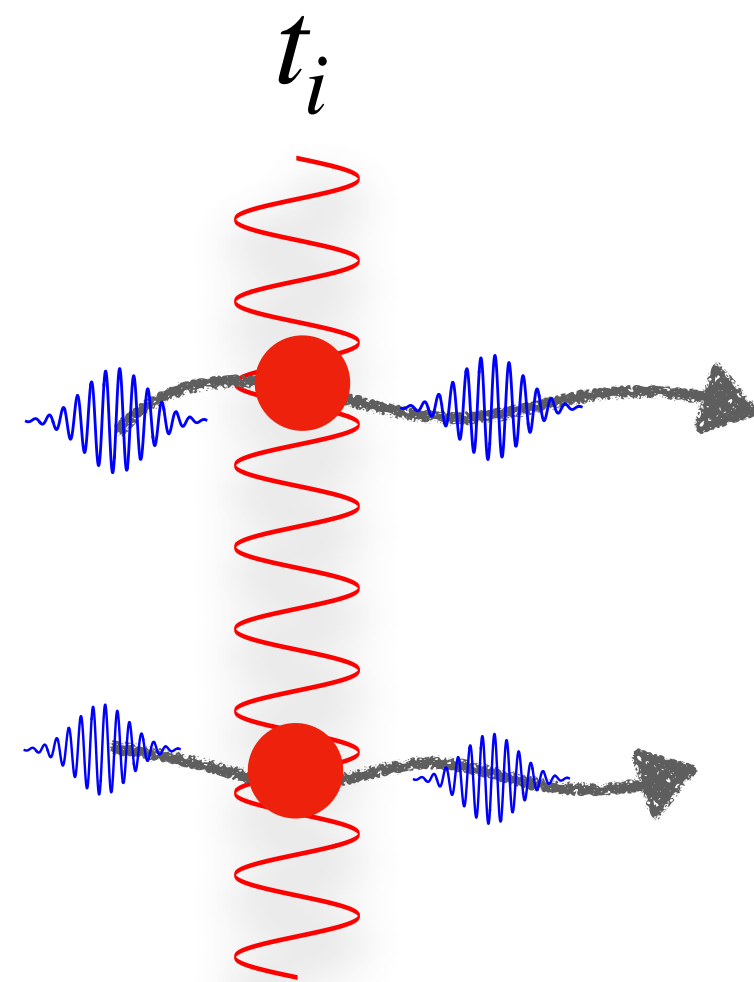
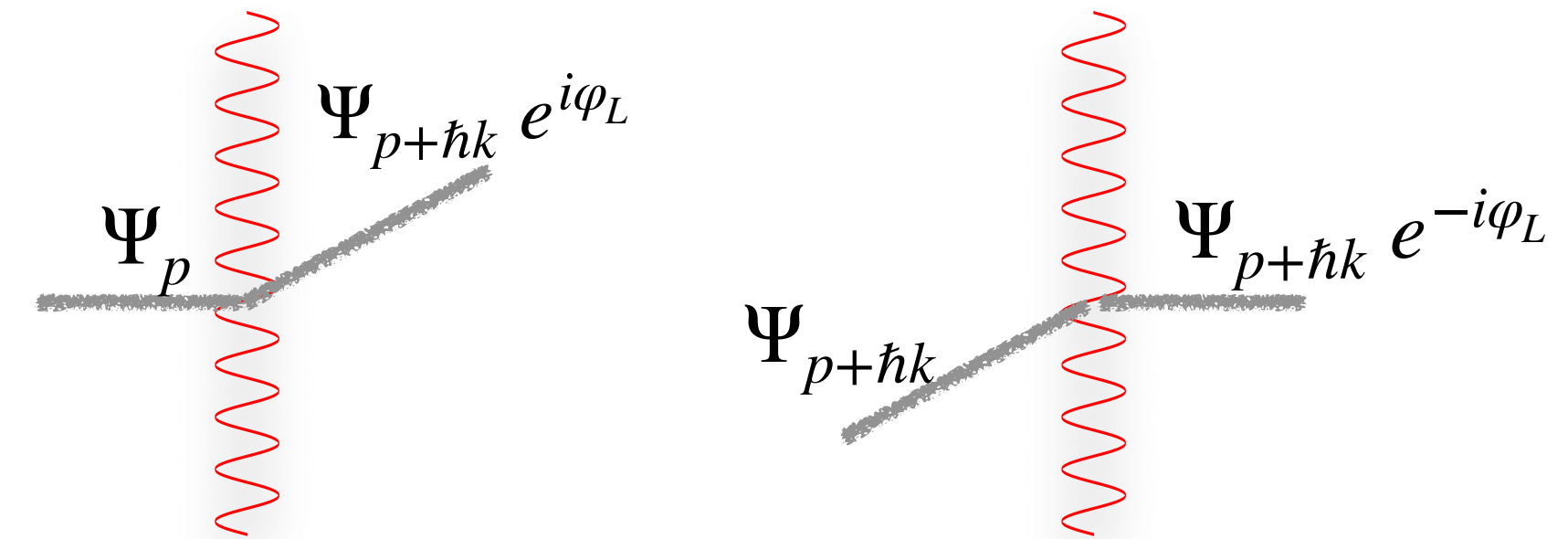
$$\Delta\Phi_p = \frac{1}{\hbar} \sum_i \left(S_B(t_{i+1}, t_i) - S_A(t_{i+1}, t_i) \right) + \frac{p_{B,d} + p_{A,d}}{2\hbar} (r_{B,d} - r_{A,d})$$

Interaction with laser

δ -like pulses

- Interaction preserve the wave packet shape + short pulses
- Neglect the effect of additional forces during the pulses
- Laser phase printed on the diffracted part of the wave-function

$$\varphi_i = k_i \cdot r_i - \omega_i t_i - \phi_i$$



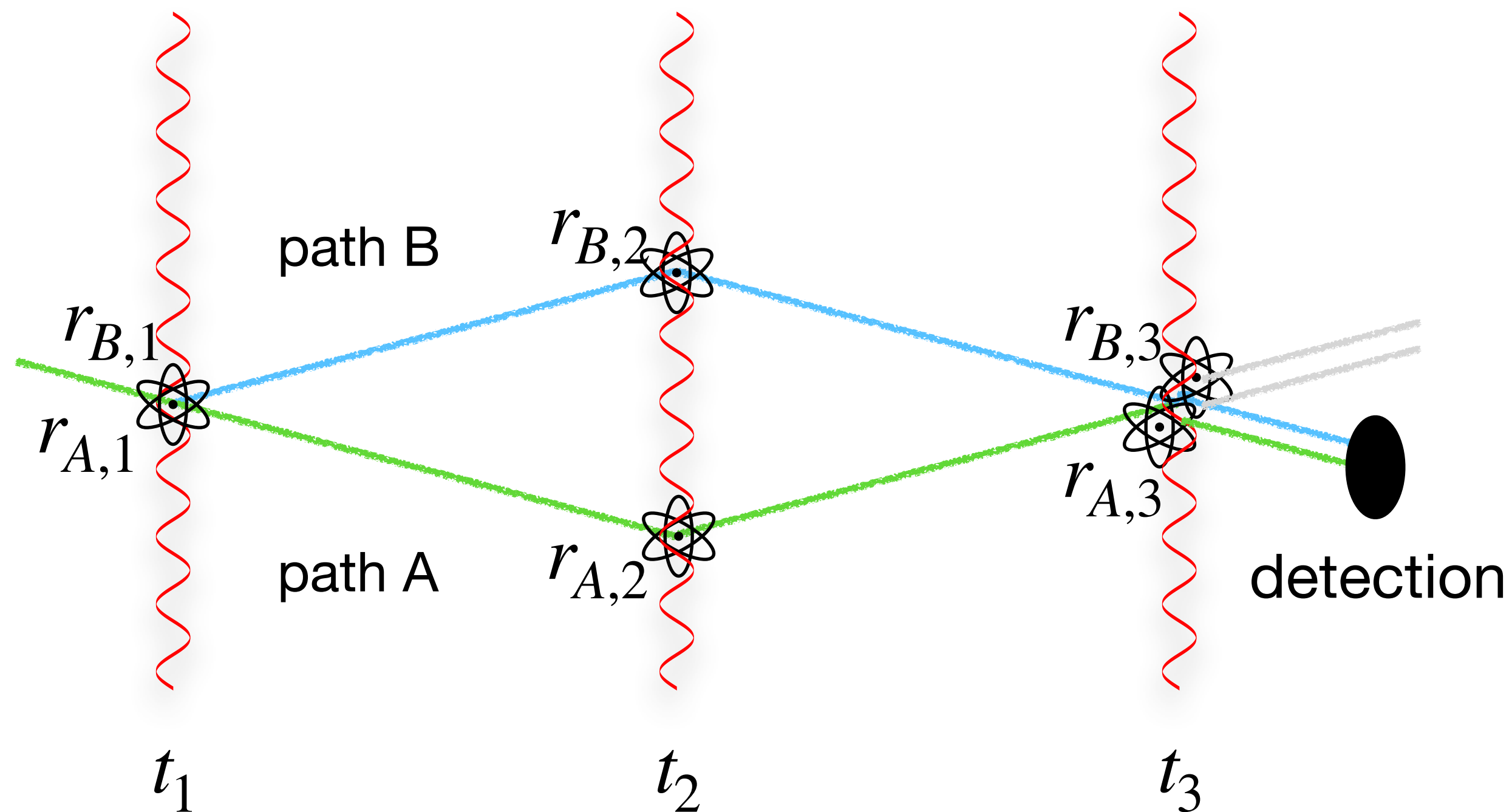
laser pulse

$$\Delta\phi_l = \sum_i \left[(k_{B,j} r_{B,j} - k_{A,j} r_{A,j}) - (\omega_{B,j} - \omega_{A,j}) t_j - (\phi_{B,j} - \phi_{A,j}) \right]$$

Interaction with laser

Example of a symmetric interferometer

$$\Delta\phi_l = \sum_{i=1}^3 \left[(k_{B,i}r_{B,i} - k_{A,i}r_{A,i}) - (\omega_{B,i} - \omega_{A,i})t_i - (\phi_{B,i} - \phi_{A,i}) \right]$$

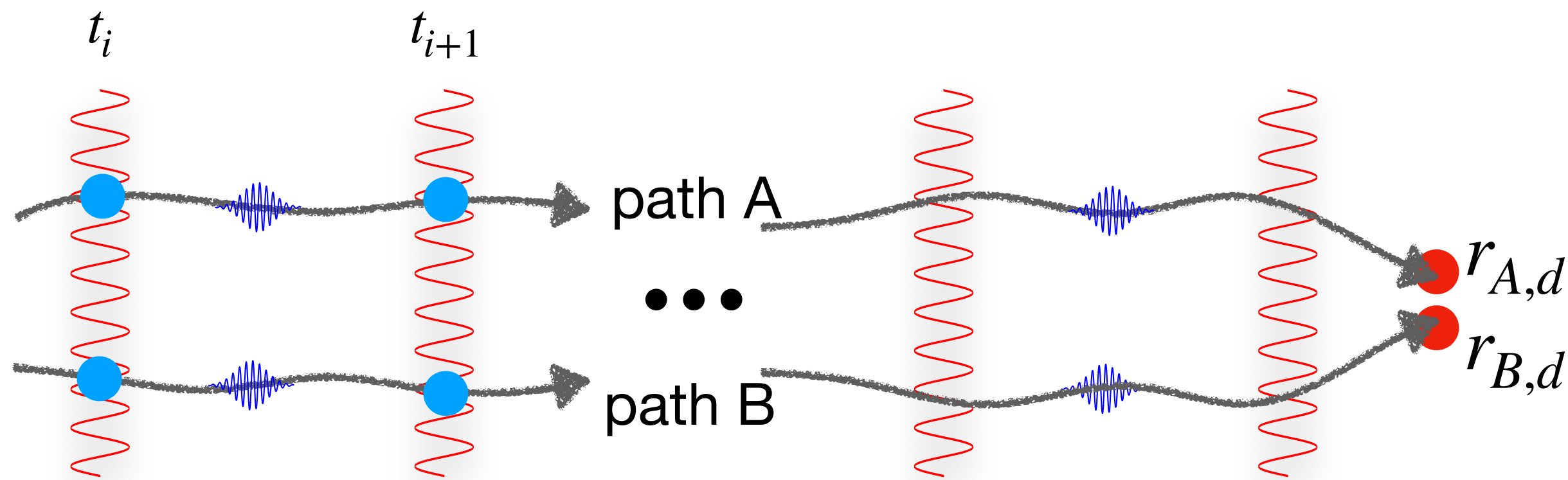


	k_{Aj}	k_{Bj}	ω_{Aj}	ω_{Bj}	ϕ_{Aj}	ϕ_{Bj}
$t_1 = 0$	0	$+k$	0	$-\omega$	0	$-\phi_1$
$t_2 = T$	$+k$	$-k$	$-\omega$	$+\omega$	$-\phi_2$	$+\phi_2$
$t_3 = 2T$	$-k$	0	$+\omega$	0	$+\phi_3$	0

2-photon transitions :

$$\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2 ; \omega = \omega_1 - \omega_2 ; \phi = \phi_1 - \phi_2$$

Interferometer phase shift



$$\Delta\Phi = \frac{1}{\hbar} \sum_i \left(S_B(t_{i+1}, t_i) - S_A(t_{i+1}, t_i) \right) + \sum_i (k_{Bj}r_{Bj} - k_{Aj}r_{Aj}) - (\omega_{Bj} - \omega_{Aj})t_j - (\phi_{Bj} - \phi_{Aj}) + \frac{p_{B,d} + p_{A,d}}{2\hbar} (r_{B,d} - r_{A,d})$$

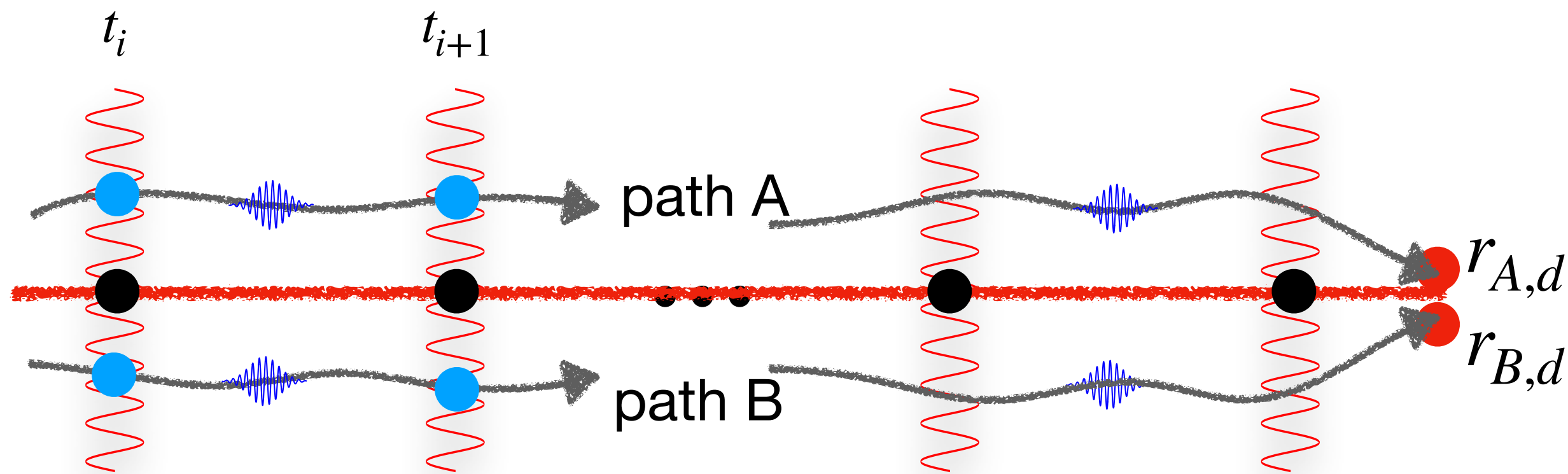
$$\Delta\phi = \phi_{\text{propagation}} + \phi_{\text{laser}} + \phi_{\text{separation}}$$

★ For at most quadratic Hamiltonian : 4 end-points theorem

$$S_B(t_{i+1}, t_i) - S_A(t_{i+1}, t_i) = \frac{p_{B,i+1} + p_{A,i+1}}{2} (r_{B,i+1} - r_{A,i+1}) - \frac{p_{B,i} + \hbar k_{B,i} + p_{A,i} + \hbar k_{B,i}}{2} (r_{B,i} - r_{A,i}) - (E_{B,i} - E_{A,i})(t_{i+1} - t_i)$$

Mid-point theorem

Ch Antoine and Ch J Bordé J. Opt. B: Quantum Semiclass. Opt. 5 S199, (2003)



phase shift calculated by the laser phase at the mid-point trajectory

$$\Delta\Phi = \sum_i (k_{Bi} - k_{Ai}) \frac{r_{Bi} + r_{Ai}}{2} - (\omega_{Bj} - \omega_{Aj})t_j - (\phi_{Bj} - \phi_{Aj}) - \frac{(E_{Bi} - E_{Ai})(t_{i+1} - t_i)}{\hbar}$$

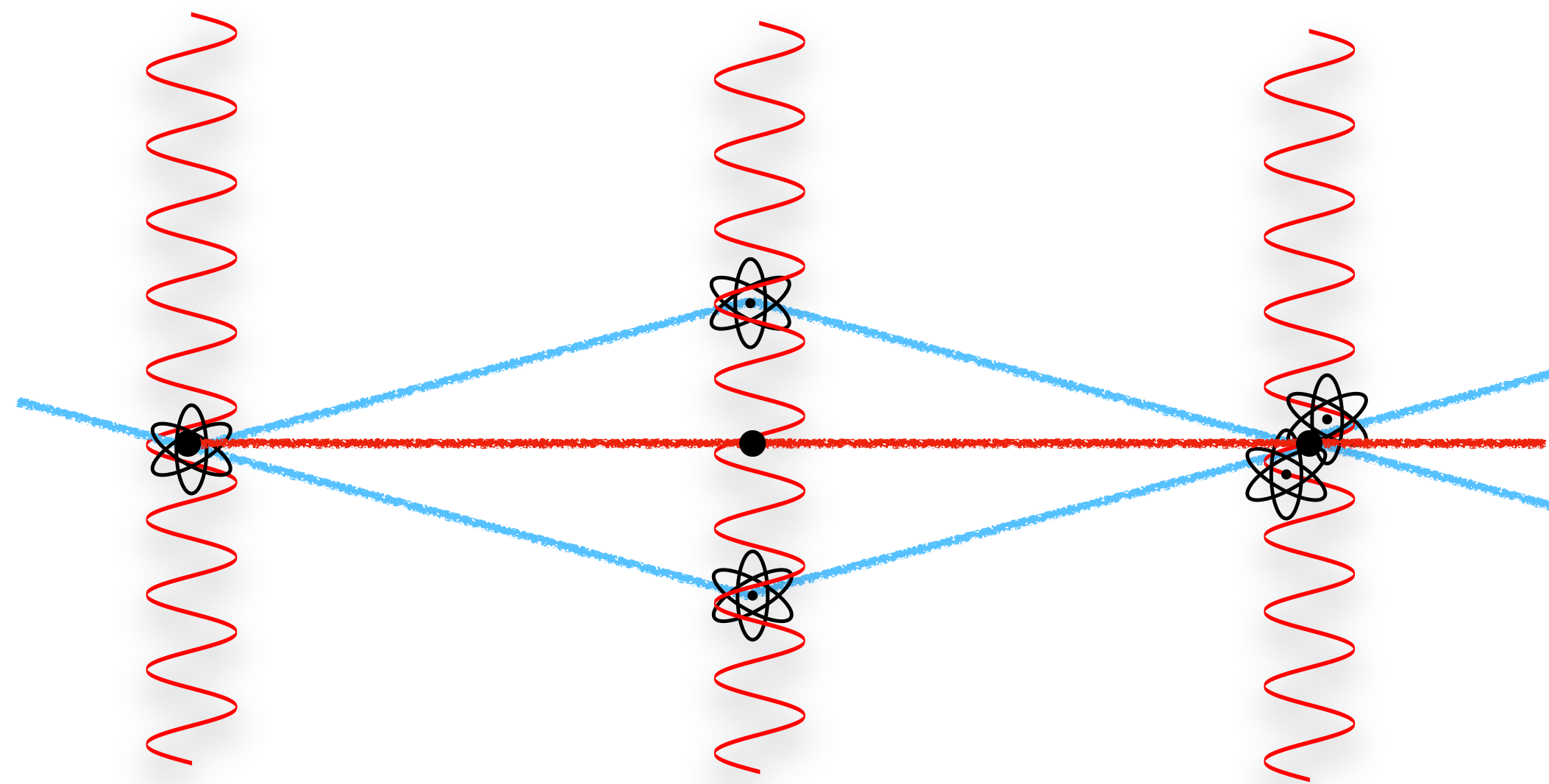
averaged trajectory

frequency and laser phase

rest mass energy
of the atom

Beyond quadratic hamiltonian

Chris Overstreet, Peter Asenbaum, Mark A. Kasevich. *Am. J. Phys.* ; 89 (3): 324, (2021).



$$\text{Laser: } kr(t_i) + \delta t_i + \phi_i$$

$$\text{Propagation: } \frac{1}{\hbar} \int \mathcal{L}(r(t), p(t), t) dt$$

$$\text{Separation: } \frac{\Delta r \Delta p}{\hbar}$$

Mid-Point Phase

$$\Delta\phi = k(\bar{r}_1 - 2\bar{r}_2 + \bar{r}_3) + \delta_1 t_1 - 2\delta_2 t_2 + \delta_3 t_3 +$$

Exact for quadratic $H(r,p)$

Assymmetric Forces

$$\int \frac{\Delta r}{2\hbar} \left(\frac{\partial U(r_A, t)}{\partial r} + \frac{\partial U(r_B, t)}{\partial r} \right) dt -$$

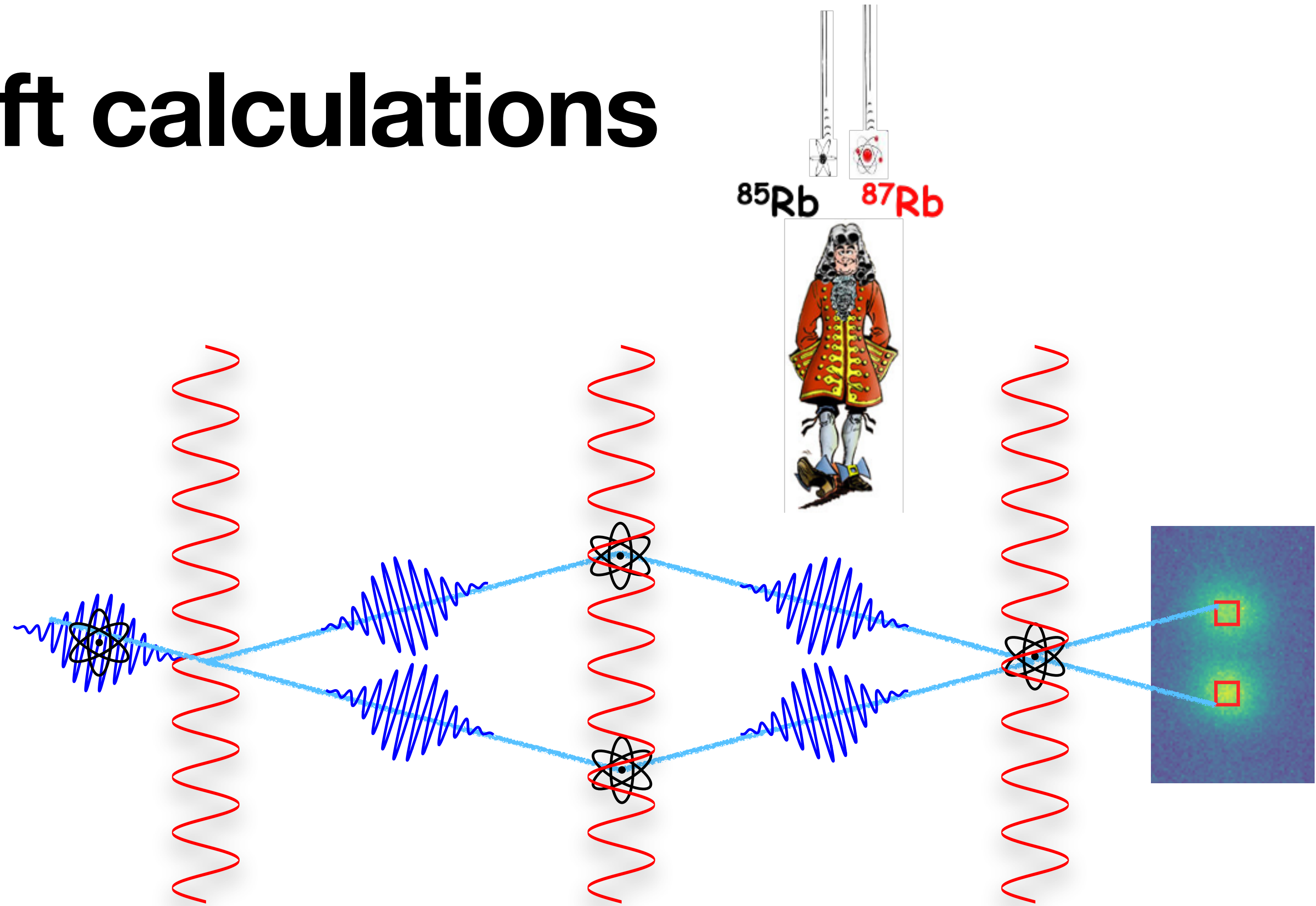
Scalar A-B Phase

$$\int \frac{U(r_A, t) - U(r_B, t)}{\hbar} dt$$

Separated arm interferometers

Outline: Phase shift calculations

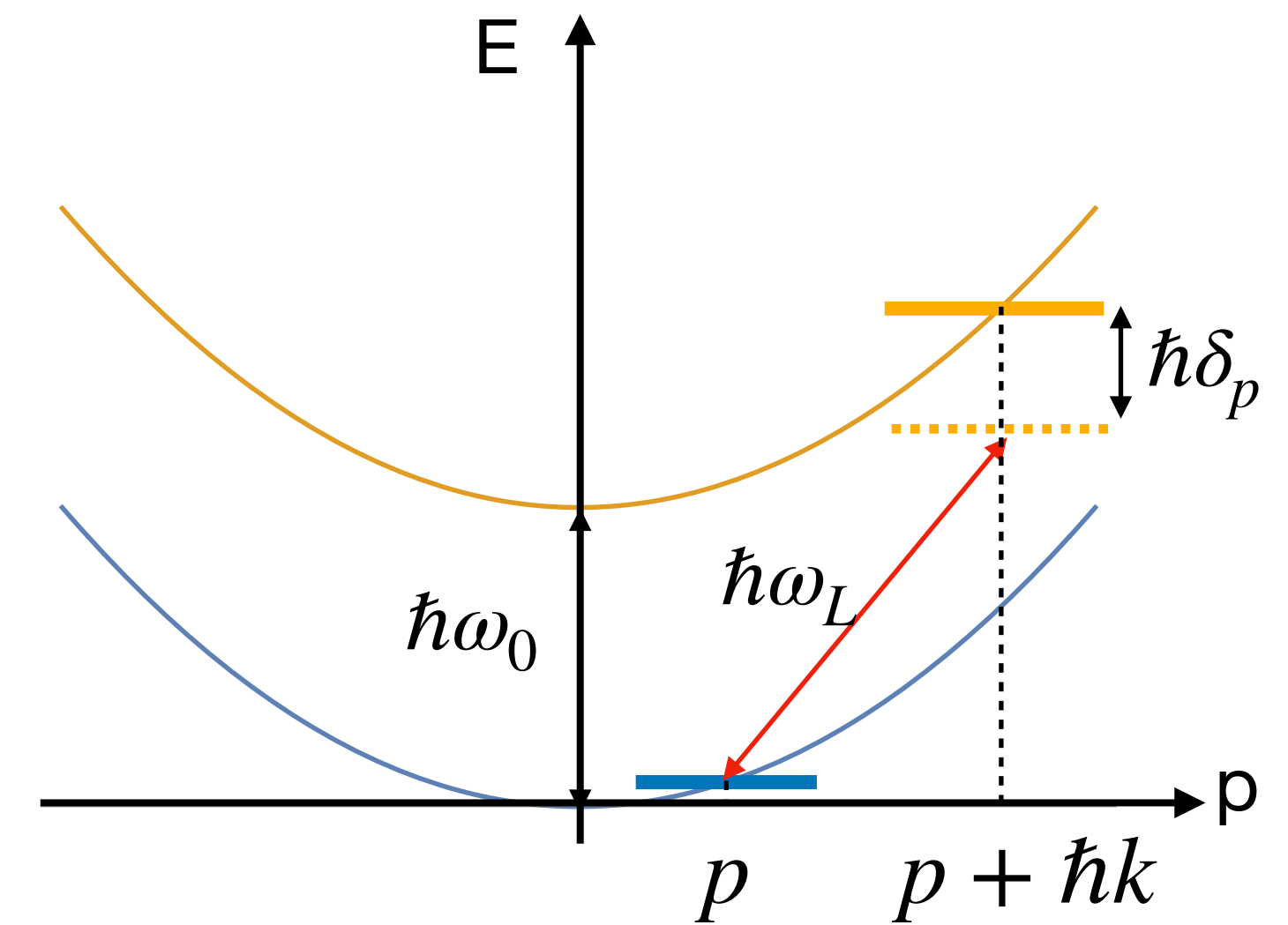
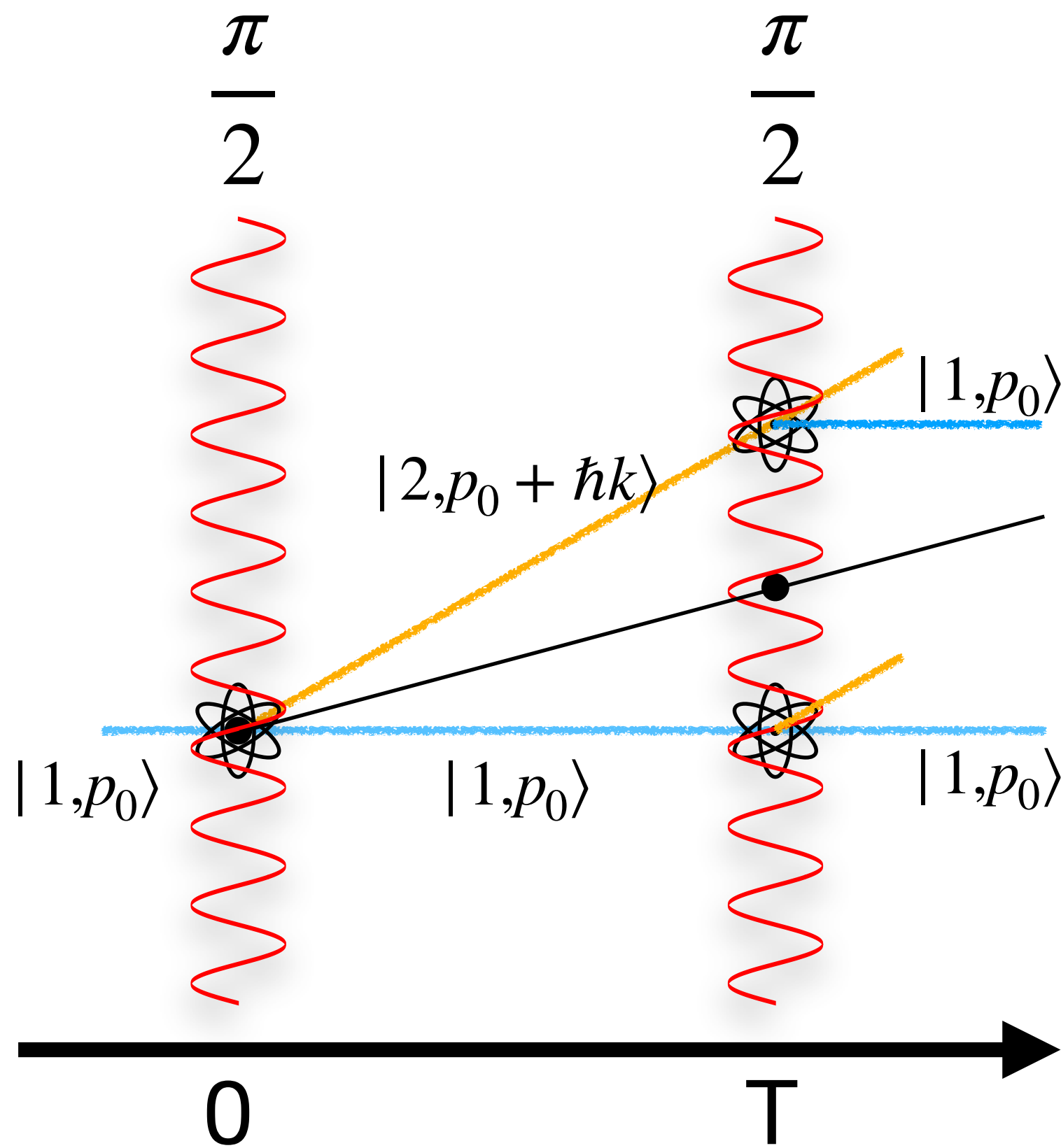
1. Atom Optics
2. Wave Packet propagation
3. Phase shift formulae
4. Applications



Ramsey interferometer

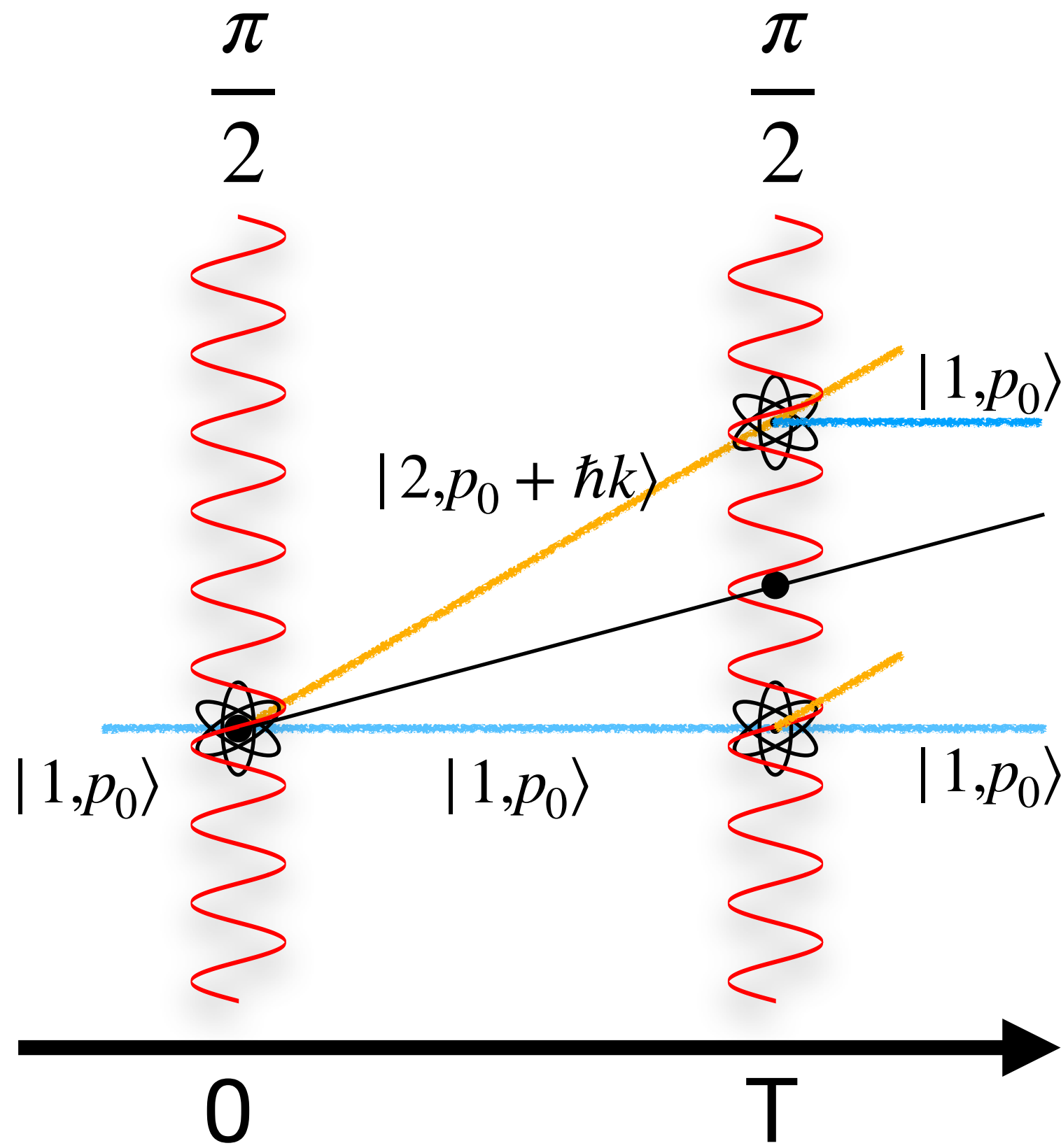
Clock effect

$$\Delta\Phi = \sum_{i=1}^2 (k_{Bi} - k_{Ai}) \frac{r_{Bi} + r_{Ai}}{2} - (\omega_{Bj} - \omega_{Aj})t_j - (\phi_{Bj} - \phi_{Aj}) - \frac{(E_{Bi} - E_{Ai})(t_{i+1} - t_i)}{\hbar}$$



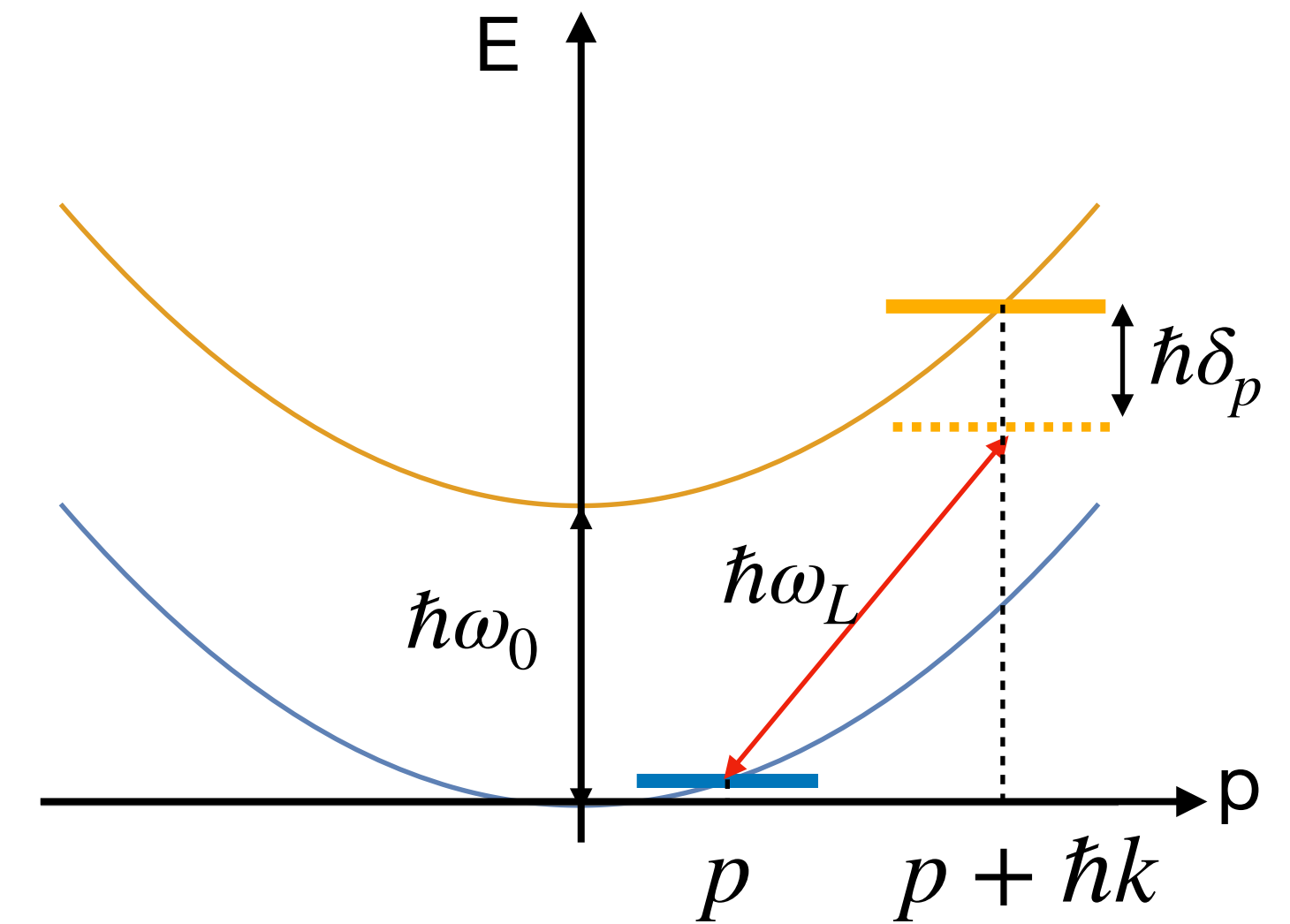
Ramsey interferometer

Clock effect



$$\Delta\Phi = \sum_{i=1}^2 (k_{Bi} - k_{Ai}) \frac{r_{Bi} + r_{Ai}}{2} - (\omega_{Bj} - \omega_{Aj})t_j - (\phi_{Bj} - \phi_{Aj}) - \frac{(E_{Bi} - E_{Ai})(t_{i+1} - t_i)}{\hbar}$$

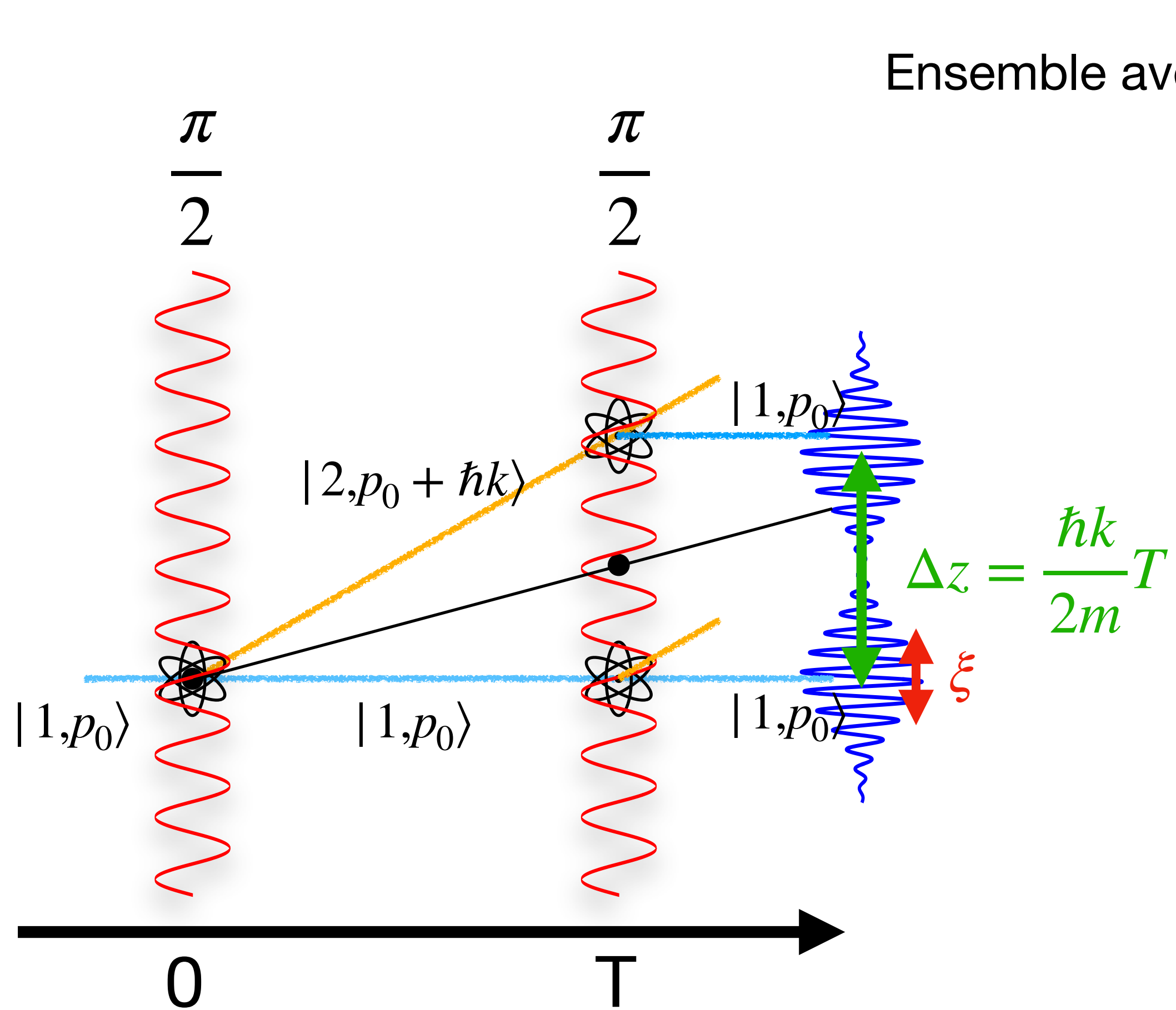
- Detection in port $|1, p_0\rangle$
- $k_A = 0 ; k_{B1} = k ; k_{B2} = -k$
- $\bar{r}_1 = 0 ; \bar{r}_2 = \left(\frac{p_0}{m} + \frac{\hbar k}{2m}\right)T$
- $\omega_{B2} = -\omega_L ; \omega_A = 0$



$$\Delta\phi = -k\left(\frac{p_0}{m} + \frac{\hbar k}{2m}\right)T + \omega_L T - \omega_0 T = \left(\omega_L - \left(\omega_0 + k \cdot v_0 + \frac{\hbar k^2}{2m}\right)\right)T = \delta_p T$$

Ramsey interferometer

Open interferometer = contrast reduction

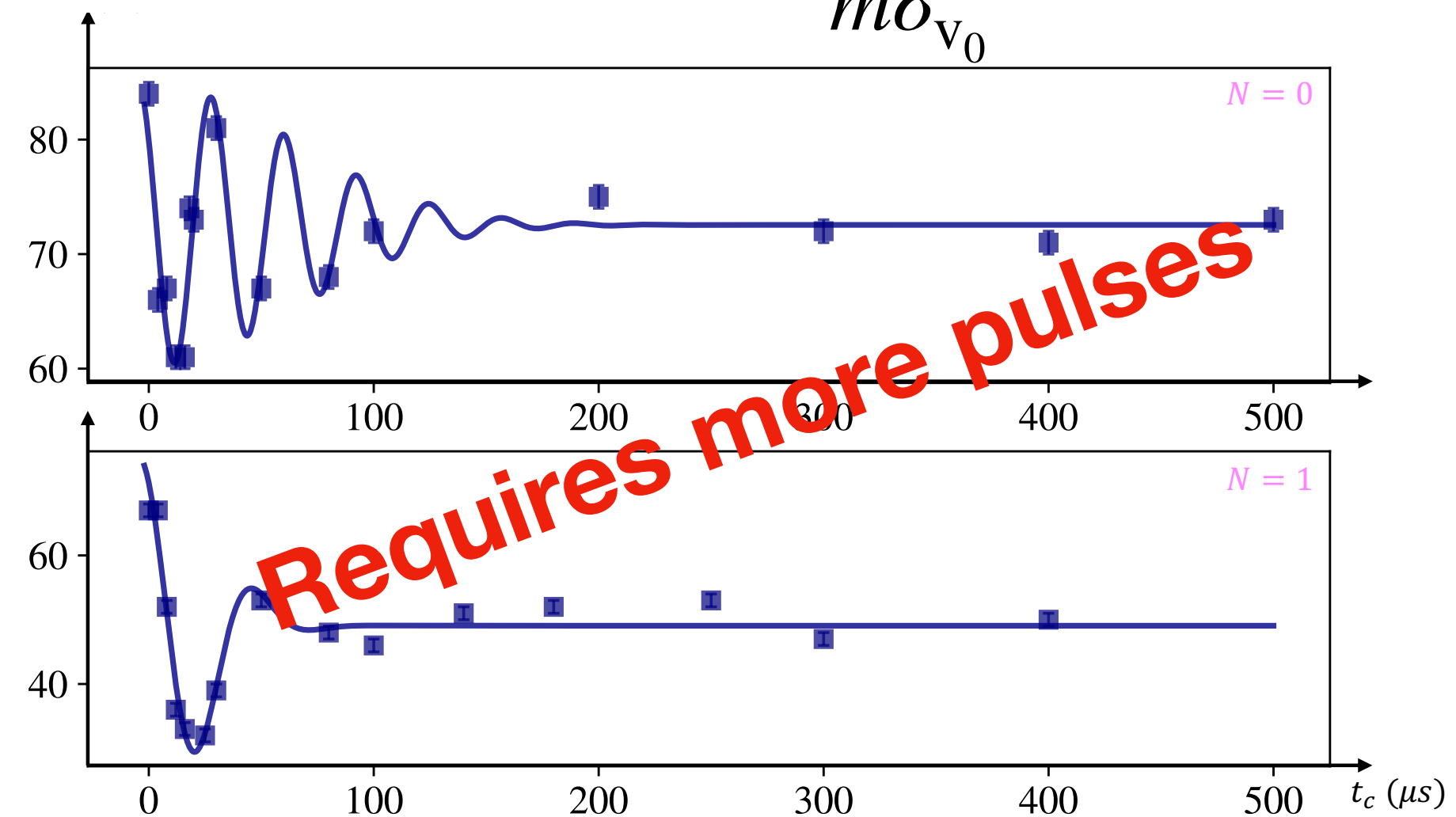


$$\text{Ensemble averaging: } \langle S \rangle \propto \int V \cos((\delta_0 + \omega_R)T - kv_0 T) \exp\left[-\frac{v_0^2}{2\sigma_{v_0}^2}\right] dv_0$$

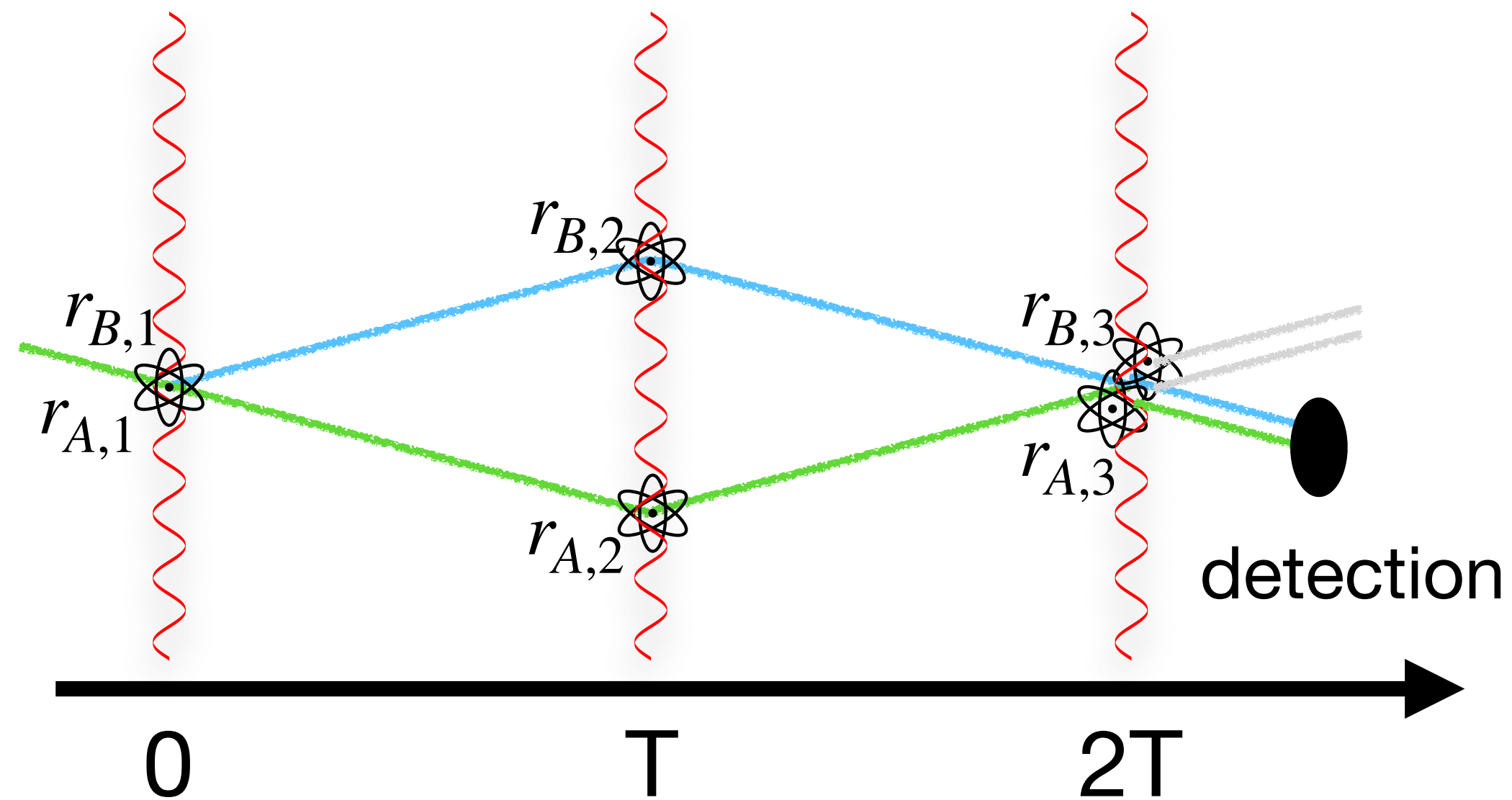
$$\propto V \exp\left(\frac{-kT\sigma_{v_0}^2}{2}\right) \cos((\delta_0 + \omega_R)T)$$

$$\propto V \exp\left(\frac{-\Delta z^2}{2\xi^2}\right) \cos((\delta_0 + \omega_R)T)$$

$$\text{Interference : } \xi = \frac{\hbar}{m\sigma_{v_0}} \gg \Delta z$$



Symmetric 3 pulses Interferometer



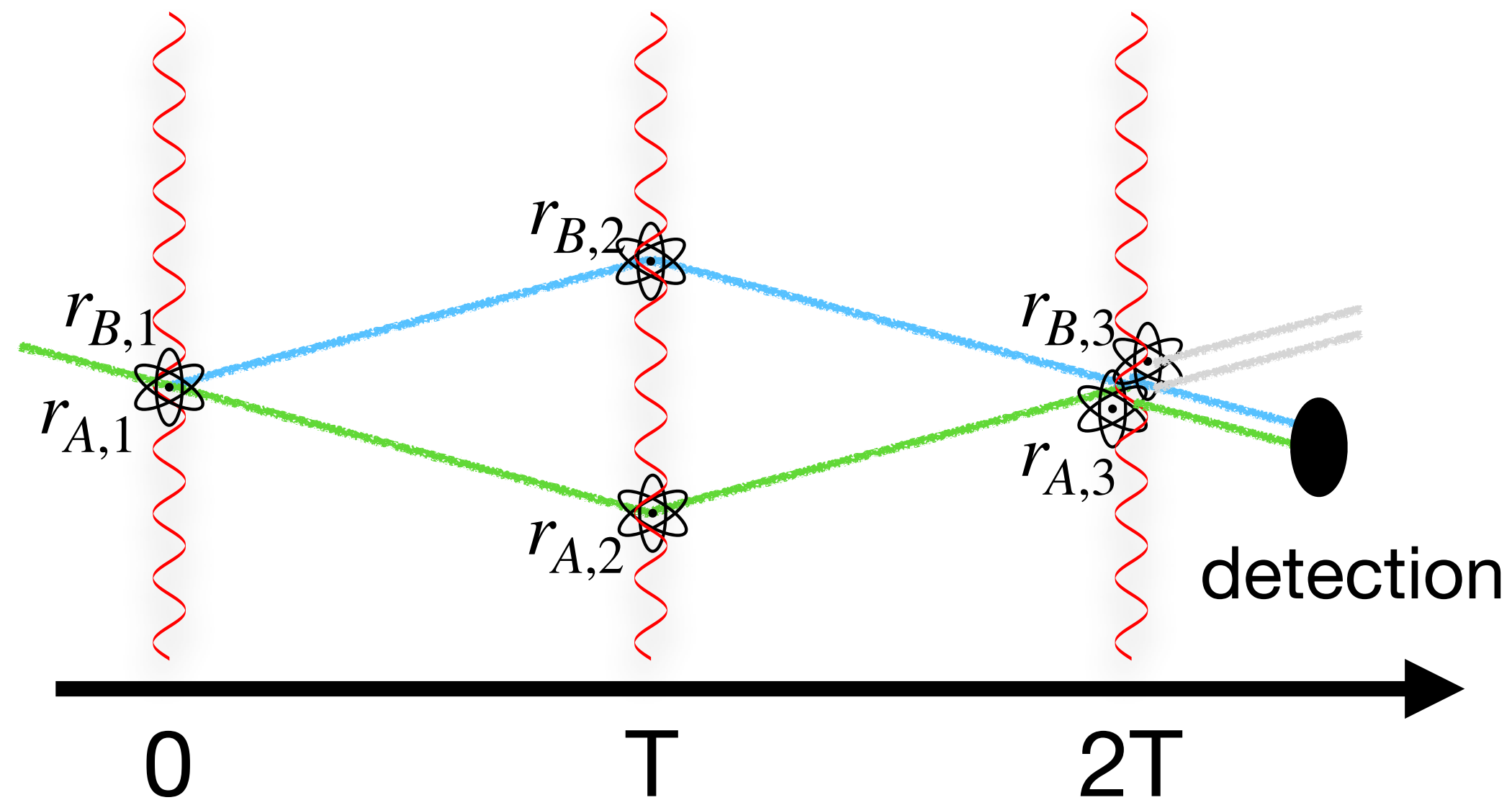
$$\Delta\Phi = \sum_{i=1}^3 (k_{Bi} - k_{Ai}) \frac{r_{Bi} + r_{Ai}}{2} - (\omega_{Bj} - \omega_{Aj})t_j - (\phi_{Bj} - \phi_{Aj})$$

	k_{Aj}	k_{Bj}	ω_{Aj}	ω_{Bj}	ϕ_{Aj}	ϕ_{Bj}
$t_1 = 0$	0	$+k$	0	$-\omega$	0	$-\phi_1$
$t_2 = T$	$+k$	$-k$	$-\omega$	$+\omega$	$-\phi_2$	$+\phi_2$
$t_3 = 2T$	$-k$	0	$+\omega$	0	$+\phi_3$	0

$$\Delta\Phi = k \cdot (\bar{r}_1 - 2\bar{r}_2 + \bar{r}_3) \longrightarrow \text{Need to calculate the classical trajectory}$$

Symmetric 3 pulses Interferometer

Gravity



Hamiltonian: $H = \frac{p^2}{2m} - mgz$

Trajectory: $z = z_0 + v_0 t - \frac{gt^2}{2}$

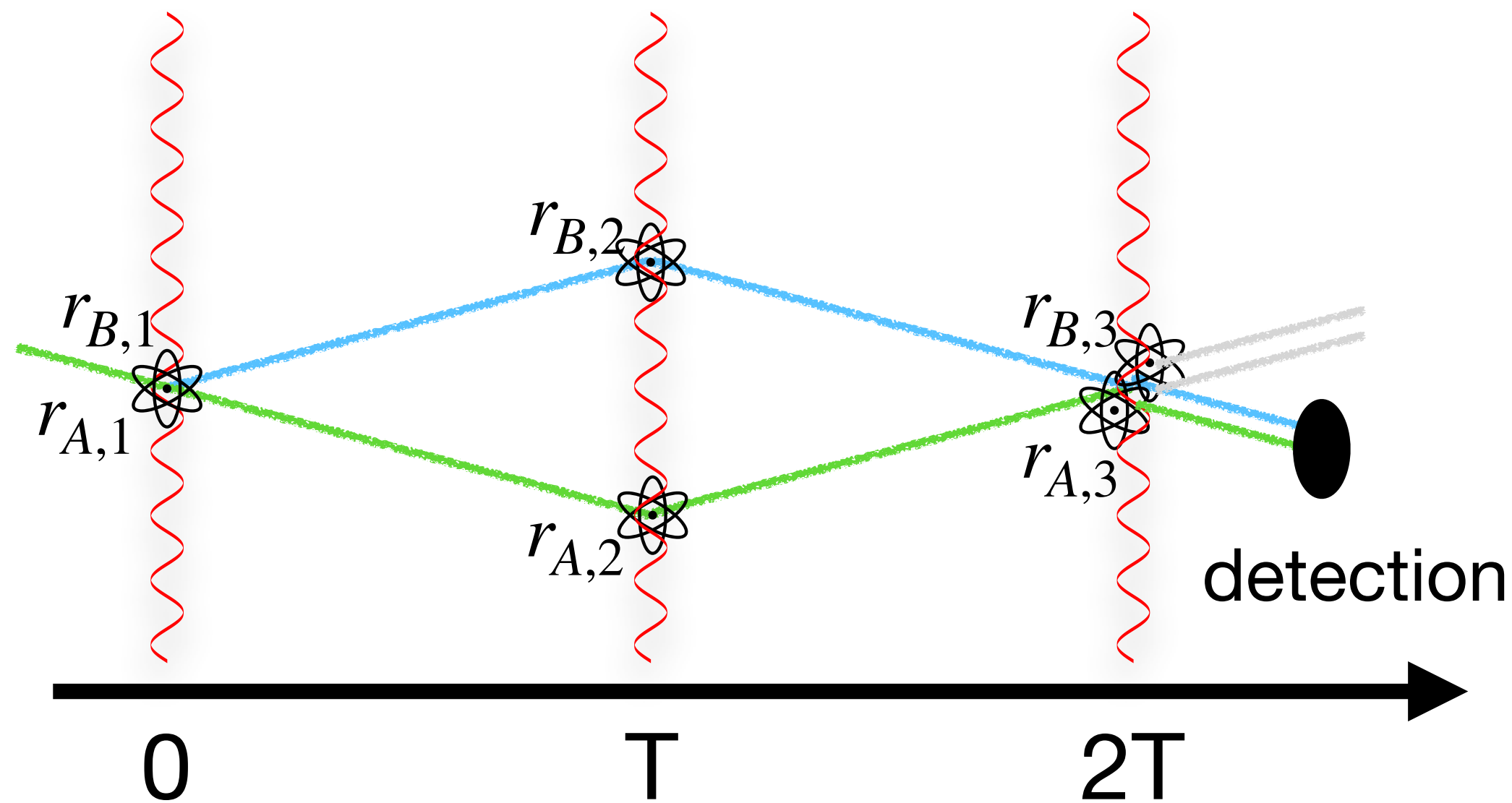
Mid point-trajectory with $v_0 = v_i + \frac{\hbar k}{2m}$

$$\Delta\Phi = k \cdot (z_1 - 2z_2 + z_3) = k \cdot \left(z_0 - 2z_0 - 2\left(v_i + \frac{\hbar k}{2m}\right)T - 2\frac{gT^2}{2} + z_0 + \left(v_i + \frac{\hbar k}{2m}\right)2T + \frac{g(2T)^2}{2} \right)$$

$$\Delta\phi = kgT^2$$

Symmetric 3 pulses Interferometer

Gravity + gradient



Hamiltonian: $H = \frac{p^2}{2m} - mgz + \frac{m\Gamma z^2}{2}$

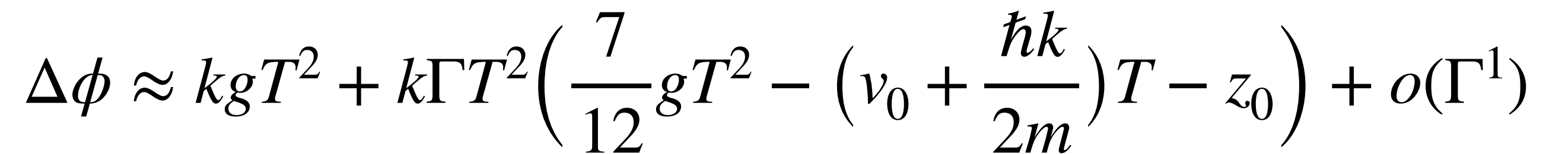
Trajectory: $z = \frac{g + (z_0\Gamma - g)\cosh(\sqrt{\Gamma}t) + v_0\sqrt{\Gamma} \sinh(\sqrt{\Gamma}t)}{\Gamma}$

Mid point-trajectory with $v_0 = v_i + \frac{\hbar k}{2m}$

$$\Delta\phi = \frac{k}{\sqrt{\Gamma}} \left[\left(v_i + \frac{\hbar k}{2m} \right) (\sinh(2T\sqrt{\Gamma}) - 2 \sinh(T\sqrt{\Gamma})) + \sqrt{\Gamma} \left(z_0 - \frac{g}{\Gamma} \right) (1 + \cosh(2T\sqrt{\Gamma}) - 2 \cosh(T\sqrt{\Gamma})) \right]$$

$$\approx kgT^2 + k\Gamma T^2 \left(\frac{7}{12}gT^2 - \left(v_i + \frac{\hbar k}{2m} \right) T - z_0 \right) + o(\Gamma^1)$$

Gravity + gradient



 $v_0 \approx 0$

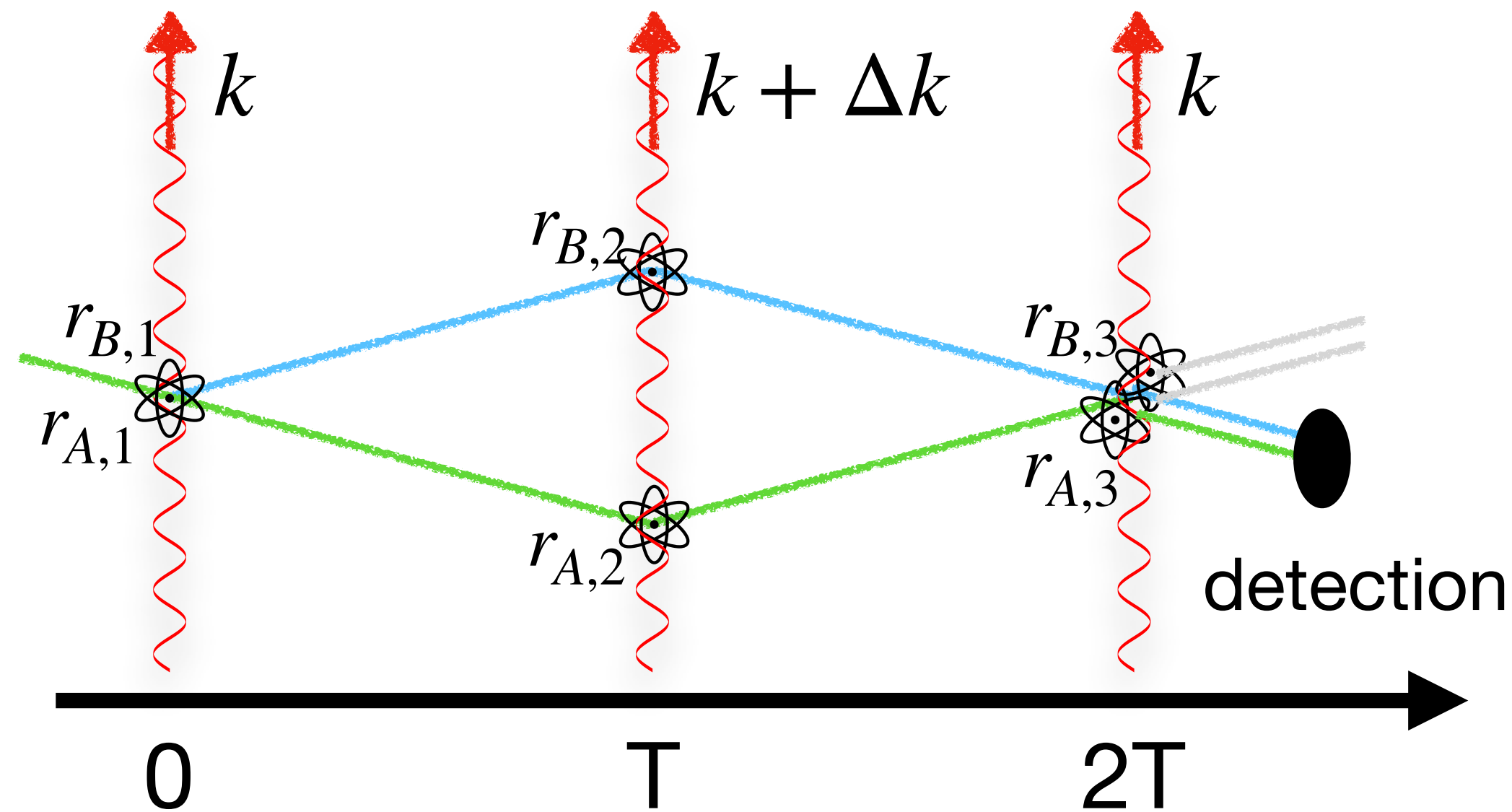
$$\frac{\phi_g}{\phi_\Gamma} \approx 10^{-7}$$

- Constant gravity gradient needs to be considered in accuracy budget

- $\Delta\phi$ depends on v_0 can decrease visibility: $\sigma_{v_0} \rightarrow \sigma_\phi \rightarrow V \propto e^{-\frac{\sigma_\phi^2}{2}}$

Symmetric 3 pulses Interferometer

Gravity + gradient



$$\Delta k = \frac{1}{2}\Gamma T^2 k$$

$$\Delta\phi \approx kgT^2 + k\Gamma T^2 \left(\frac{7}{12}gT^2 - \left(v_0 + \frac{\hbar k}{2m} \right) T - z_0 \right) + o(\Gamma^1)$$

A constant gravity gradient can be reasonably canceled !

- ☑ Visibility improvement
- ☑ Relax knowledge on v_0 in fountain
- ☑ Measurement defined at the mirror position

Symmetric 3 pulses Interferometer

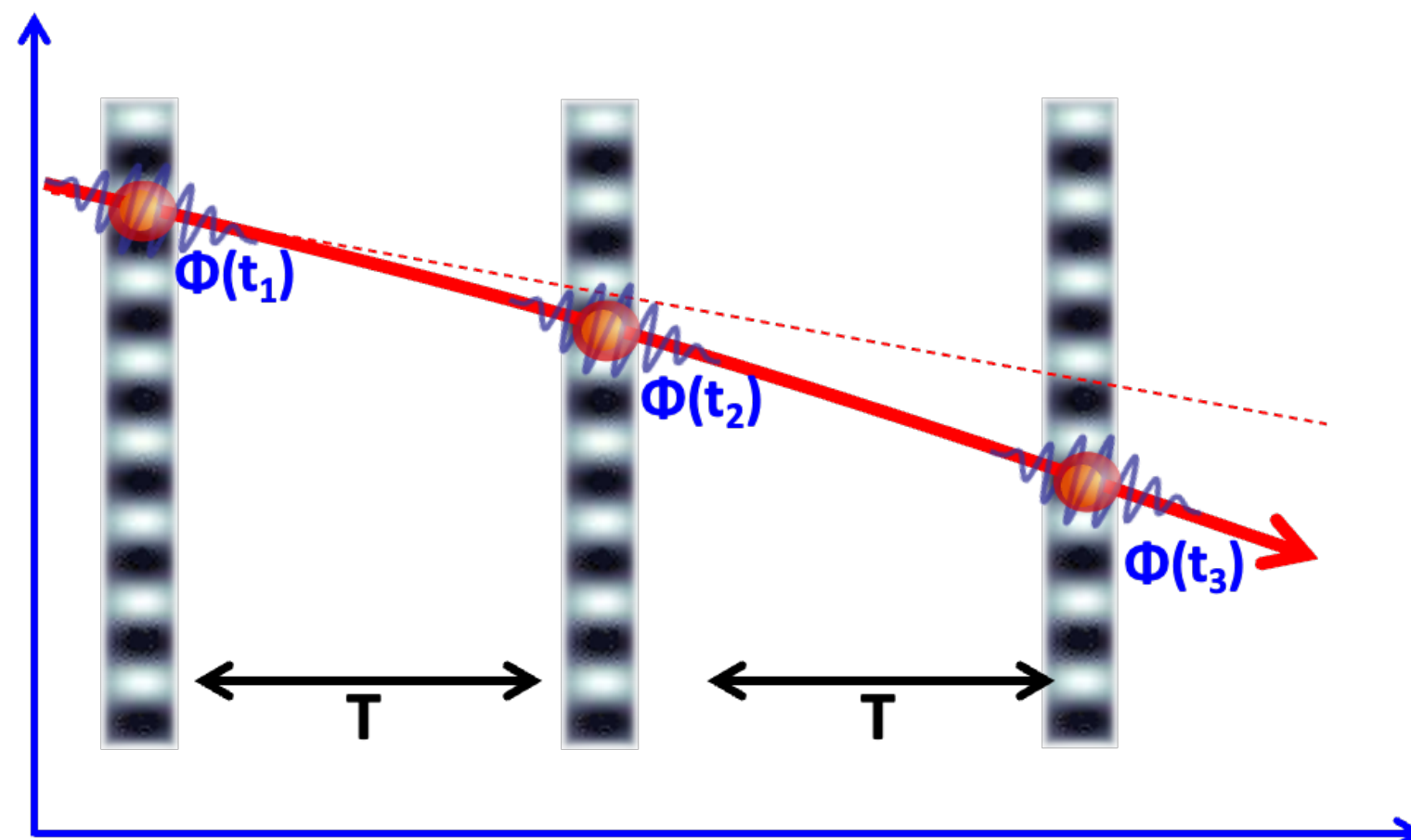
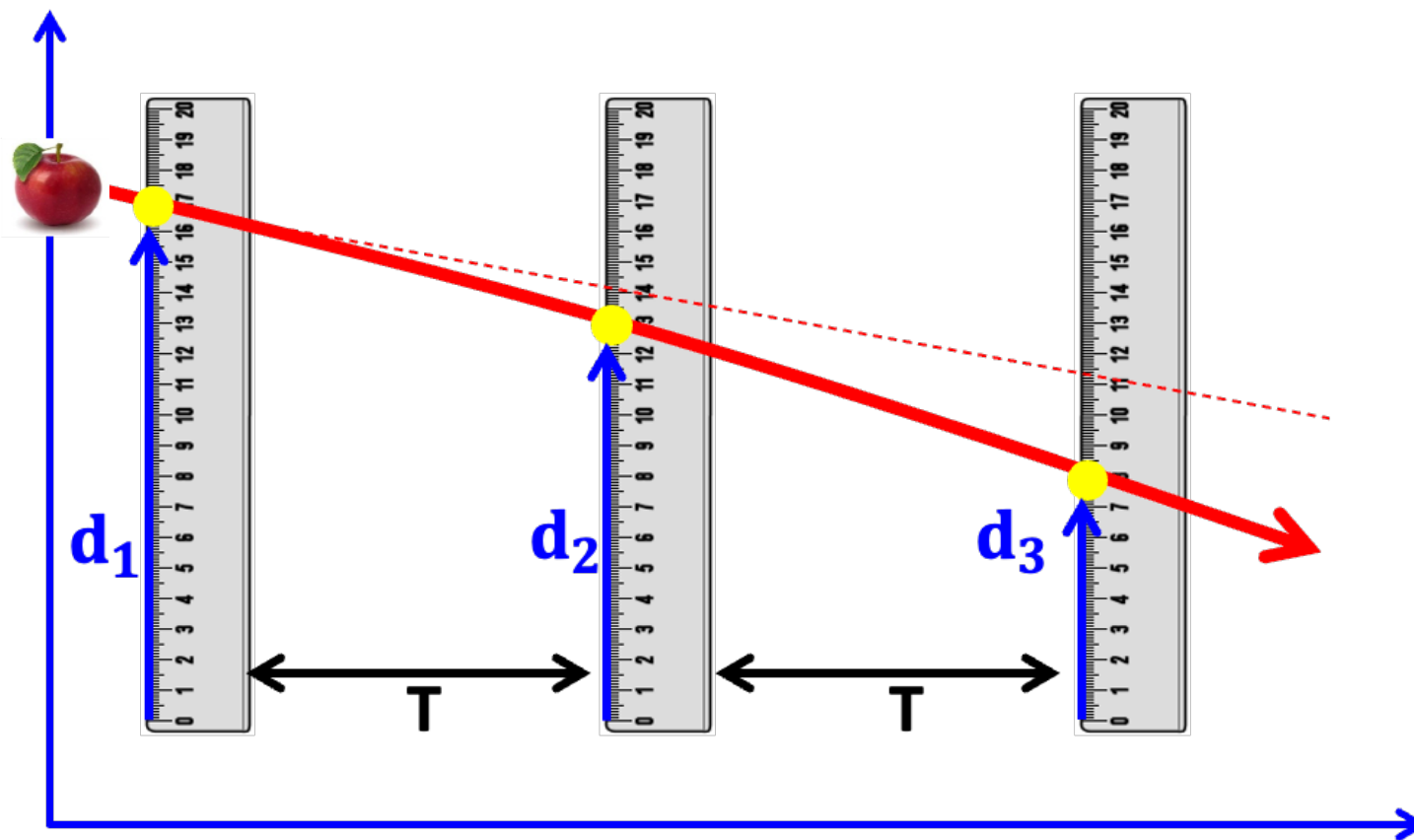
Applications: absolute gravimeter



Absolute g = trajectory of a free-falling test mass

$$a = \frac{d_1 - 2d_2 + d_3}{T^2}$$

- Laser = absolute ruler
- Atom = ideal mass test



Atom Interferometer = atom + laser

$$\Delta\Phi = \frac{2\pi}{\lambda} \cdot (\bar{r}_1 - 2\bar{r}_2 + \bar{r}_3) \quad a = \frac{d_1 - 2d_2 + d_3}{T^2} = \frac{\lambda}{2\pi} \frac{\Delta\phi}{T^2}$$

Displacement of free-falling atoms using an optical ruler.

Gravimetry

Gravimeter « Observatoire de Paris »

1m vertical motion

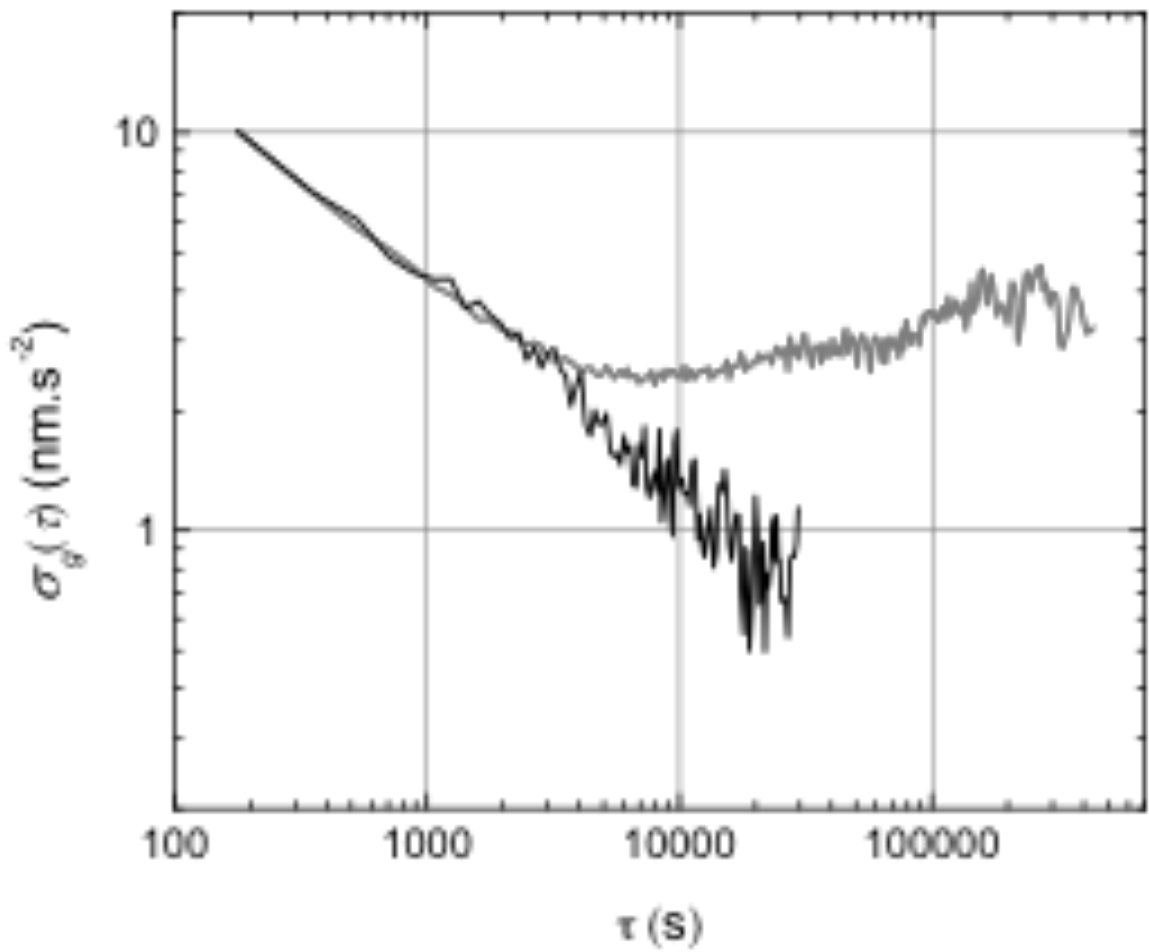
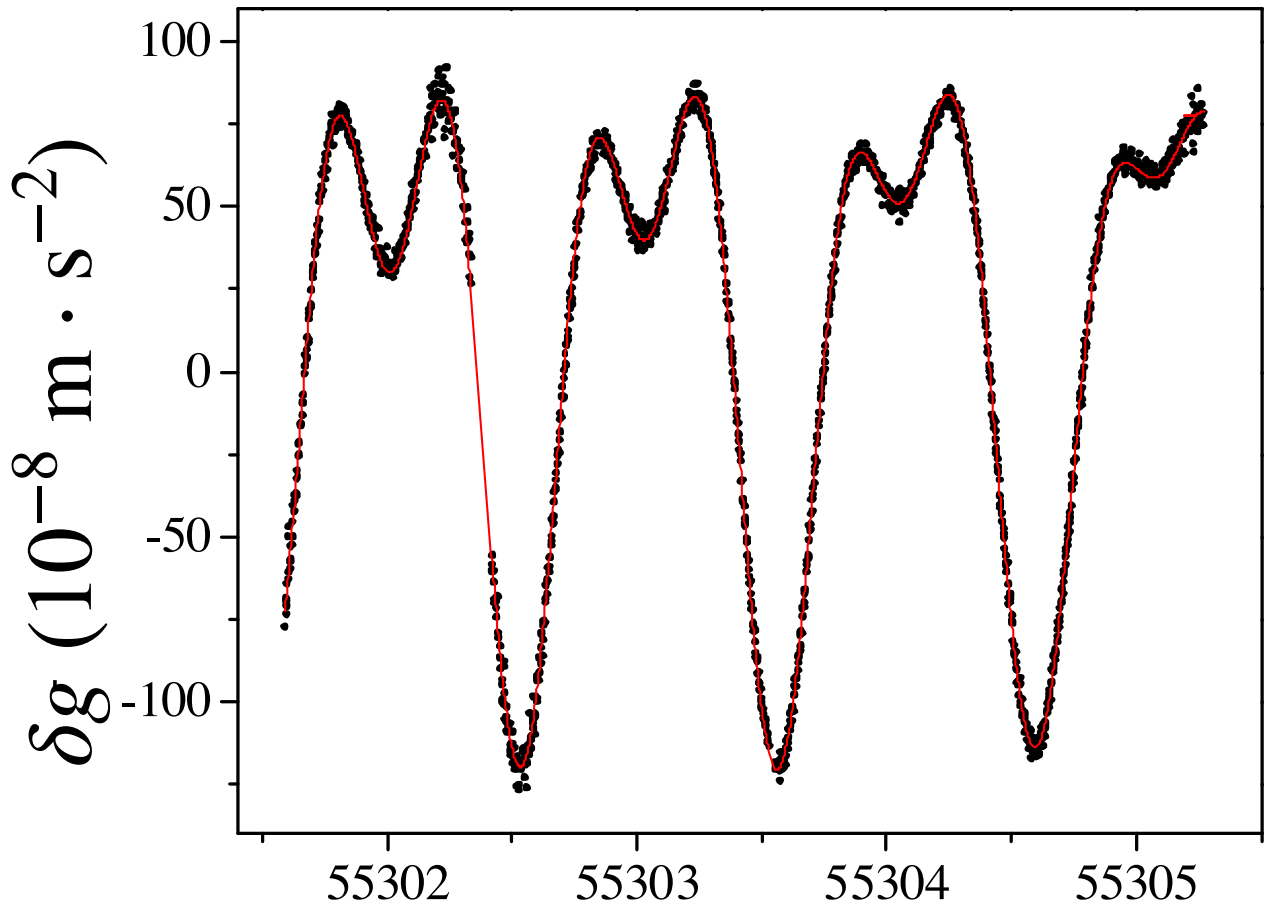
$$g(Tlse) = 9.804\,312\,268\,\text{m} \cdot \text{s}^{-2}$$

Earth Flatenning 150kg at few m

Effect	Bias μGal	u μGal
Alignments	0.3	0.5
Frequency reference	0.5	<0.1
RF phase shift	0.0	<0.1
<i>vgg</i>	-13.4	<0.1
Self gravity effect	-2.1	0.1
Coriolis	-5.3	0.8
Wavefront aberrations	-5.6	1.3
LS1	0.0	<0.1
Zeeman	0.0	<0.1
LS2	-3.6	0.8
Detection offset	0.0	0.5
Optical power	0.0	0.5
Cloud indice	0.4	<0.1
Cold collisions	<0.1	<0.1
CPT	0.0	<0.1
Raman α LS	0.3	<0.1
Finite Speed of Light	0.0	<0.1
TOTAL	-28.5	2.0

Best short term stability: $\sim 6 \times 10^{-8} \text{ m} \cdot \text{s}^{-2} @ 1 \text{ s}$

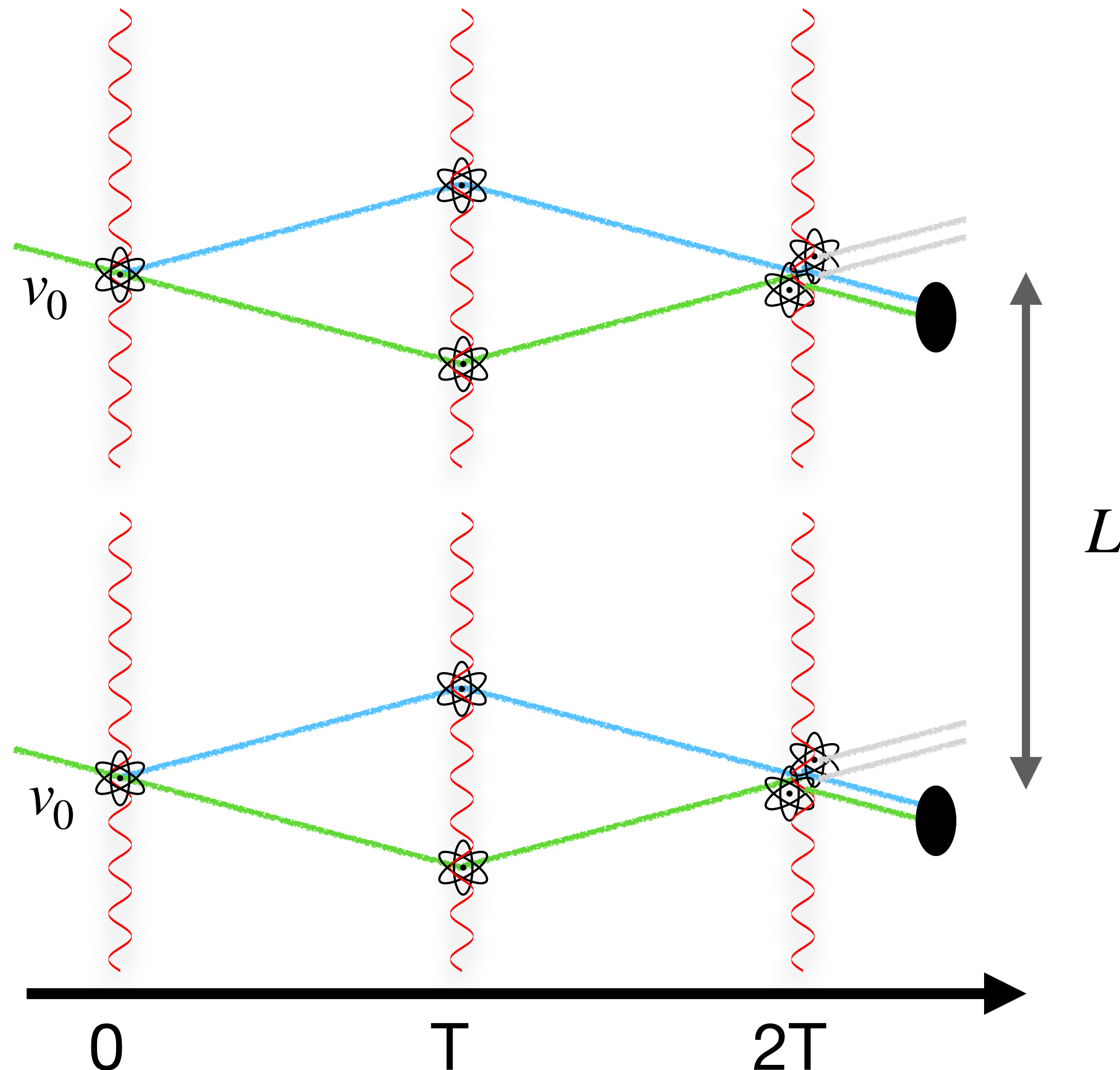
Best long term stability: $\sim 6 \times 10^{-10} \text{ m} \cdot \text{s}^{-2}$



Accuracy budget $10^{-9}g$

Symmetric 3 pulses Interferometer

Gradiometer configuration



Gravity variations at high frequency

$$\Delta\phi^{up} - \Delta\phi^{down} \sim LkT^3\Gamma$$

Rejecting correlated noise:

- EP-tests with atoms
- Gravitational Wave
- Dark Matter
- G constant measurement

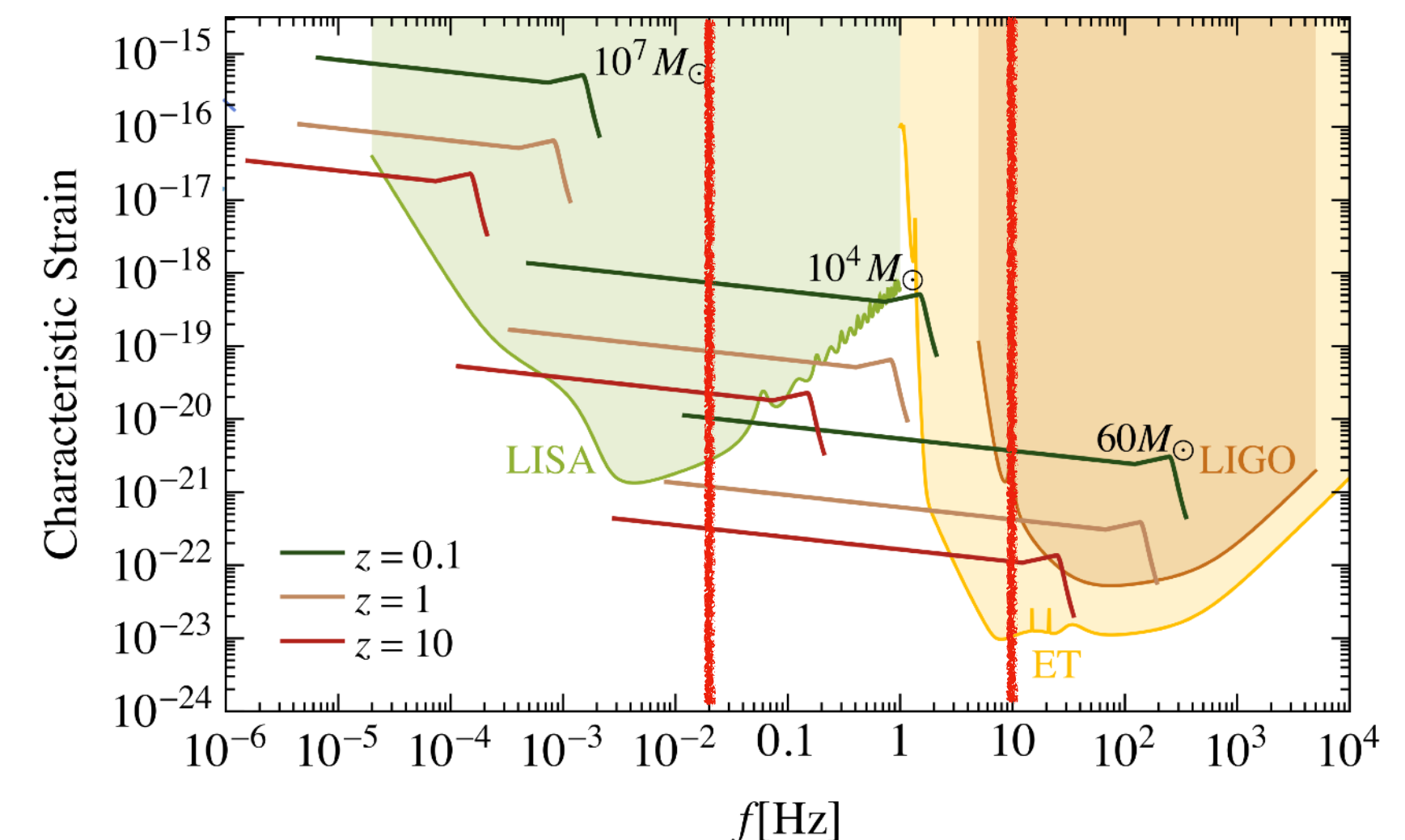
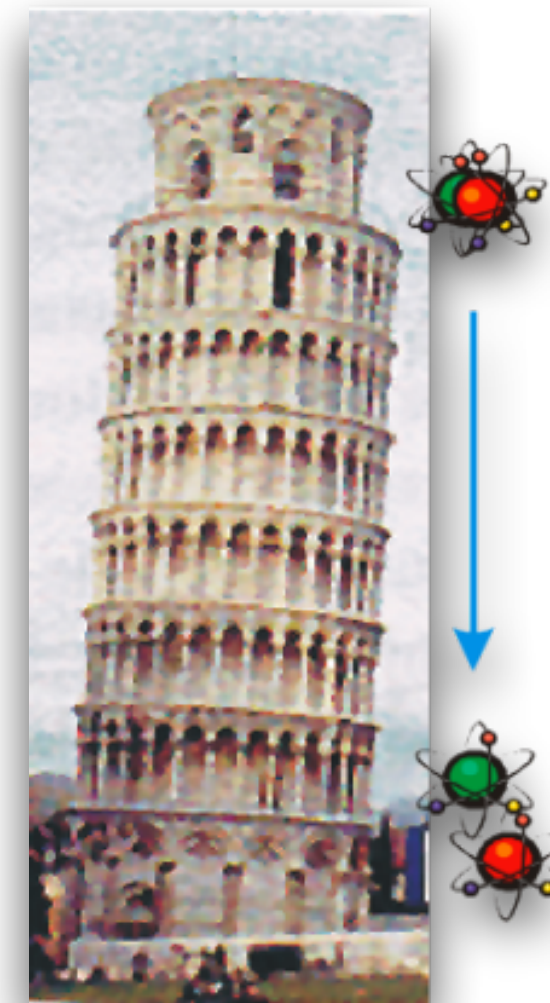
Symmetric 3 pulses Interferometer

Applications: fundamental tests

Acceleration proportional to spacetime area enclosed: $\Delta\phi = kT^2 \cdot a$

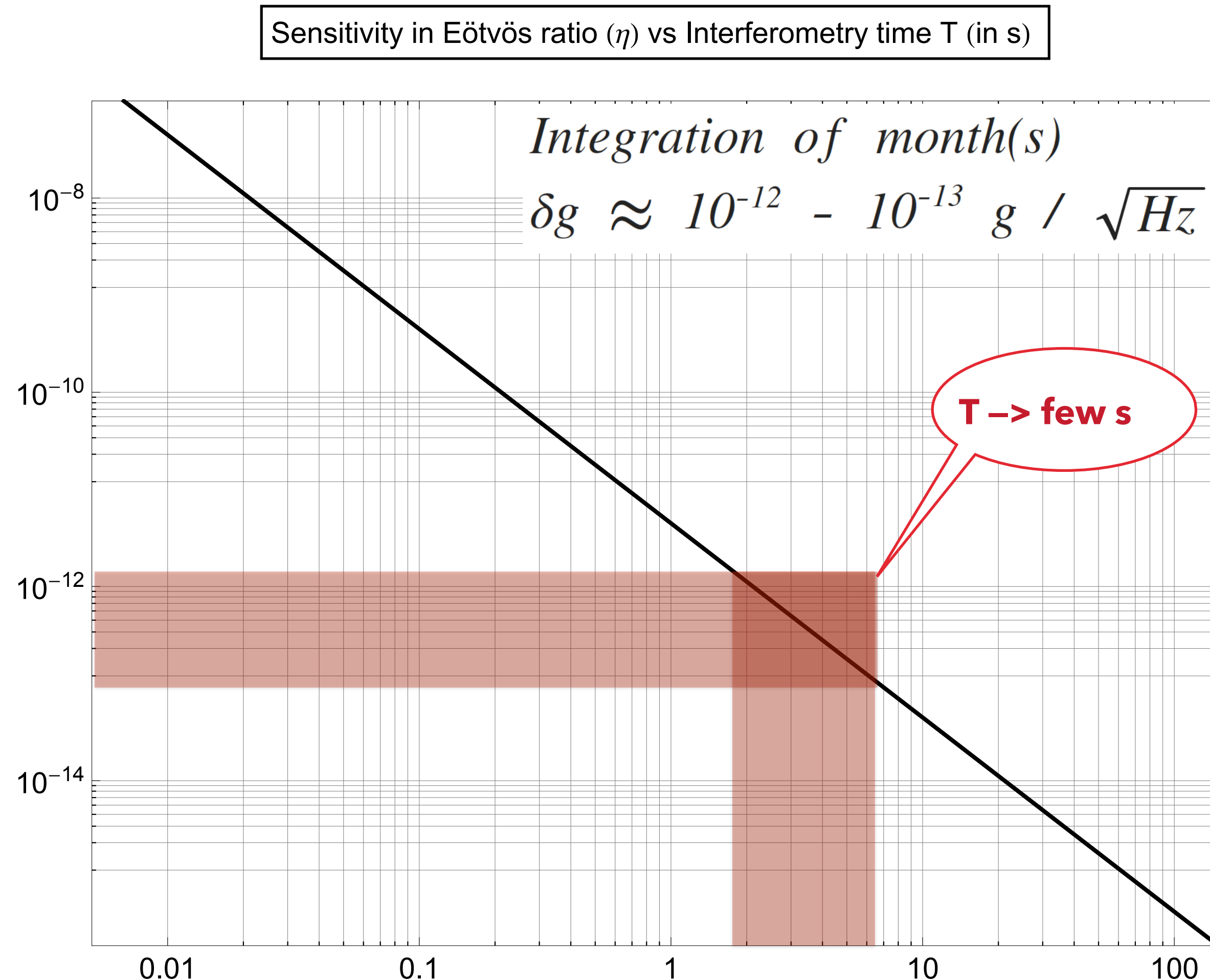
Sensitivity: $\frac{\sigma_a}{a} = \frac{\sigma_{\Delta\phi}}{kT^2} \longrightarrow$ long duration, large momentum transfer

- Measurement of relativistic effects.
- Equivalence principle.
- Gravitational waves.



Symmetric 3 pulses

Required sensitivity for challenging State-of-The Art



EP test at $\eta = 10^{-15}$ as an example
 $^{87}\text{Rb} / ^{85}\text{Rb}$ as test pair

$$\delta g \propto \frac{1/\sqrt{N}}{k_{\text{eff}} T^2} \quad \text{Quantum Projection Noise}$$

$$N \approx 10^6 \text{ atoms}$$

$$k_{\text{eff}} \approx 2 \cdot 2\pi / 780 \text{ nm}$$

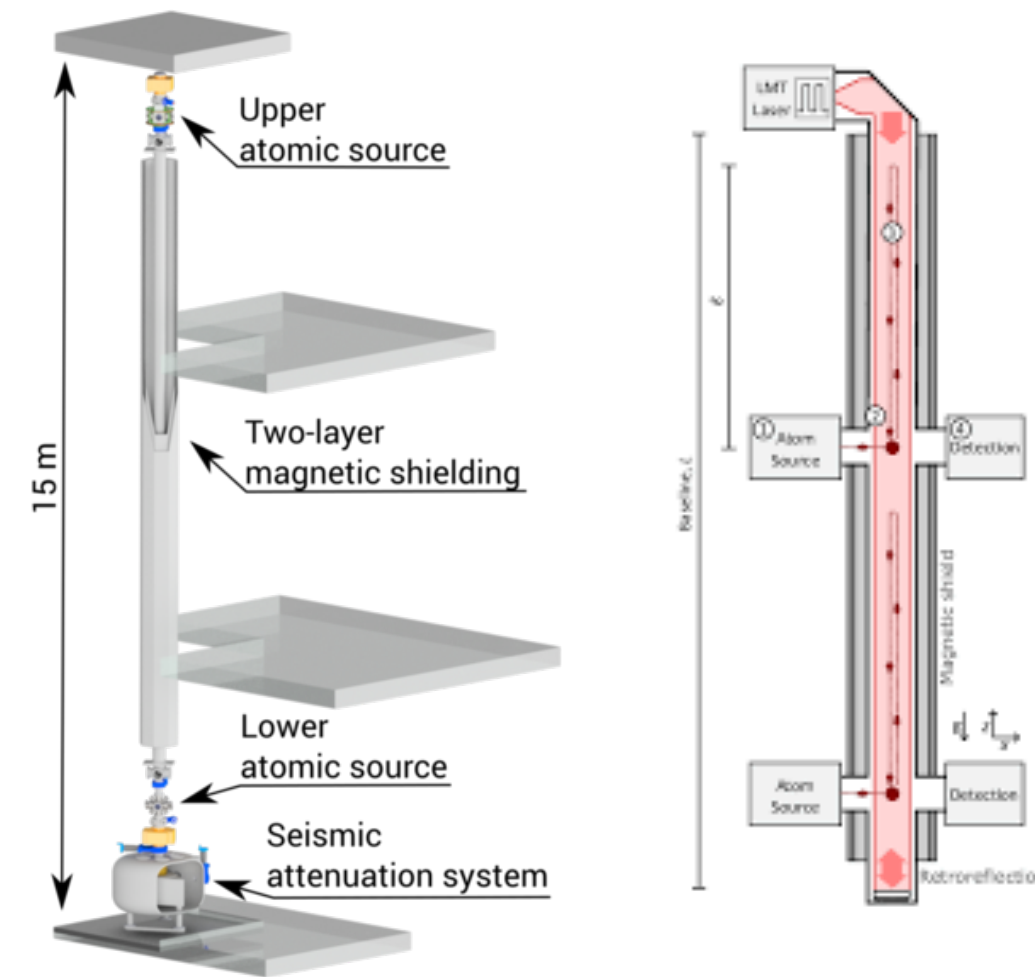
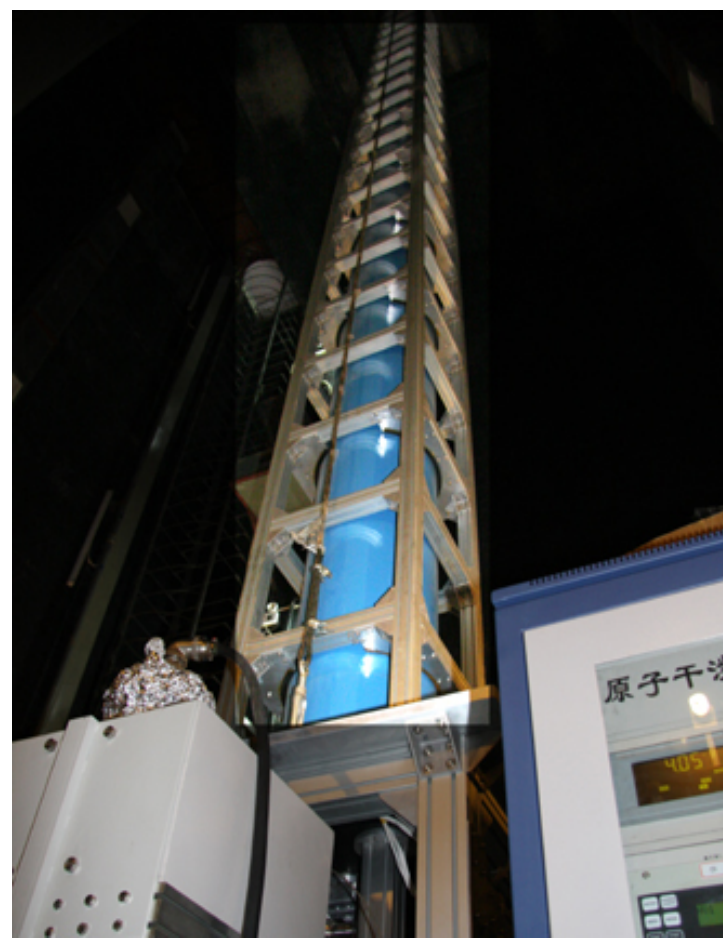
Long interferometry time

Courtesy of N. Gaaloul

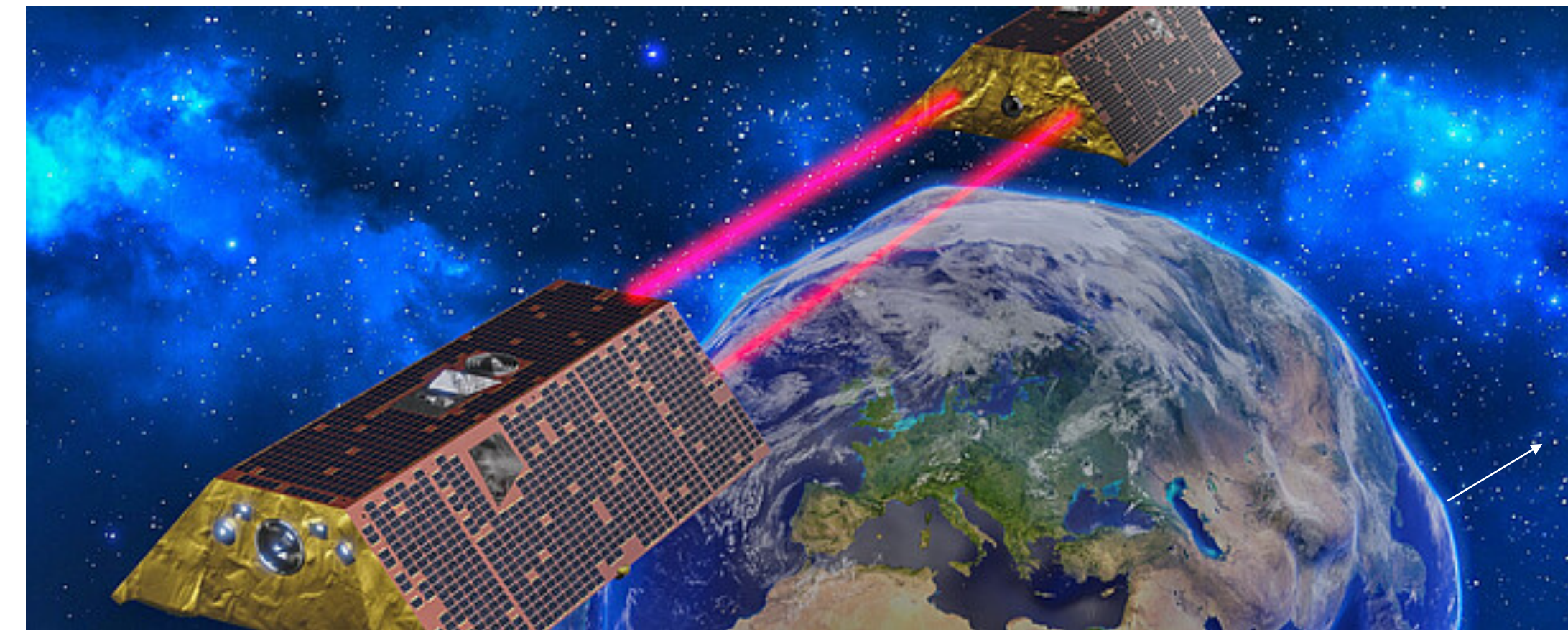
Symmetric 3 pulses

New large Terrestrial Infrastructure and space missions

Large fountain: (Stanford, AION, Wuhan, VLBAI ...)



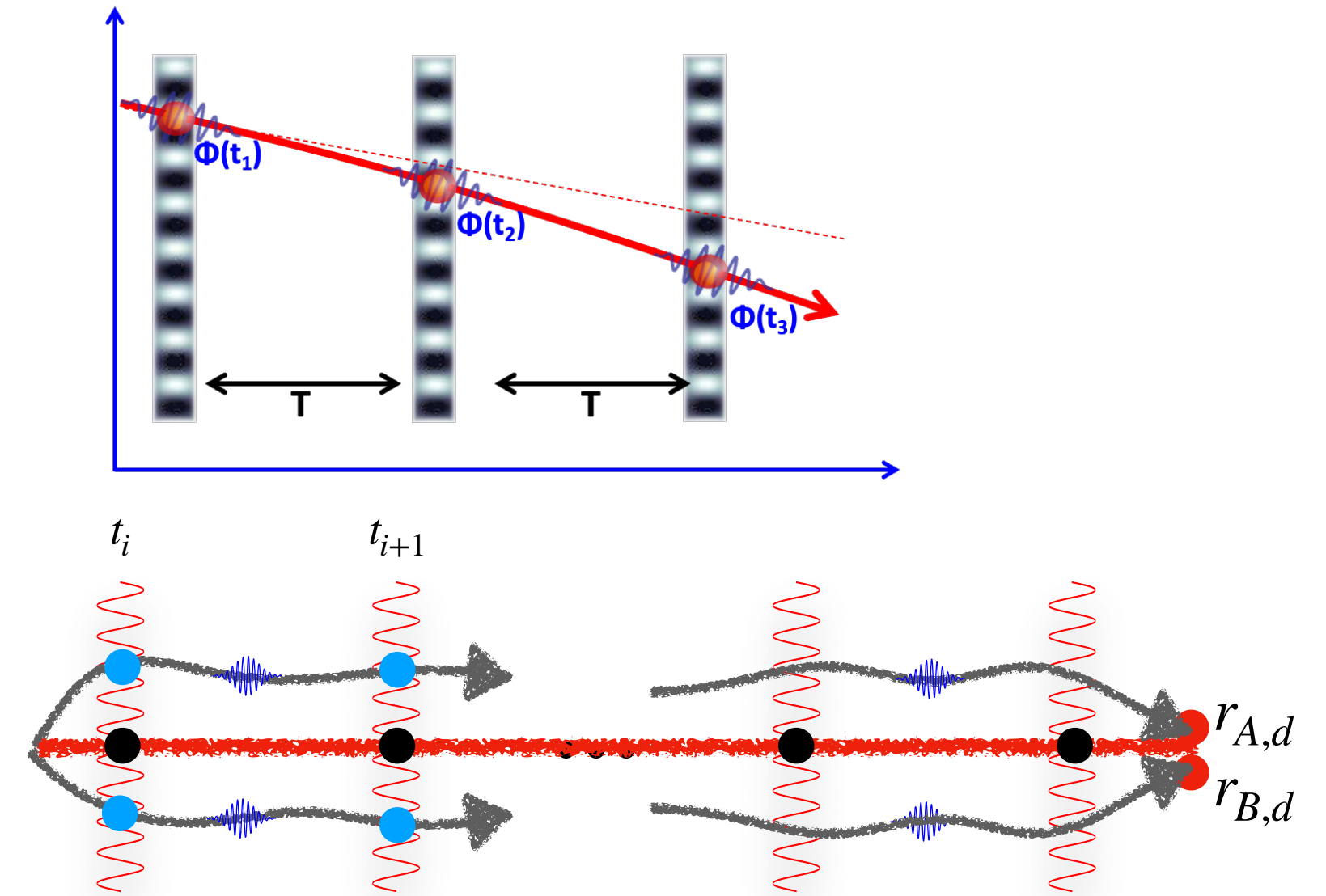
Space missions



Conclusion

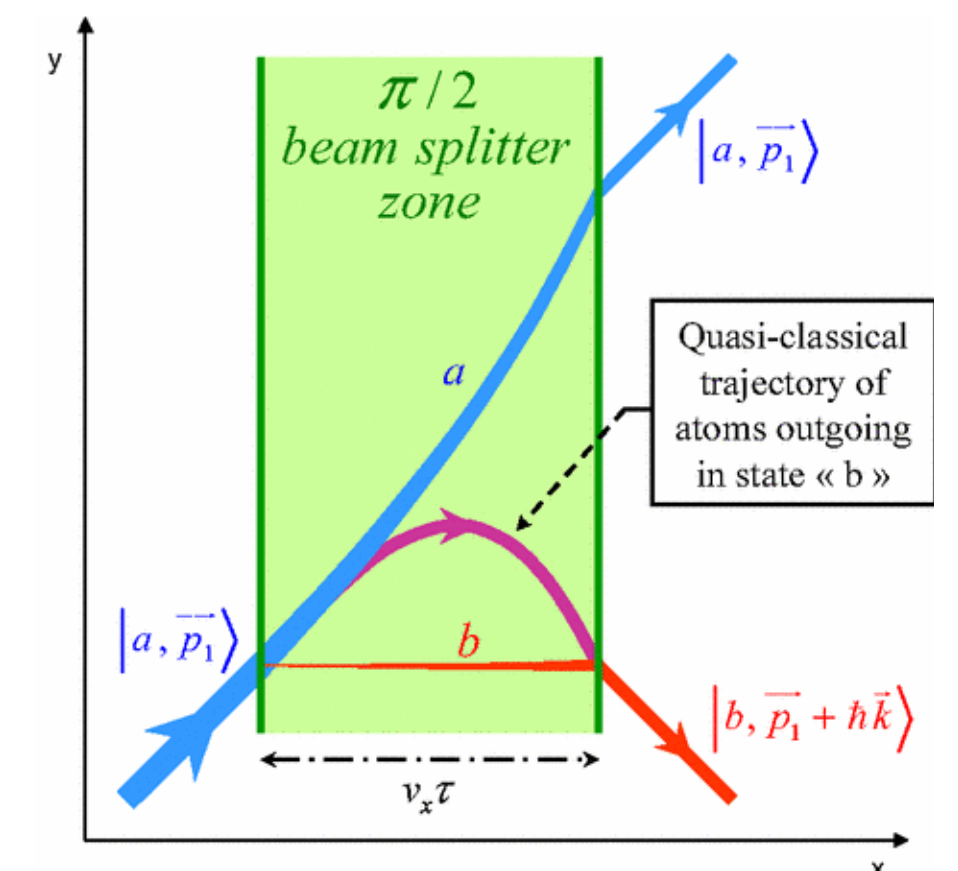
- ✓ Free fall Atom interferometer = absolute sensor
- ✓ Exact phase shift calculation for quadratic hamiltonian and δ -like pulse:

The phase shift is the laser phase printed at the centre of the wave packet.



Suggestions for further studies:

- Considering inertial forces during the pulses
- Impact of atom interactions
- Relativistic framework unification of atom and light optics

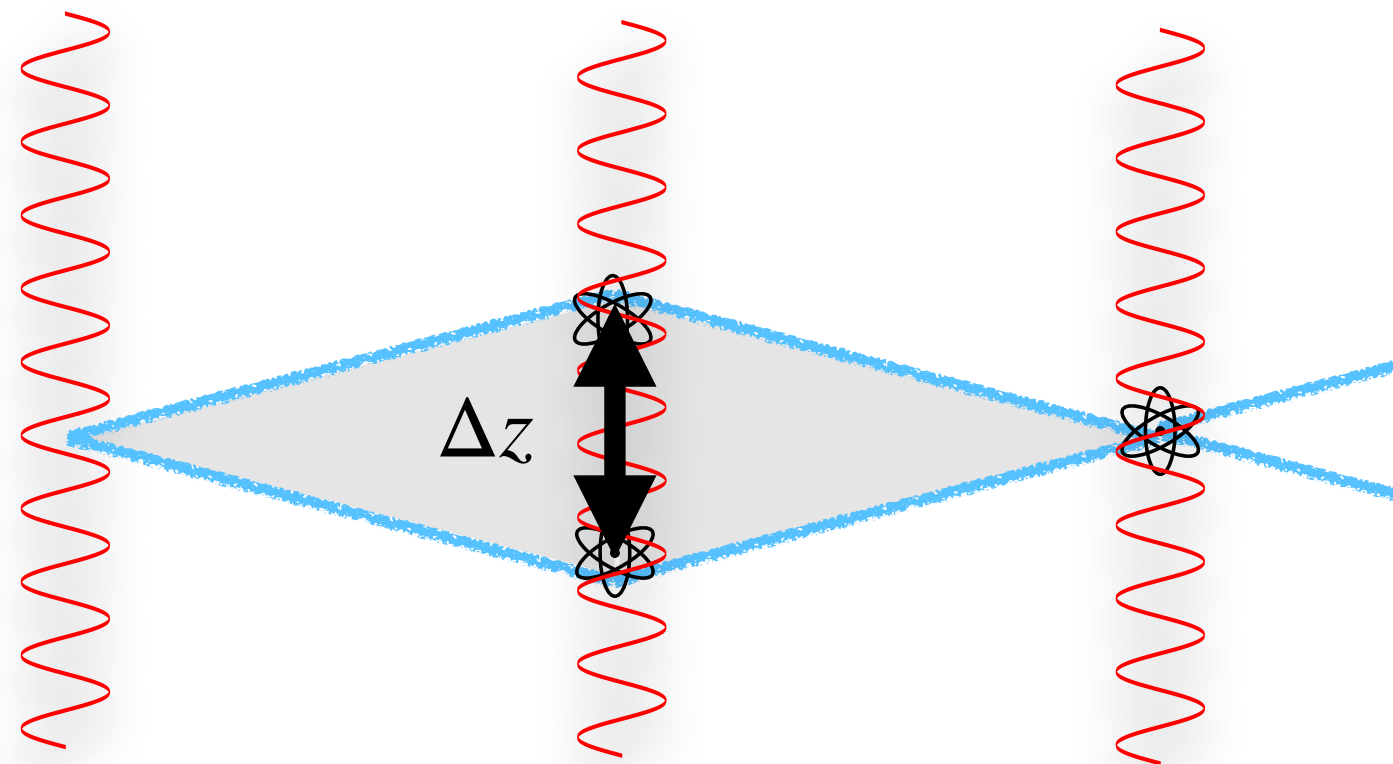


Conclusion

Force sensitivity scales with space-time area

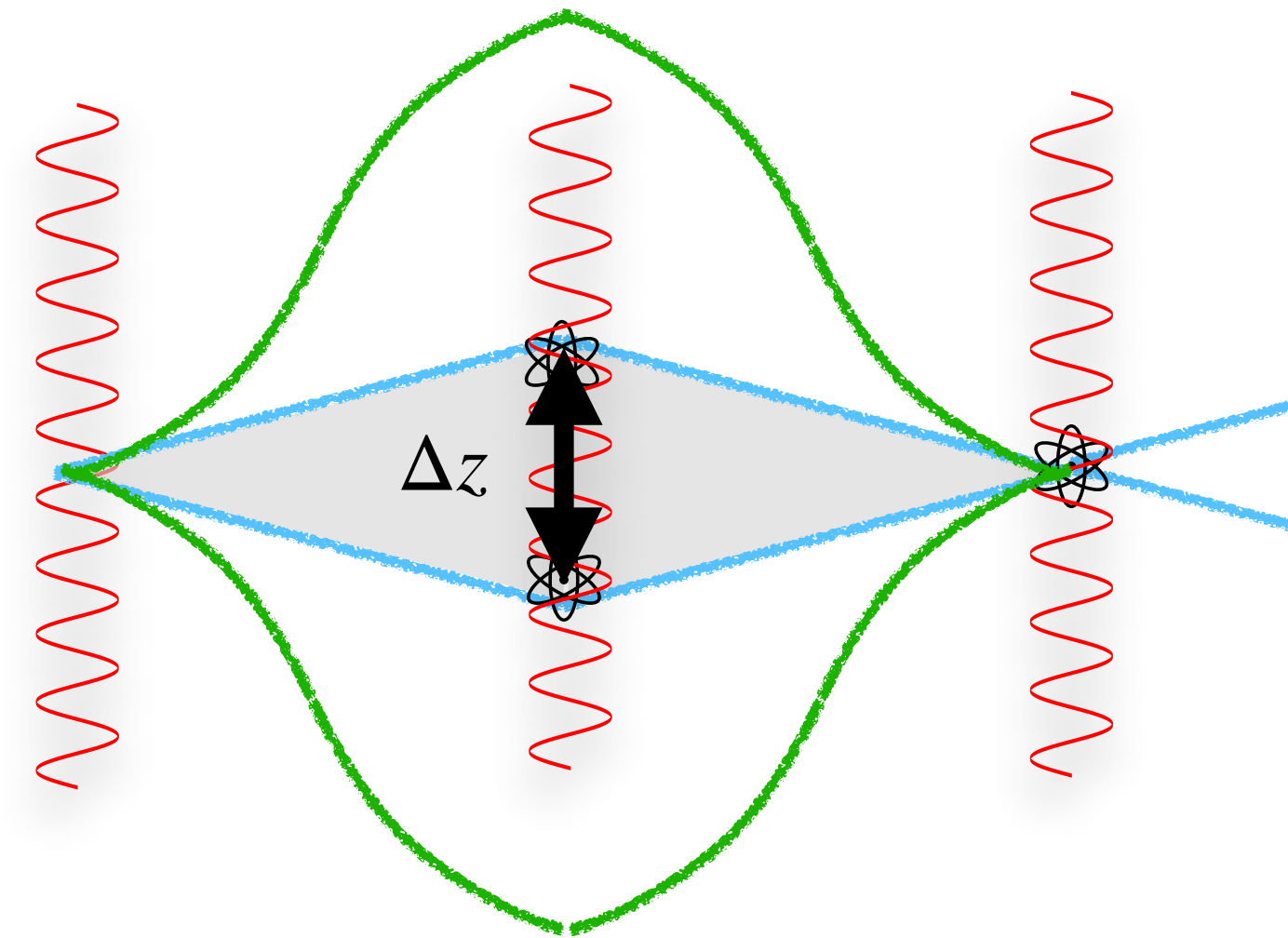
Free Fall interferometers

$$\mathcal{A} \propto T \cdot \Delta z \propto k_{eff} \cdot T^2$$



Optimal sensitivity: Continuous acceleration in a lattice

$$\mathcal{A} \propto \Delta z \times T \propto a_{\text{LMT}} T^3$$



Long interaction time + Large Momentum Transfer + high acceleration

Large Momentum Transfer Atom Interferometer: LMT Interferometer

Lecture 2

A. Gauguet - FOMO Summer School 2026 - Piombino Italy

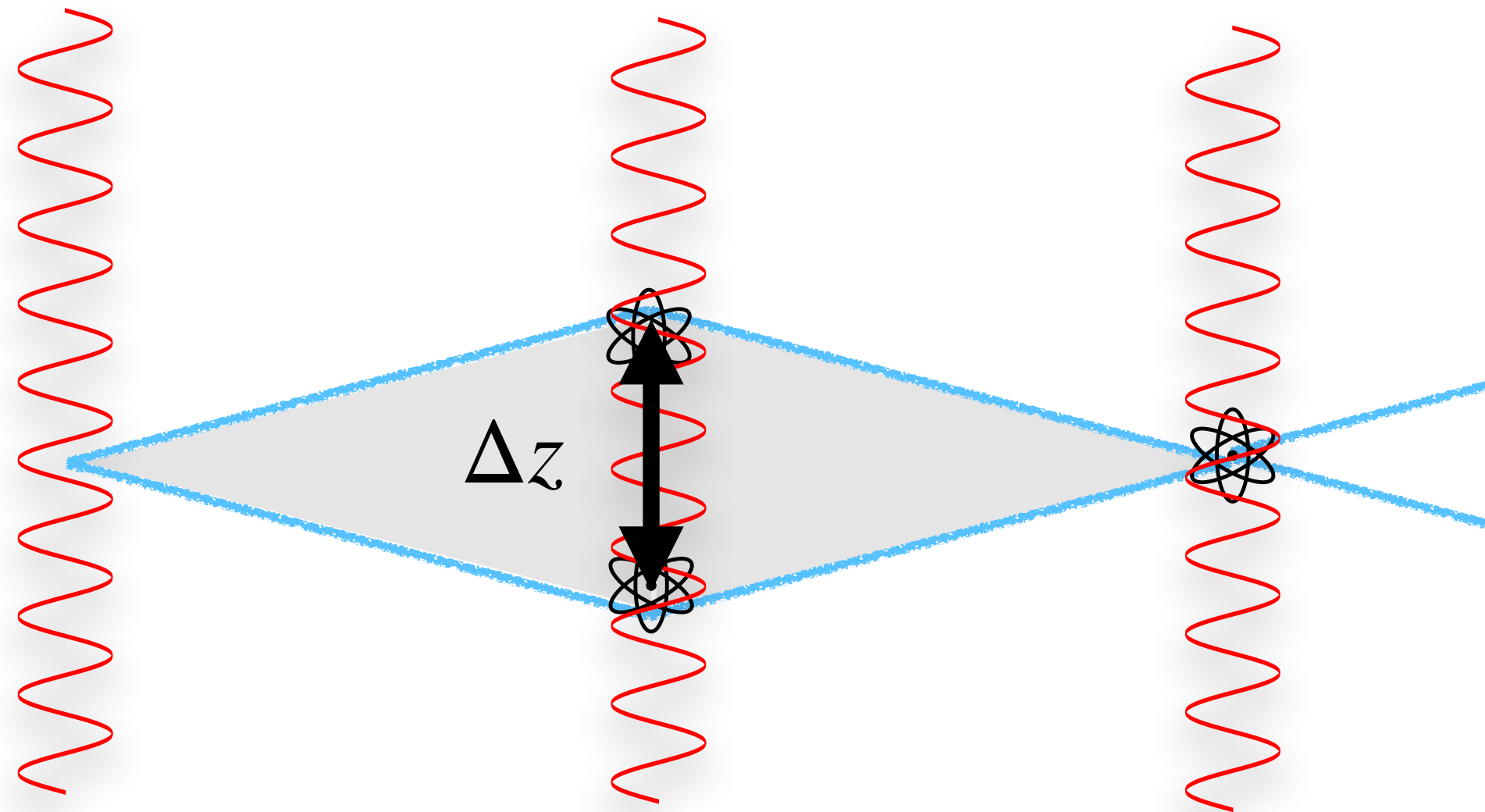
Part I accuracy: Phase shift calculations

Part II sensitivity: LMT atom interferometer

Large Scale atom interferometers

Needs for Large Momentum Transfert Nk

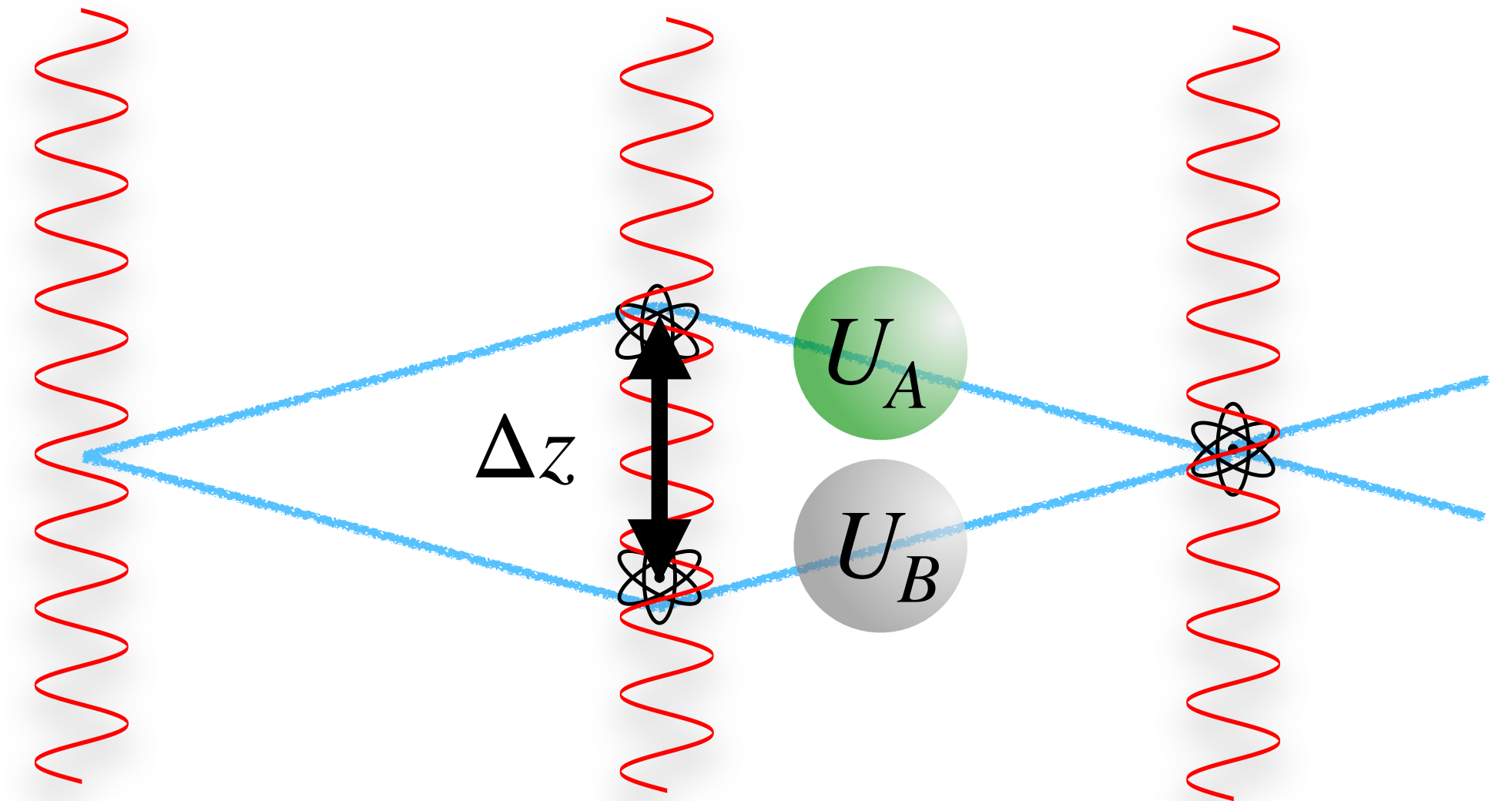
Increased acceleration sensitivity



$$\sigma_a = \frac{\sigma_{\Delta\phi}}{NkT^2}$$

Increased
space-time area

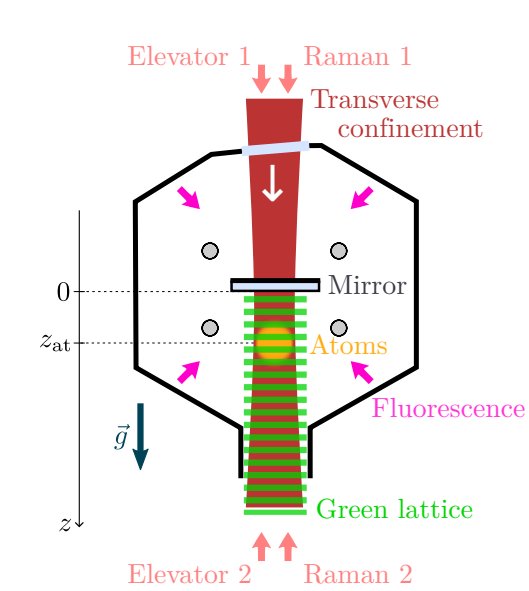
Separated arms interferometers



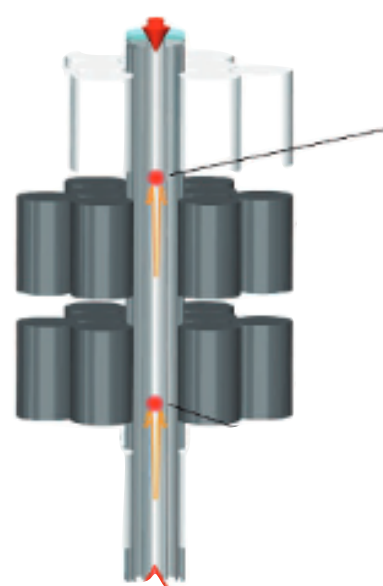
Aharonov-Bohm related phase shift

$$\Delta\phi = - \int \frac{U(r_A, t) - U(r_B, t)}{\hbar} dt$$

Applications

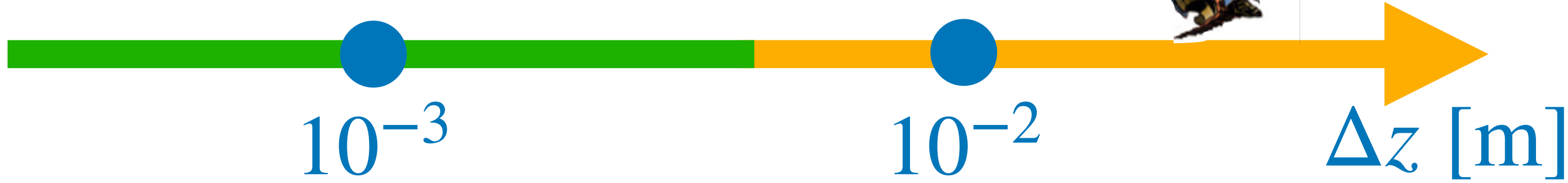


Forces at short distances



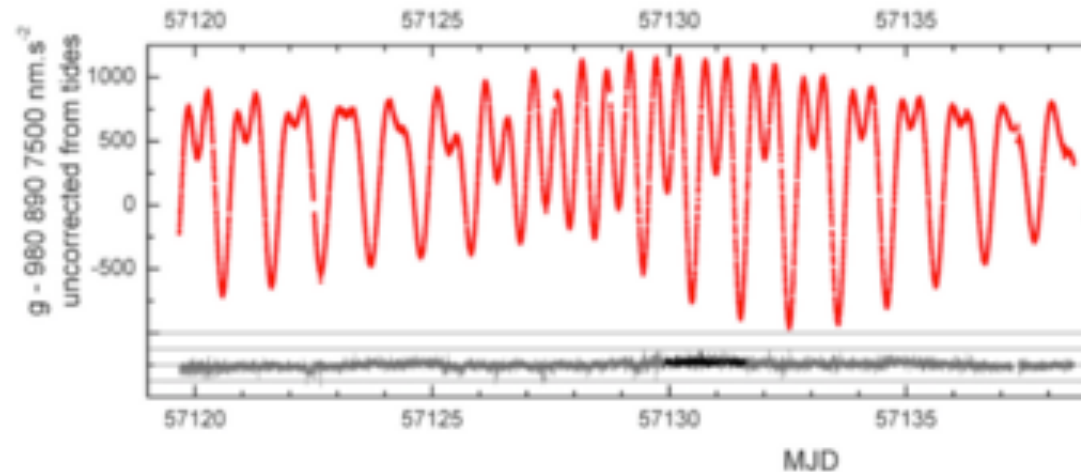
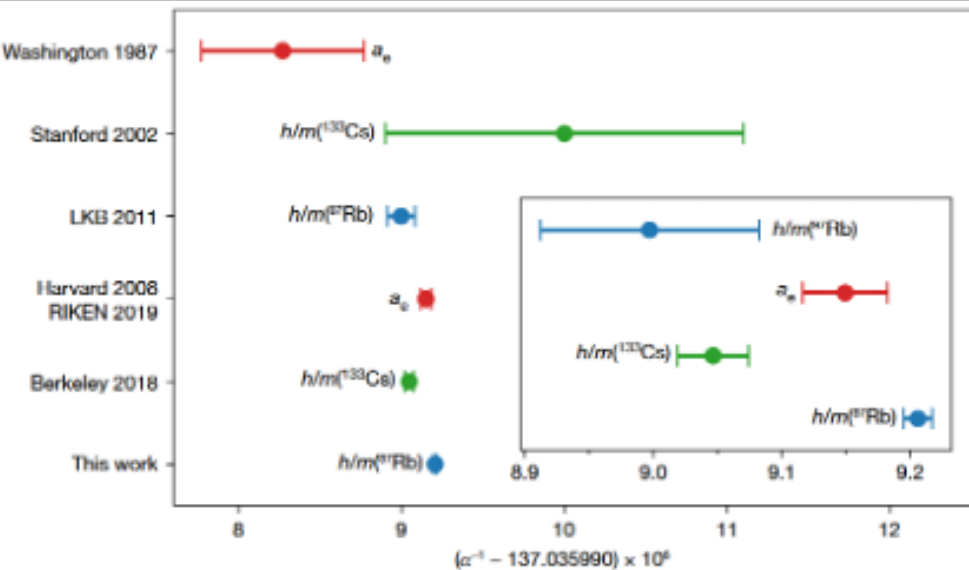
Newton G

equivalence principle test

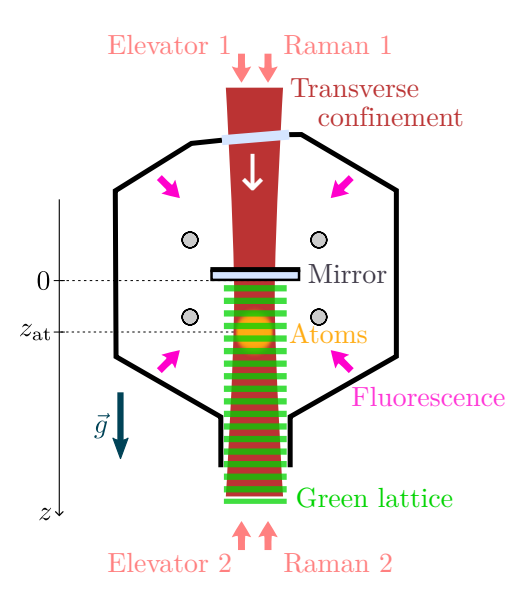


Fine structure constant

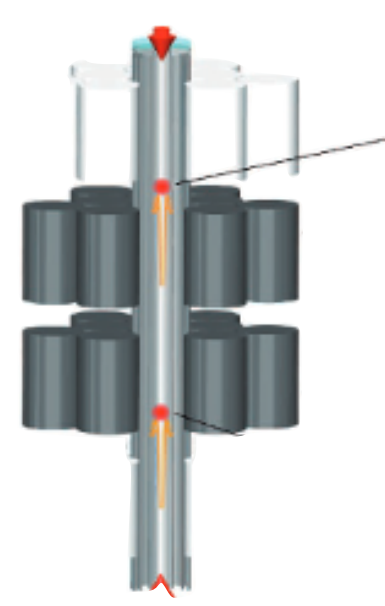
Gravimeter



Applications in fundamental physics



Forces at short distances

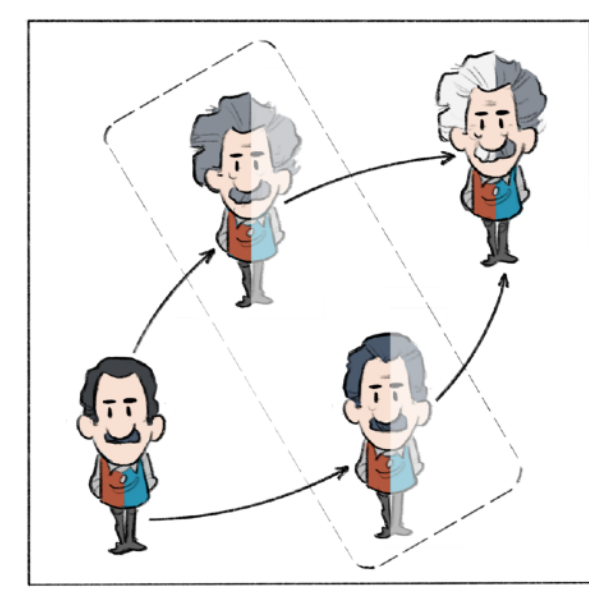


Newton G

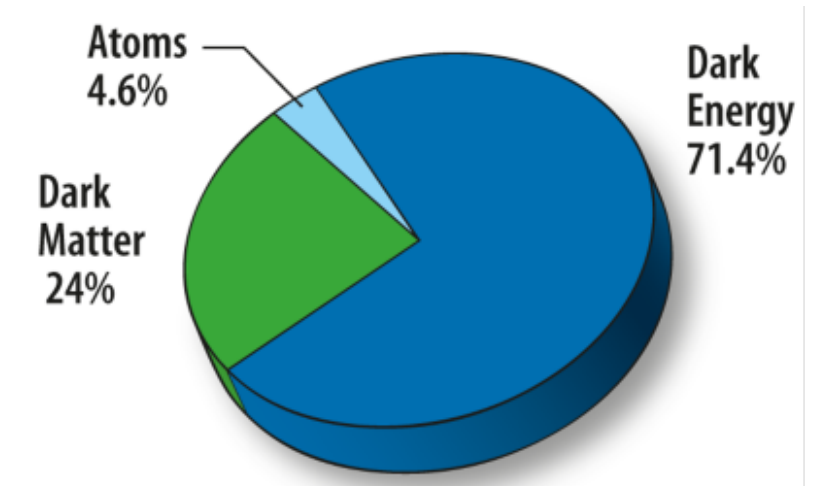
equivalence principle test



New class of atom interferometers



Relativity



Dark matter

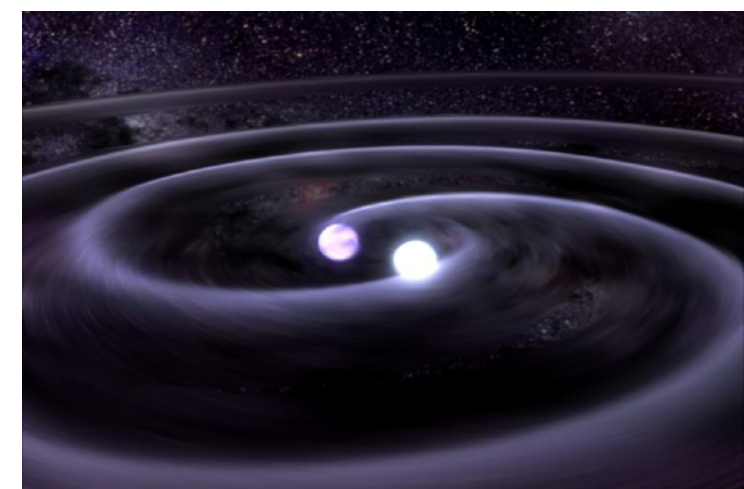
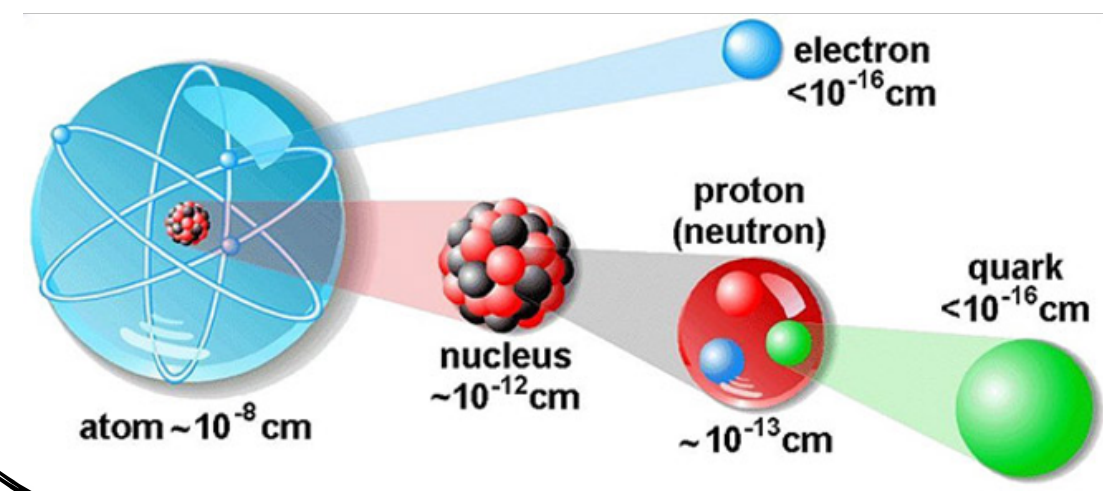
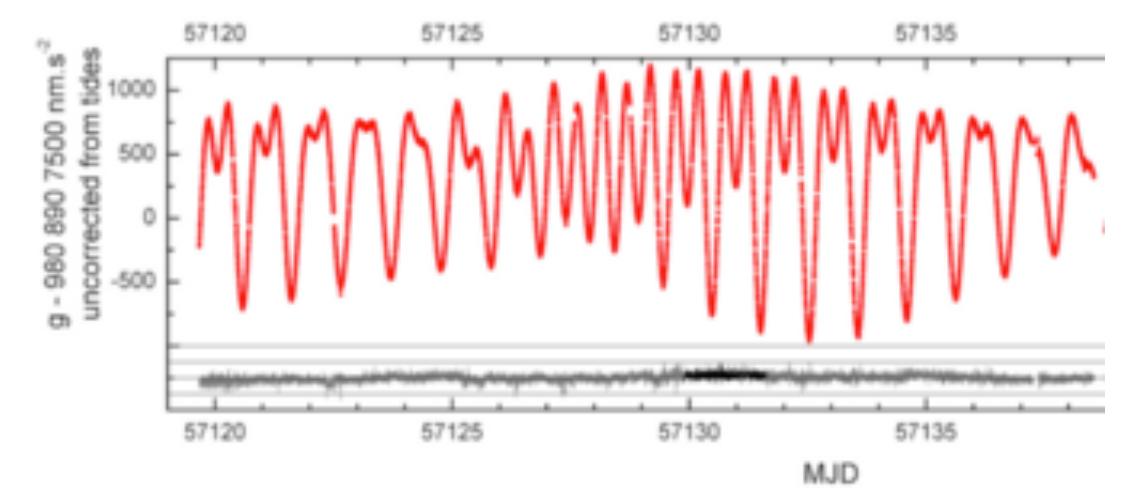
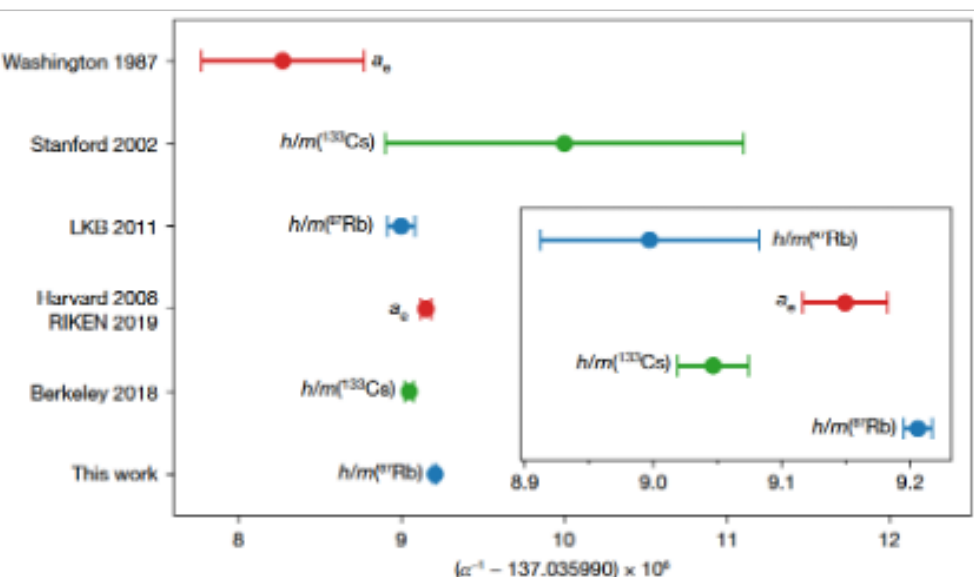


Fine structure constant

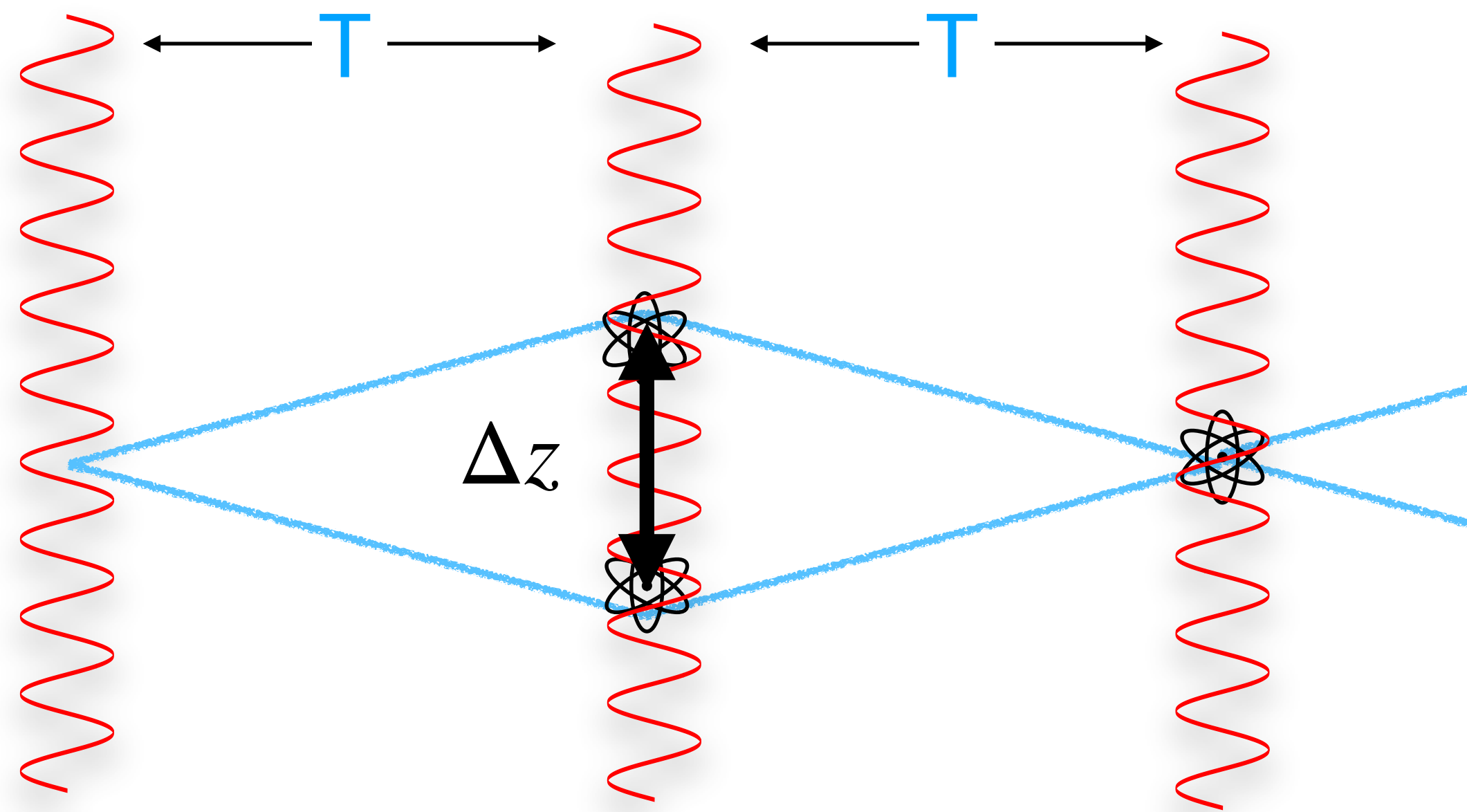
Gravimeter

Atom neutrality

Gravitational Waves



Next-generation atom interferometer



$$\Delta z = \frac{N\hbar k}{m} T$$

- Large Momentum Transfer : N
- Long time of flight : T

Kovachy et al. Nature (2015)

State of the art: $\Delta z = 54$ cm

$90\hbar k$

et

$T=1$ s



Goal: $\Delta z > 1$ m



$T > 1$ s

$> 1000\hbar k$!?

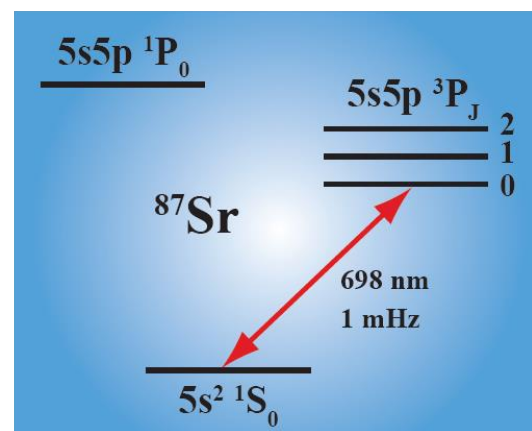
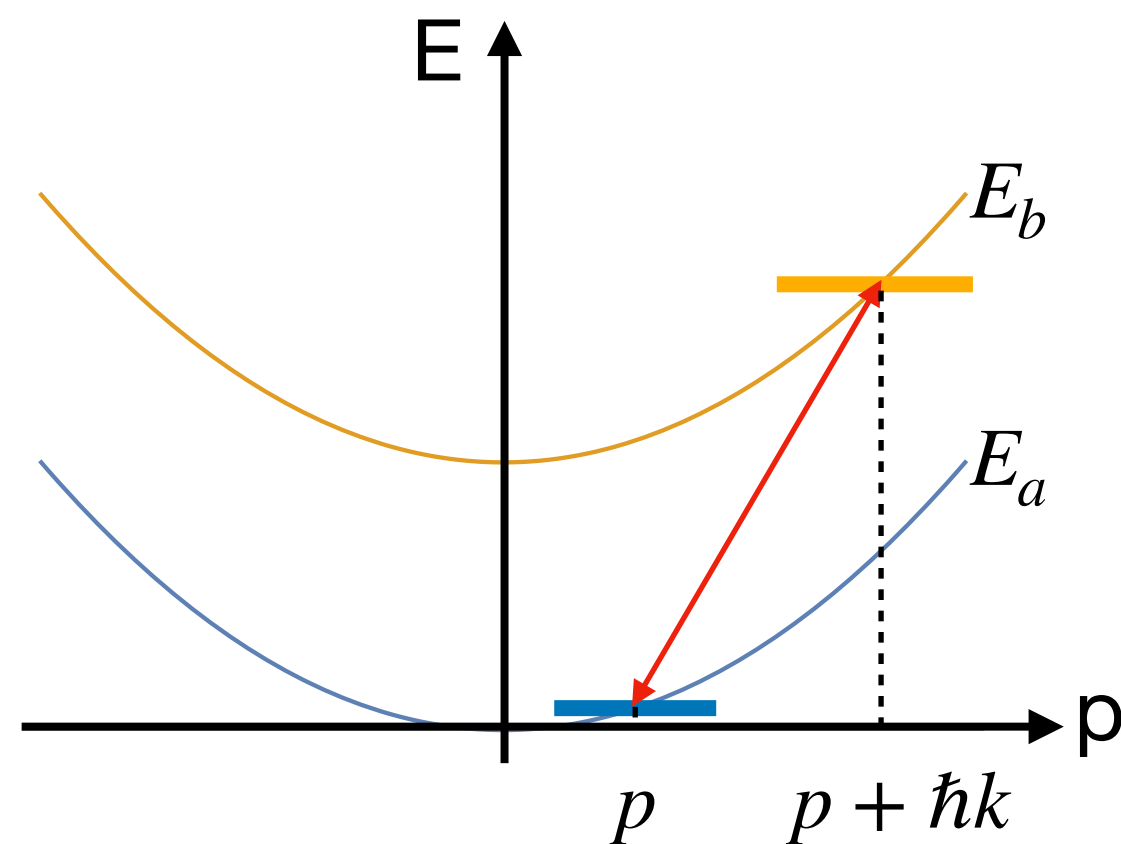
Outline: LMT interferometer

1. The basic building block for LMT interferometry
 - Raman transitions
 - Bragg and Quasi-Bragg transitions
2. LMT on sequences of pulses: Bragg - Bloch - Floquet et al.

Atom optics

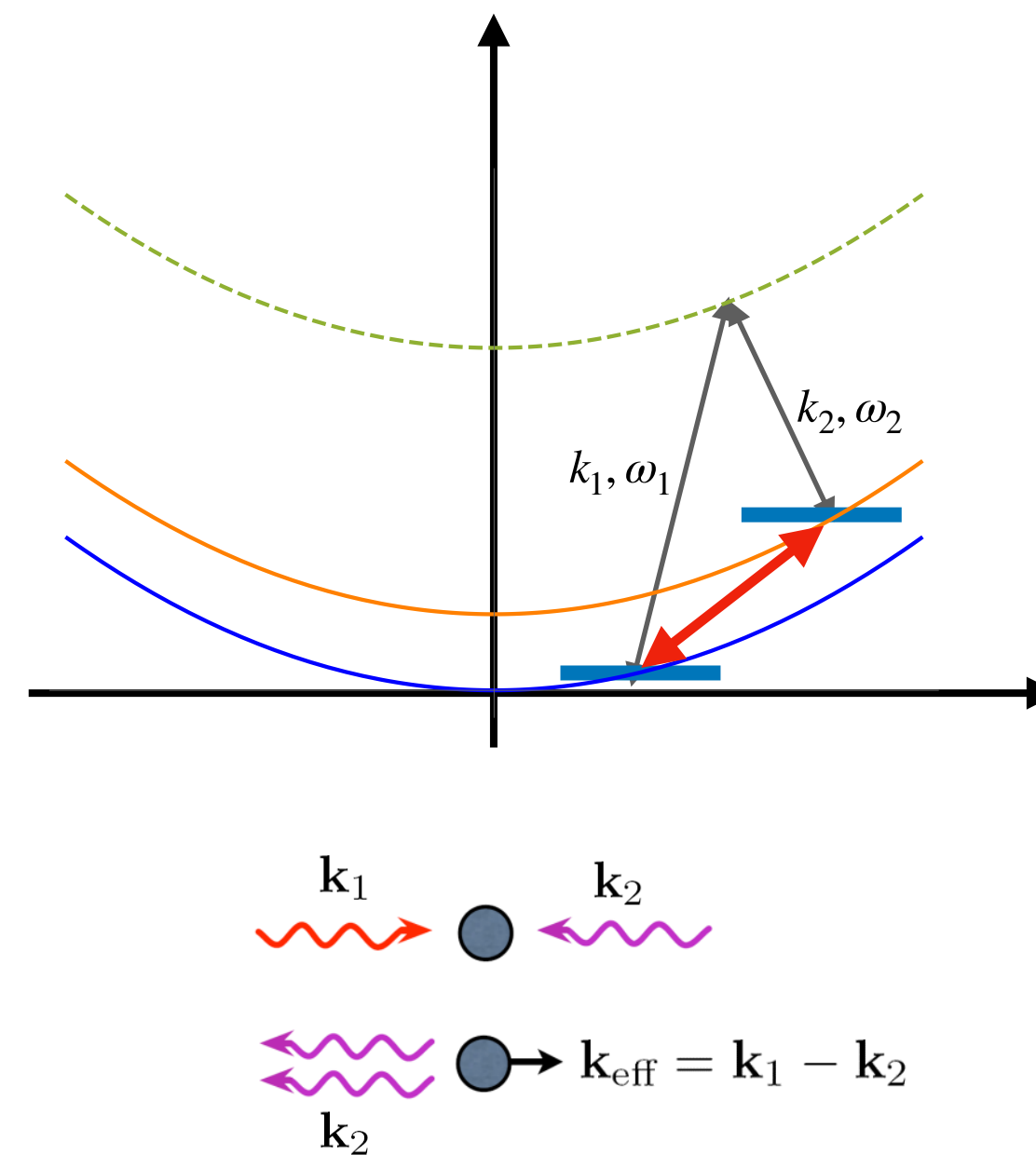
Single and multi-photon process

Single photon



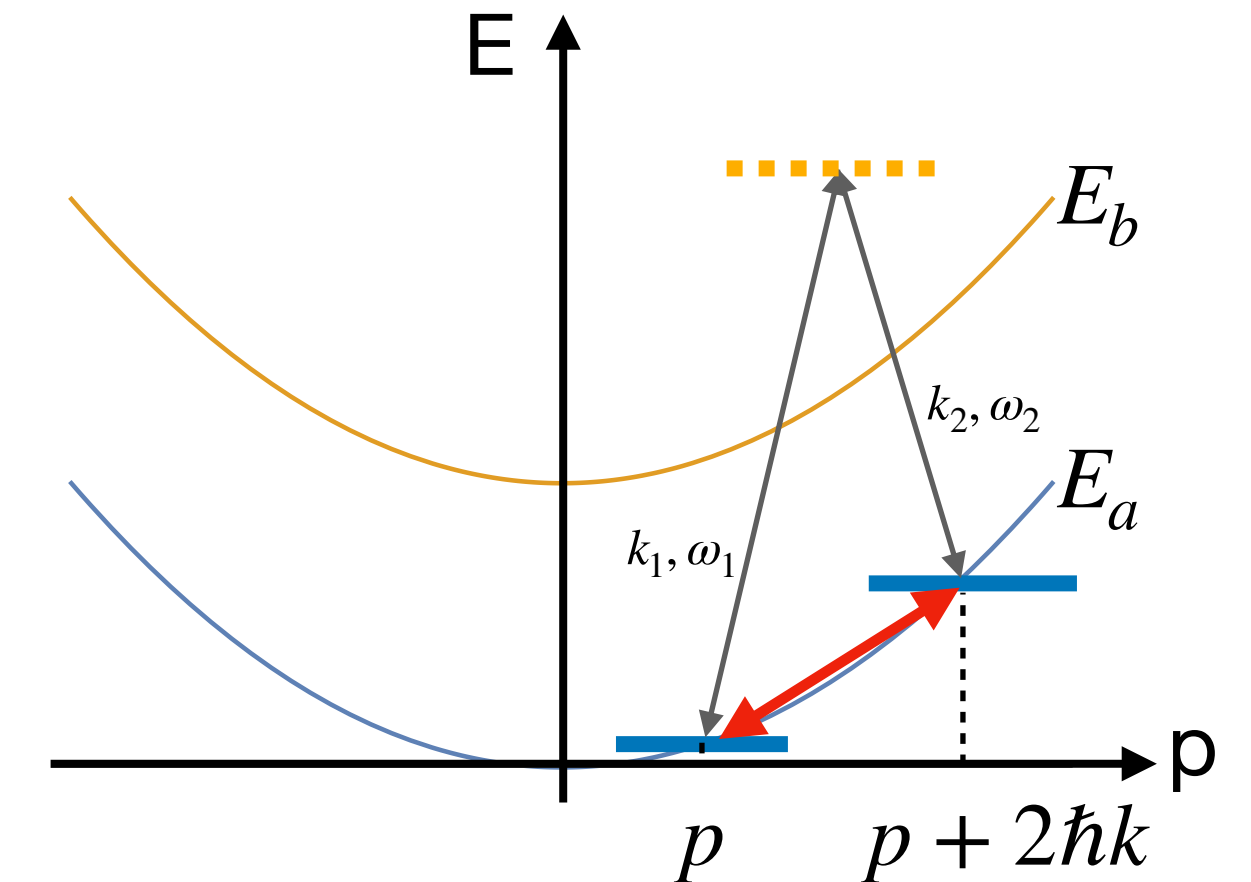
$$|\Psi\rangle = \alpha |E_a, p\rangle + \beta |E_b, p + \hbar k\rangle e^{i\varphi_{\text{opt}}(t)}$$

Two photons : Raman



$$|\Psi\rangle = \alpha |p\rangle + \beta |p + \hbar k\rangle e^{i\varphi_{\text{rf}}(t)}$$

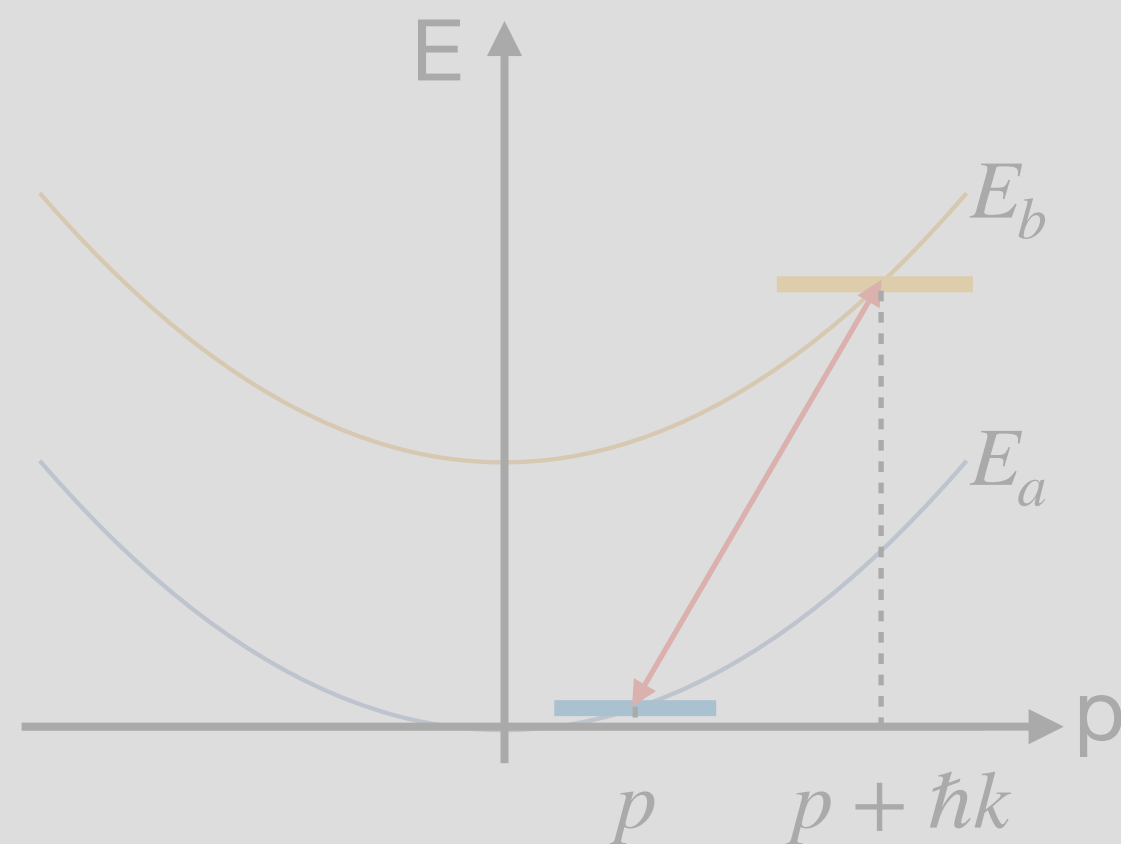
Two photons : Bragg



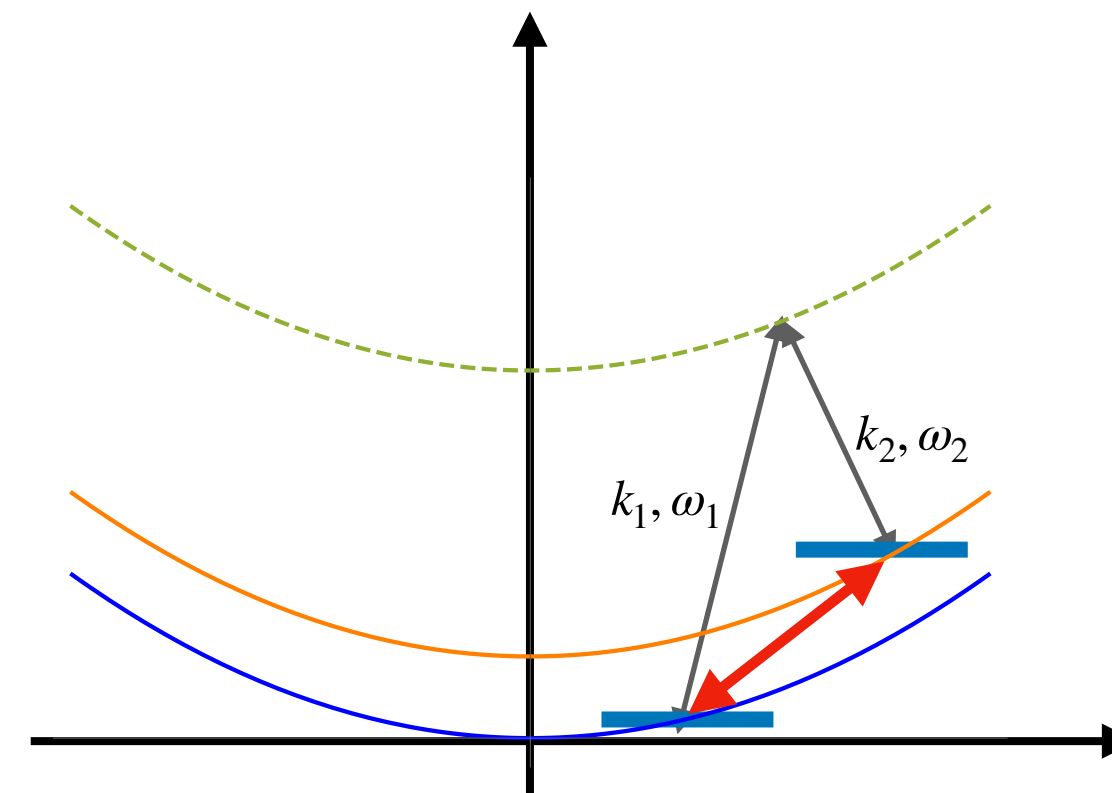
Atom optics

Single and multi-photon process

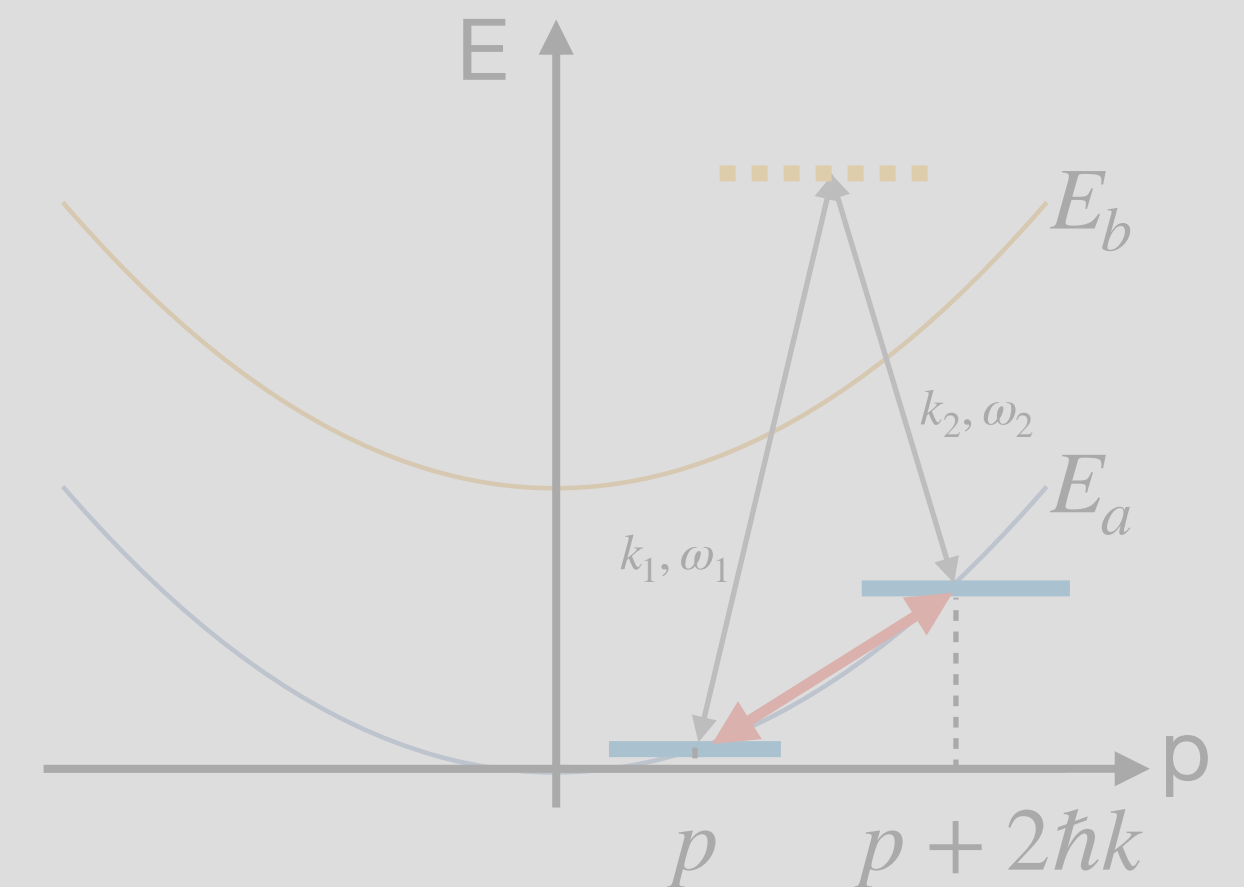
Single photon



Two photons : Raman

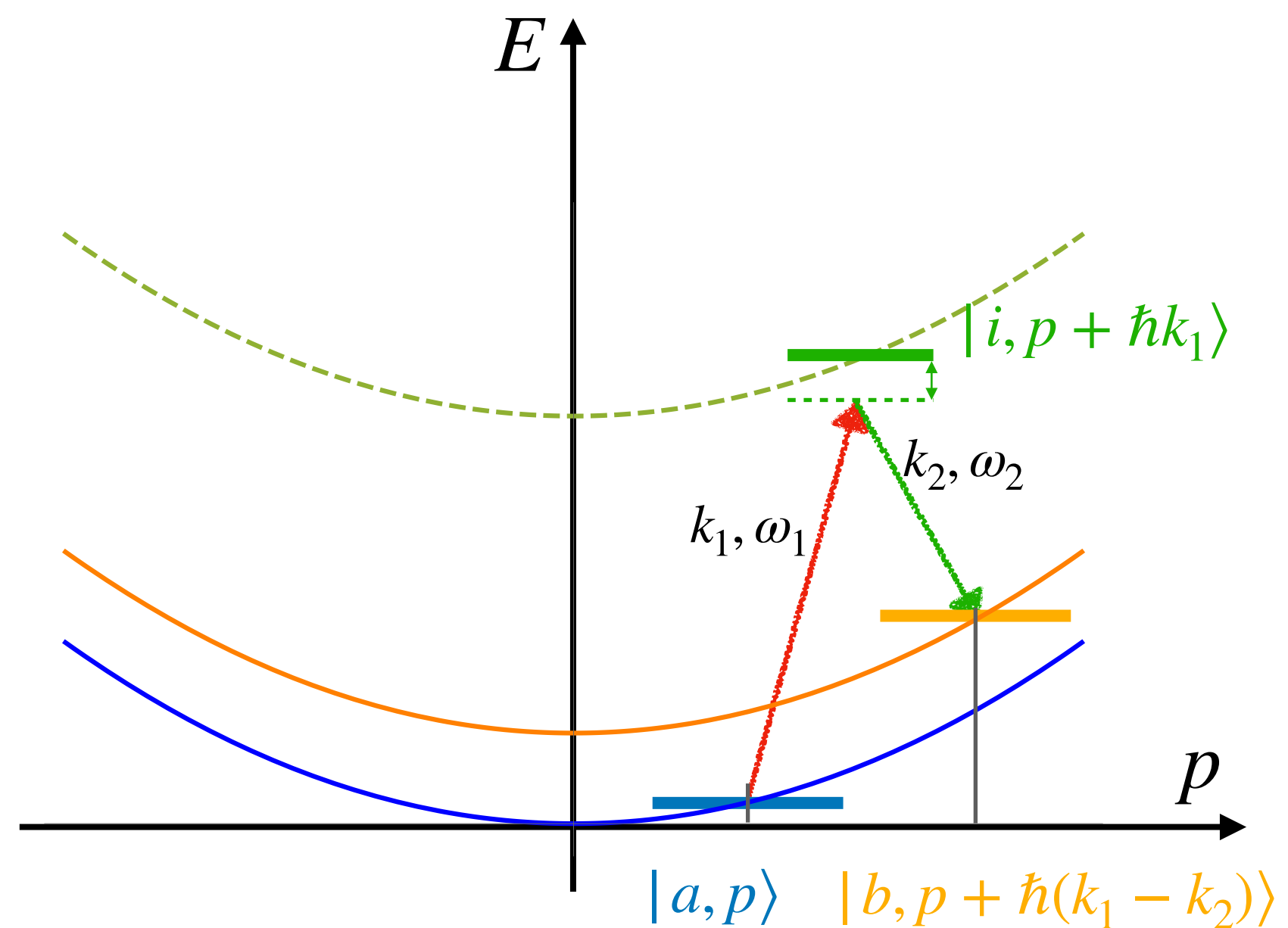


Two photons : Bragg



Raman transitions

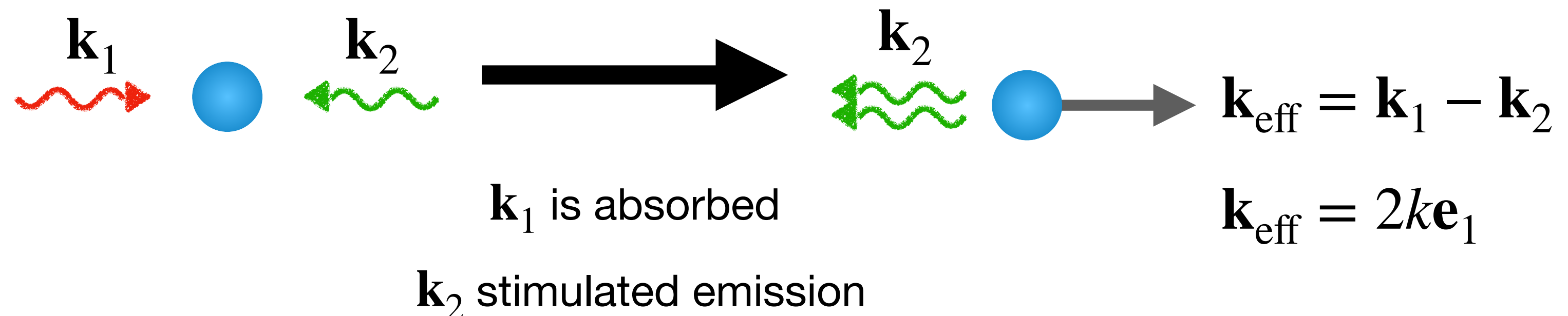
Principle: Λ -coupling



Two counter propagating laser beam:

$$\mathbf{E} = \frac{1}{2} \left(E_1 e^{i\mathbf{k}_1 \cdot \mathbf{r}} e^{-i(\omega_1 t + \phi_1)} + E_2 e^{i\mathbf{k}_2 \cdot \mathbf{r}} e^{-i(\omega_2 t + \phi_2)} \right) + c.c.$$

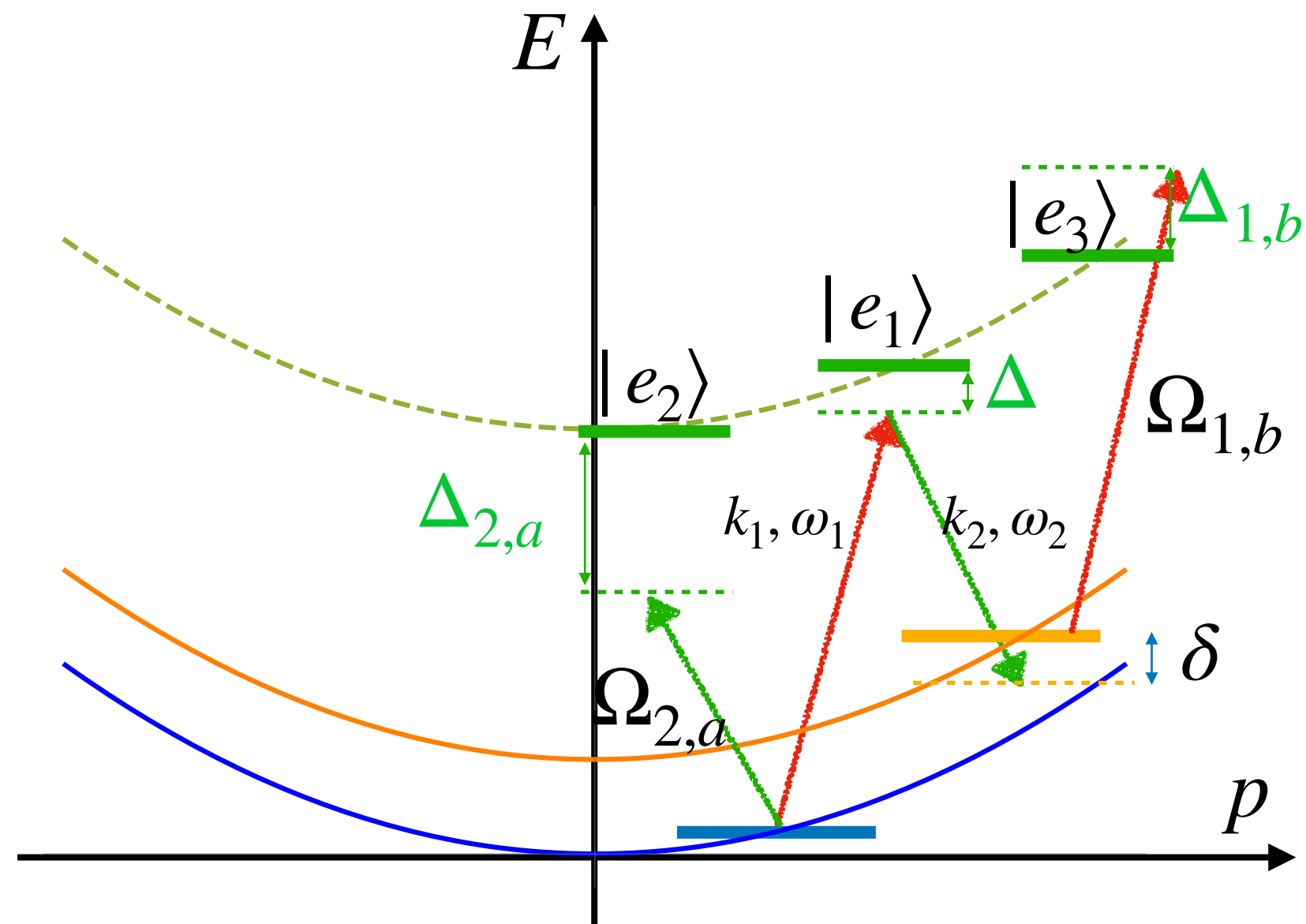
Momentum conservation:



Coupled states: $|a, p\rangle$, $|b, p + \hbar(k_1 - k_2)\rangle$, $|i\rangle$

Raman transitions

Actually: more coupled states



Detuning with excited state : $\Delta_{\text{laser,state}}$

Single photon Rabi frequency: $\hbar\Omega_{L,j} = \langle j | d_{ej} E_L | e \rangle$

2-photon detuning: $\delta = (\omega_1 - \omega_2) - (\omega_b - \omega_a)$

Coupling between 2 (ground) + 3 (excited) momentum states

$$|\Psi(t)\rangle = \gamma_a e^{-i\omega_{a,p}t} |a, p\rangle + \gamma_b e^{-i\omega_{b,p+2\hbar k}t} |b, p + 2\hbar k\rangle + \gamma_1 e^{-i\omega_{e,p+\hbar k_1}t} |e_1\rangle + \gamma_2 e^{-i\omega_{e,p+\hbar k_2}t} |e_2\rangle + \gamma_3 e^{-i\omega_{e,p+\hbar 2k+k_1}t} |e_3\rangle$$

Raman transitions

$$V_{AL} = -\hbar \left(|e\rangle\langle a| + |a\rangle\langle e| \right) \cdot \left(\frac{\Omega_{1j}}{2} e^{ik_1 r} e^{i(\omega_1 t + \phi_1)} + \frac{\Omega_{2j}}{2} e^{ik_2 r} e^{i(\omega_2 t + \phi_2)} \right) + h.c.$$



From Schrodinger equation we obtain 5 coupled ODEs:

$$\left\{ \begin{array}{l} \dot{\gamma}_a = -\frac{i}{2} \left(\Omega_{1ae}^* e^{i\Delta t} e^{i\phi_1} \gamma_{e1} + \Omega_{2ae}^* e^{i\Delta_{2ae} t} e^{i\phi_2} \gamma_{e2} \right) \\ \dot{\gamma}_b = -\frac{i}{2} \left(\Omega_{1be}^* e^{i\Delta_{1be} t} e^{i\phi_1} \gamma_{e3} + \Omega_{2be}^* e^{i(\Delta - \delta)t} e^{i\phi_2} \gamma_{e1} \right) \\ \dot{\gamma}_{e1} = -\frac{i}{2} \left(\Omega_{1ae} e^{-i\Delta_{1ae} t} e^{-i\phi_1} \gamma_a + \Omega_{2be} e^{-i(\Delta - \delta)t} e^{-i\phi_2} \gamma_b \right) \\ \dot{\gamma}_{e2} = -\frac{i}{2} \Omega_{2ae} e^{-i\Delta_{2ae} t} e^{-i\phi_2} \gamma_a \\ \dot{\gamma}_{e3} = -\frac{i}{2} \Omega_{1be} e^{-i\Delta_{1be} t} e^{-i\phi_1} \gamma_b \end{array} \right.$$

Raman transitions

Two very different timescales: Adiabatic elimination

Large detuning: $\longrightarrow$ $|e\rangle$: Fast variables $\sim \Delta^{-1}$
 $|\Delta| \gg \Omega_{li}$ $\longrightarrow$ $|g\rangle$: Slow variables $\sim \Omega_{\text{eff}}^{-1}$

Integrate $\dot{\gamma}_e$ considering $\gamma_{a,b}$ constant and insert $\gamma_e(t)$ in ODEs for $\gamma_{a,b}$

$$\gamma_{e1} = \frac{\Omega_{1ae}}{2\Delta} \gamma_a e^{-i(\Delta t + \phi_1)} + \frac{\Omega_{2be}}{2(\Delta - \delta)} \gamma_b e^{-i((\Delta - \delta)t + \phi_2)}$$

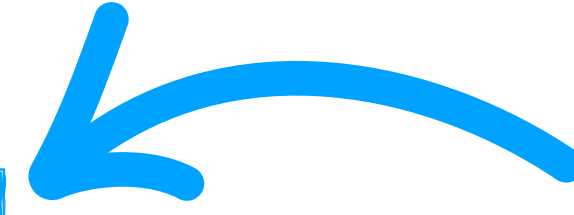
$$\gamma_{e2} = \frac{\Omega_{2ae}}{2\Delta_{2ae}} \gamma_a e^{-i(\Delta_{2ae} t + \phi_2)}$$

$$\gamma_{e3} = \frac{\Omega_{1be}}{2\Delta_{1ae}} \gamma_b e^{-i(\Delta_{1be} t + \phi_1)}$$

Raman transitions

Adiabatic elimination:

$\{\dot{\gamma}_{a,b}\}$ ODEs : $\gamma_{a,b}$ only !

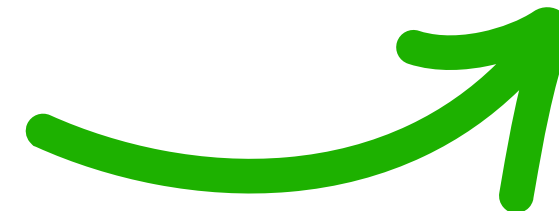


$$\left\{ \begin{array}{l} \dot{\gamma}_a = -\frac{i}{2} \left(\Omega_{1ae}^* e^{i\Delta t} e^{i\phi_1} \gamma_{e1} + \Omega_{2ae}^* e^{i\Delta_{2ae}t} e^{i\phi_2} \gamma_{e2} \right) \\ \dot{\gamma}_b = -\frac{i}{2} \left(\Omega_{1be}^* e^{i\Delta_{1be}t} e^{i\phi_1} \gamma_{e3} + \Omega_{2be}^* e^{i(\Delta-\delta)t} e^{i\phi_2} \gamma_{e1} \right) \\ \dot{\gamma}_{e1} = -\frac{i}{2} \left(\Omega_{1ae} e^{-i\Delta_{1ae}t} e^{-i\phi_1} \gamma_a + \Omega_{2be} e^{-i(\Delta-\delta)t} e^{-i\phi_2} \gamma_b \right) \\ \dot{\gamma}_{e2} = -\frac{i}{2} \Omega_{2ae} e^{-i\Delta_{2ae}t} e^{-i\phi_2} \gamma_a \\ \dot{\gamma}_{e3} = -\frac{i}{2} \Omega_{1be} e^{-i\Delta_{1be}t} e^{-i\phi_1} \gamma_b \end{array} \right.$$

$$\gamma_{e1} = \frac{\Omega_{1ae}}{2\Delta} \gamma_a e^{-i(\Delta t + \phi_1)} + \frac{\Omega_{2be}}{2(\Delta - \delta)} \gamma_b e^{-i((\Delta - \delta)t + \phi_2)}$$

$$\gamma_{e2} = \frac{\Omega_{2ae}}{2\Delta_{2ae}} \gamma_a e^{-i(\Delta_{2ae}t + \phi_2)}$$

$$\gamma_{e3} = \frac{\Omega_{1be}}{2\Delta_{1ae}} \gamma_b e^{-i(\Delta_{1be}t + \phi_1)}$$



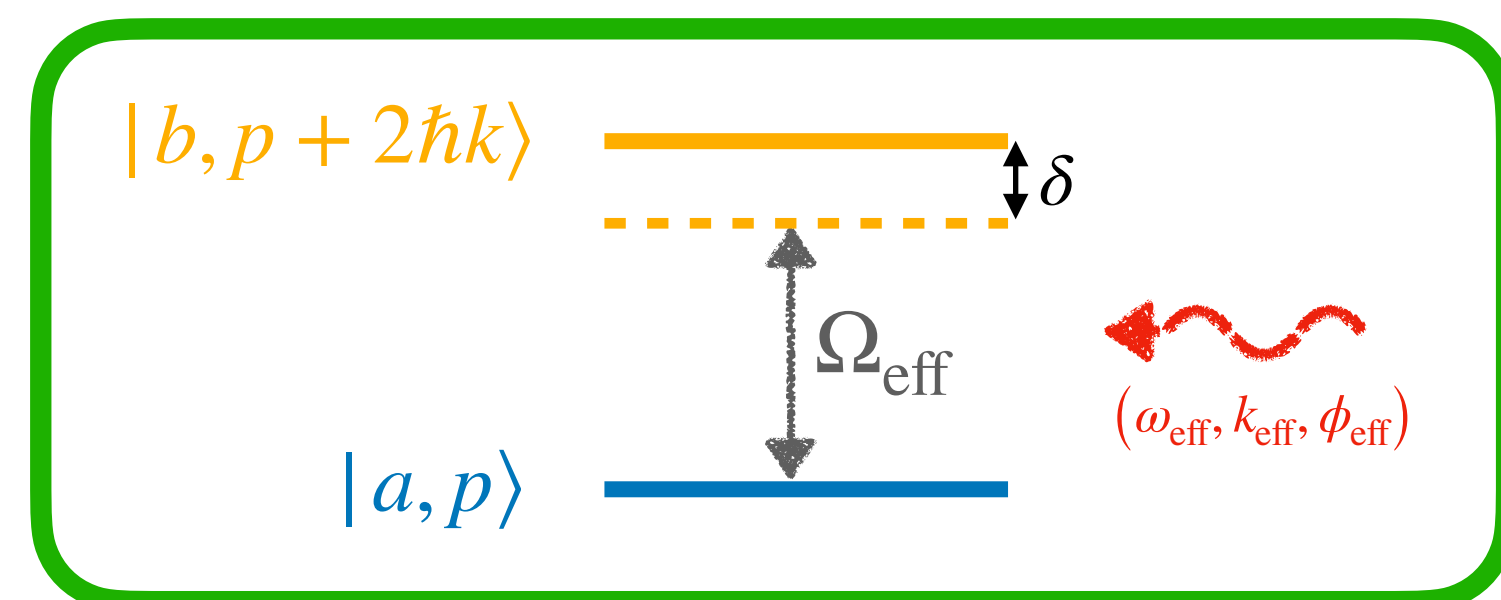
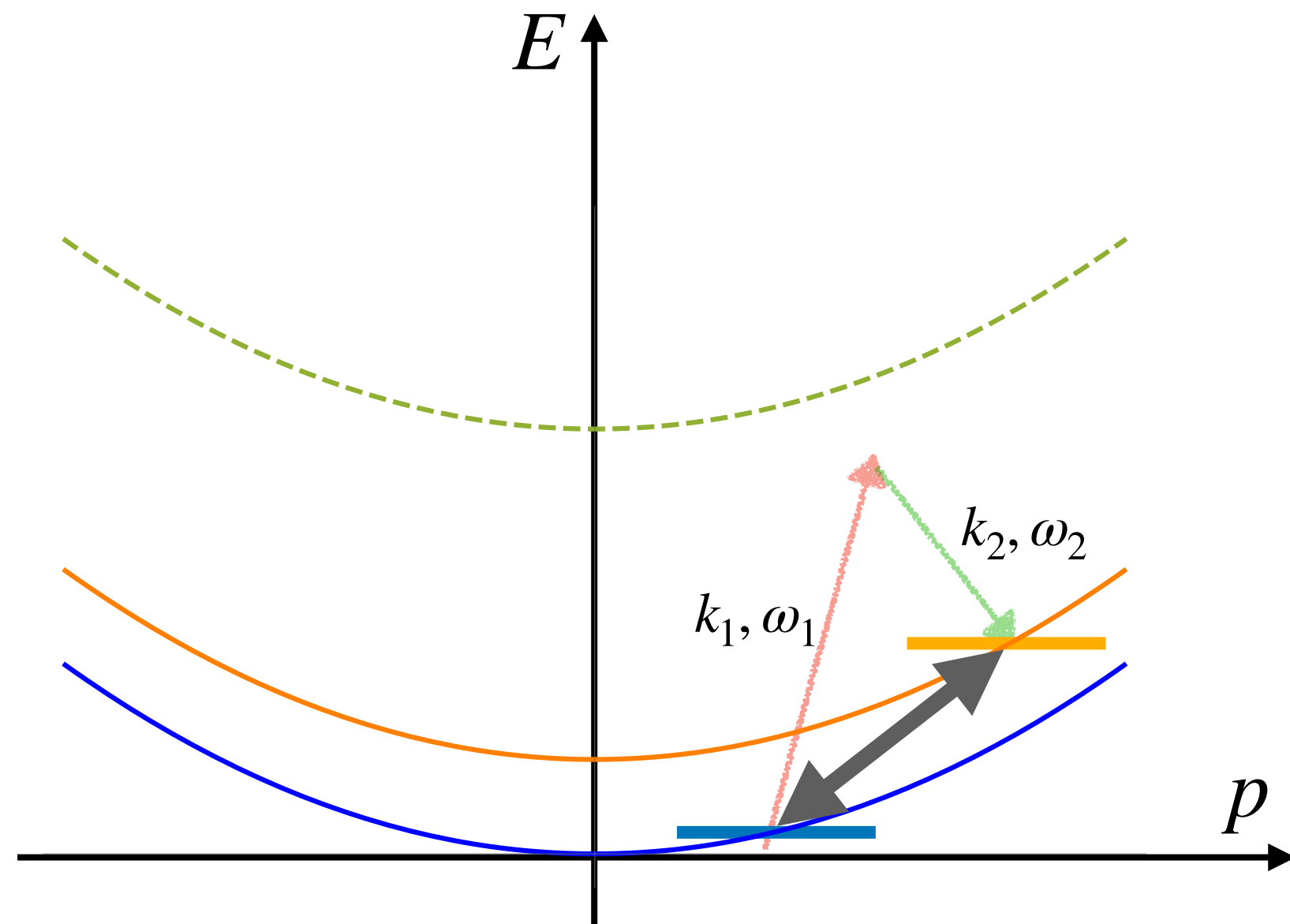
Consider $\gamma_{a,b}$ constant

Multi-photon transitions

Raman coupling

Effective 2-level system

$$\begin{cases} \dot{\gamma}_a = -i\Omega_a^{\text{LS}}\gamma_a - ie^{i\delta t}e^{i\phi_{\text{eff}}}\Omega_{\text{eff}}\gamma_b \\ \dot{\gamma}_b = -i\Omega_b^{\text{LS}}\gamma_b - ie^{-i\delta t}e^{-i\phi_{\text{eff}}}\Omega_{\text{eff}}^*\gamma_a \end{cases}$$



effective laser

$$\omega_{\text{eff}} = \omega_1 - \omega_2$$

$$\mathbf{k}_{\text{eff}} = \mathbf{k}_1 - \mathbf{k}_2$$

$$\phi_{\text{eff}} = \phi_1 - \phi_2$$

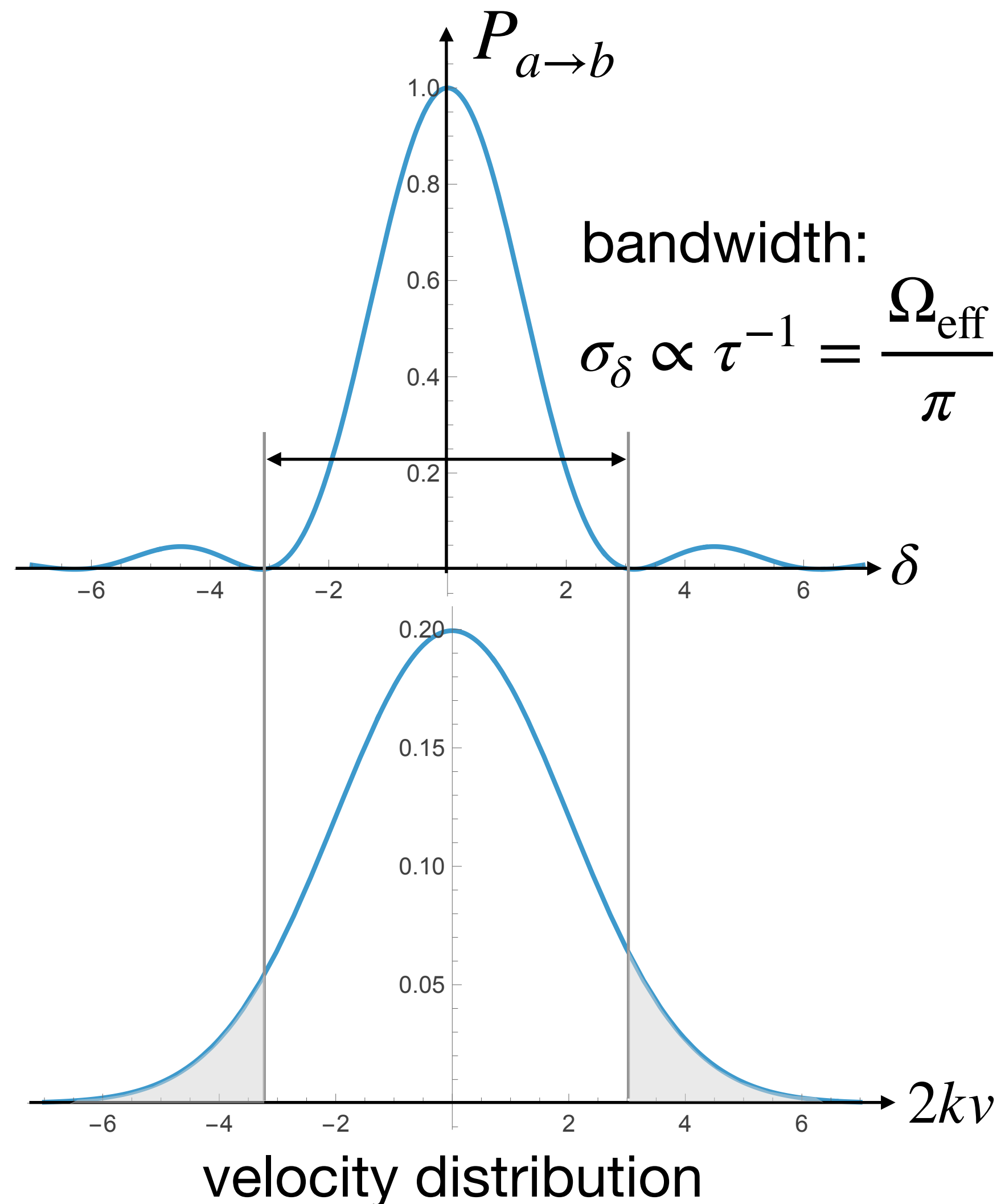
effective atom

$$\Omega_{a,b}^{\text{LS}} = \frac{\Omega_{1a}^2}{4\Delta} + \dots$$

$$\Omega_{\text{eff}} = \frac{\Omega_{1a}\Omega_{2b}}{4\Delta}$$

Multi-photon transitions

Raman coupling



Advantages:

$$|\Psi\rangle = \alpha |a, p\rangle + \beta |b, p + 2\hbar k\rangle e^{i\varphi_{\text{eff}}(t)}$$

2- internal state labelling

3- Do not need sub-recoil atomic ensembles

Disadvantages:

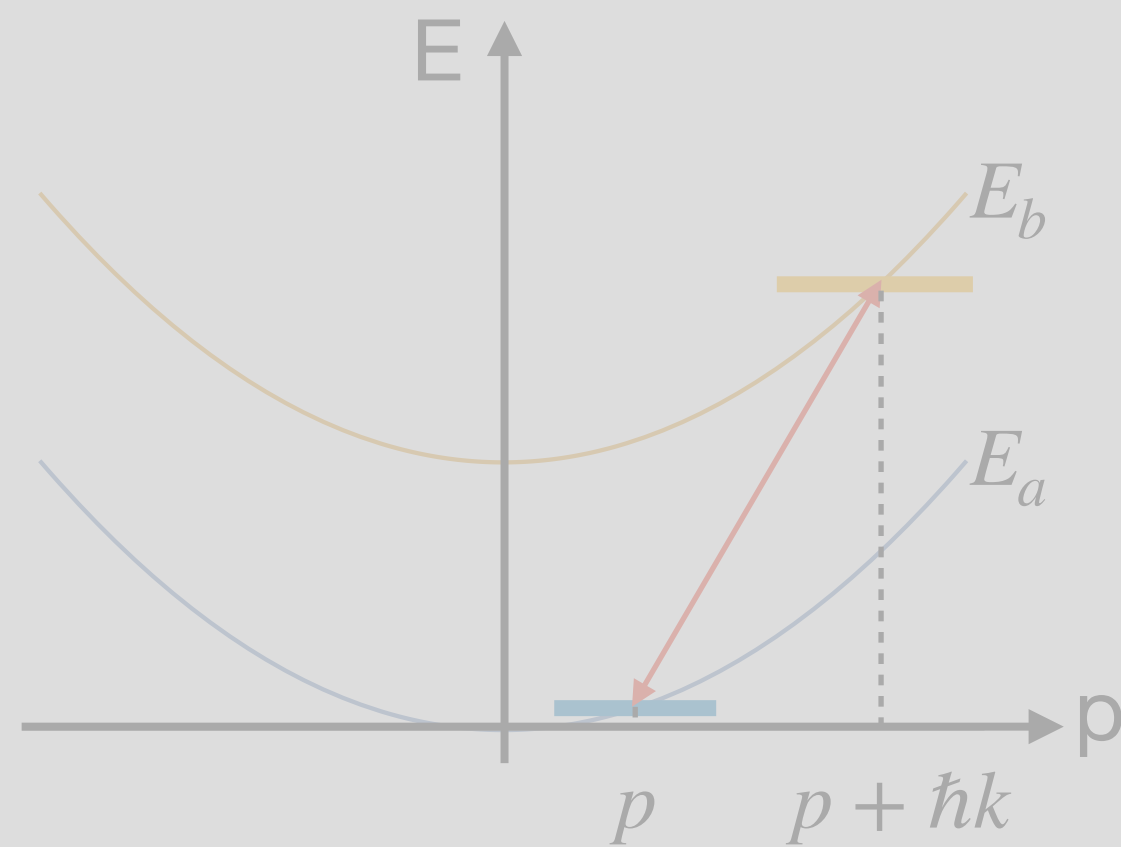
- Internal states can lead to biases (B, LS, ...),
- Large Momentum Transfer

1- relative phase in MW domain

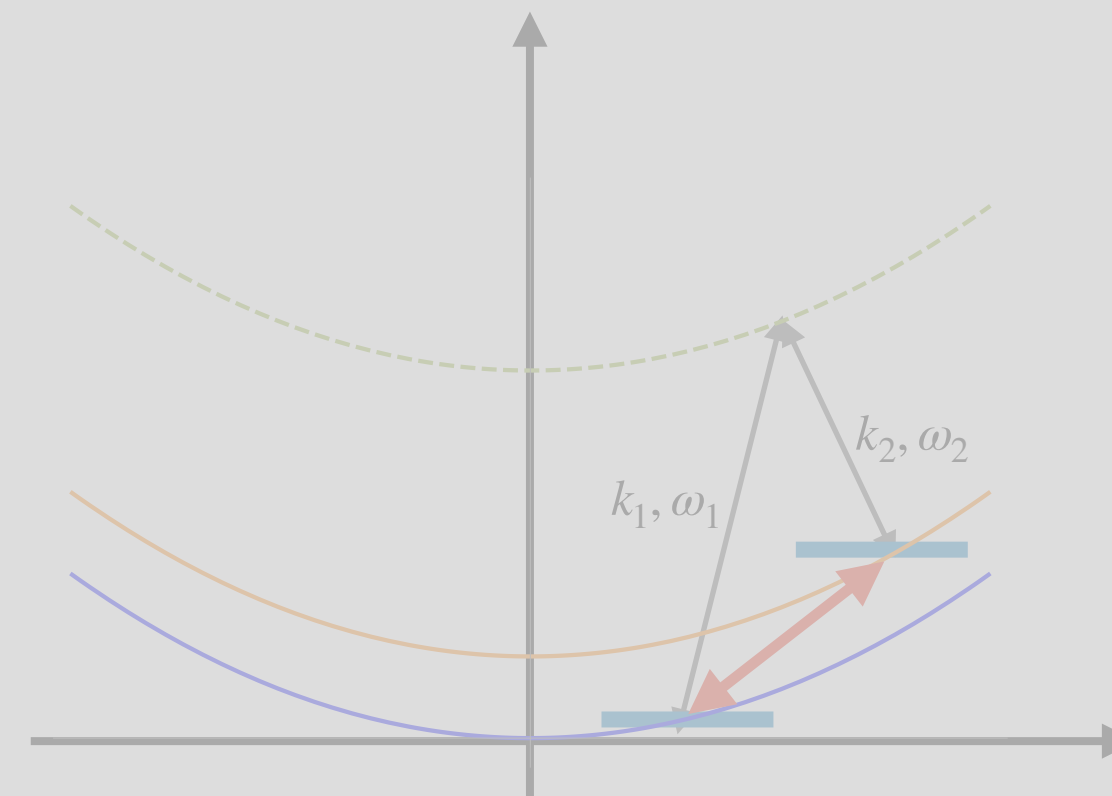
Atom optics

Single and multi-photon process

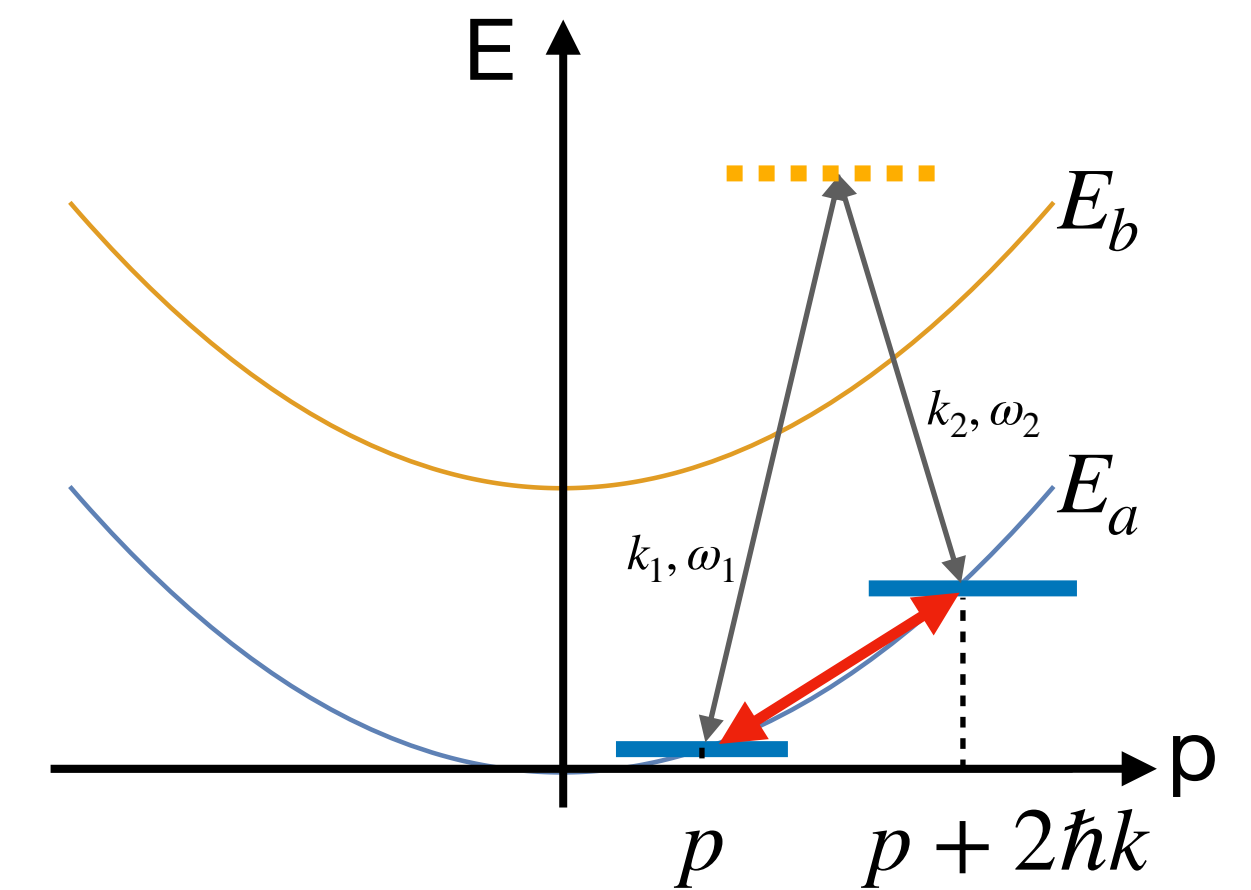
Single photon



Two photons : Raman

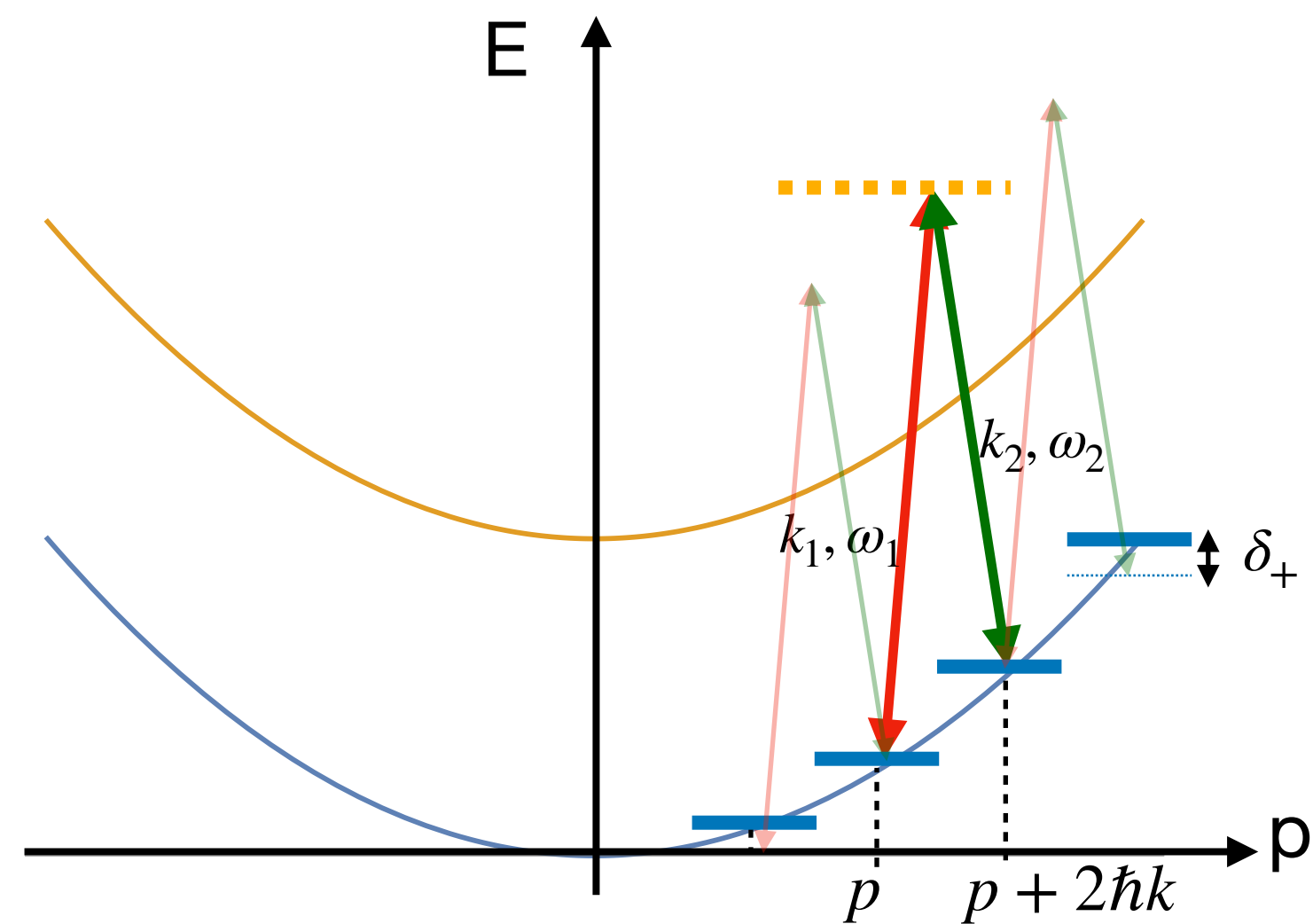


Two photons : Bragg



Bragg transitions

1st order transition



📌 Bragg coupling = Raman transitions between momentum states of the same internal state.

$$|\Psi\rangle = \alpha|p\rangle + \beta|p + 2\hbar k\rangle e^{i\varphi_{\text{rf}}(t)}$$

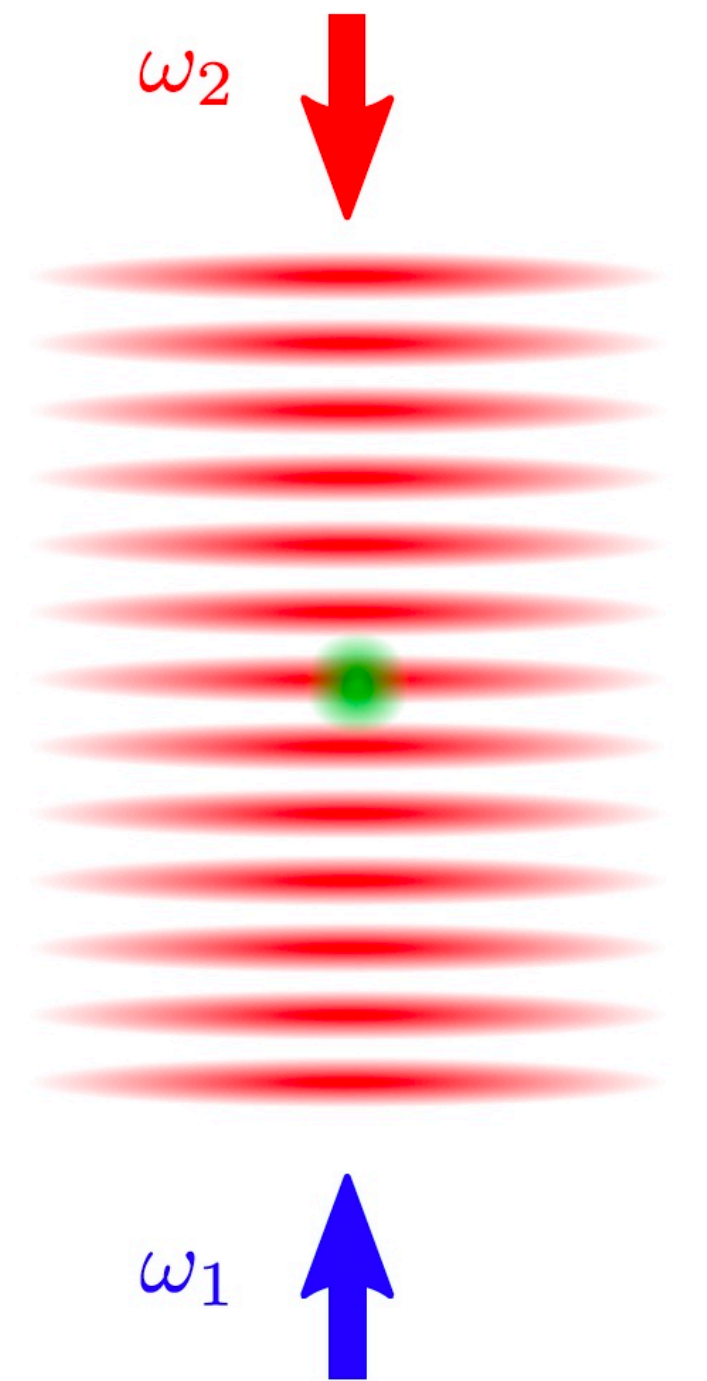
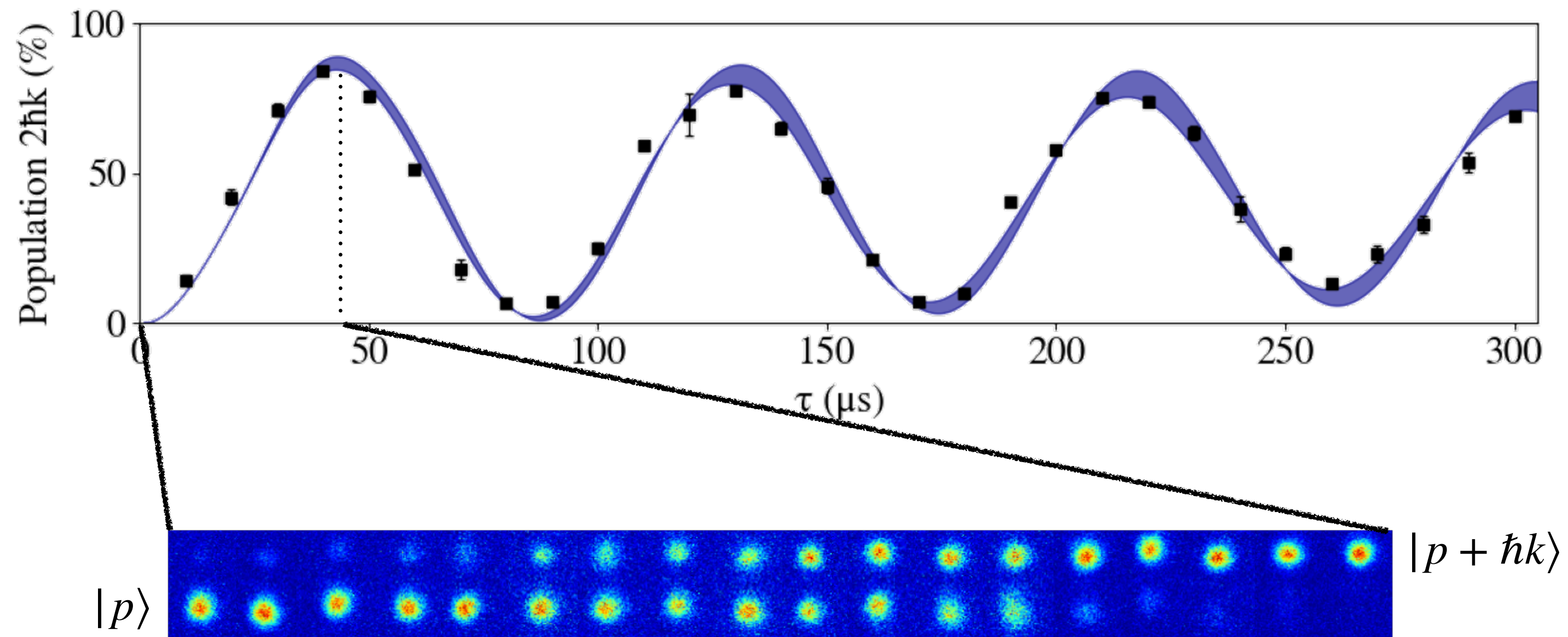
📌 Resonance condition : $\delta = (\omega_1 - \omega_2) - 2kv - 4\omega_R$

📌 Higher momentum states are closed to the 2 photon resonance.

● Bragg regime weak coupling: $\Omega \ll \delta_+ = 16\omega_R$

● Needs sub-recoil ensemble: $\sigma_p \ll \hbar k$

Rabi oscillations in Bragg regime



Bragg transitions

High order $p \rightarrow p + 2n\hbar k$

High order coupling = 2n-photons transition.

$$|\Psi\rangle = \alpha |p\rangle + \beta |p + 2n\hbar k\rangle e^{i\varphi_{\text{rf}}(t)}$$

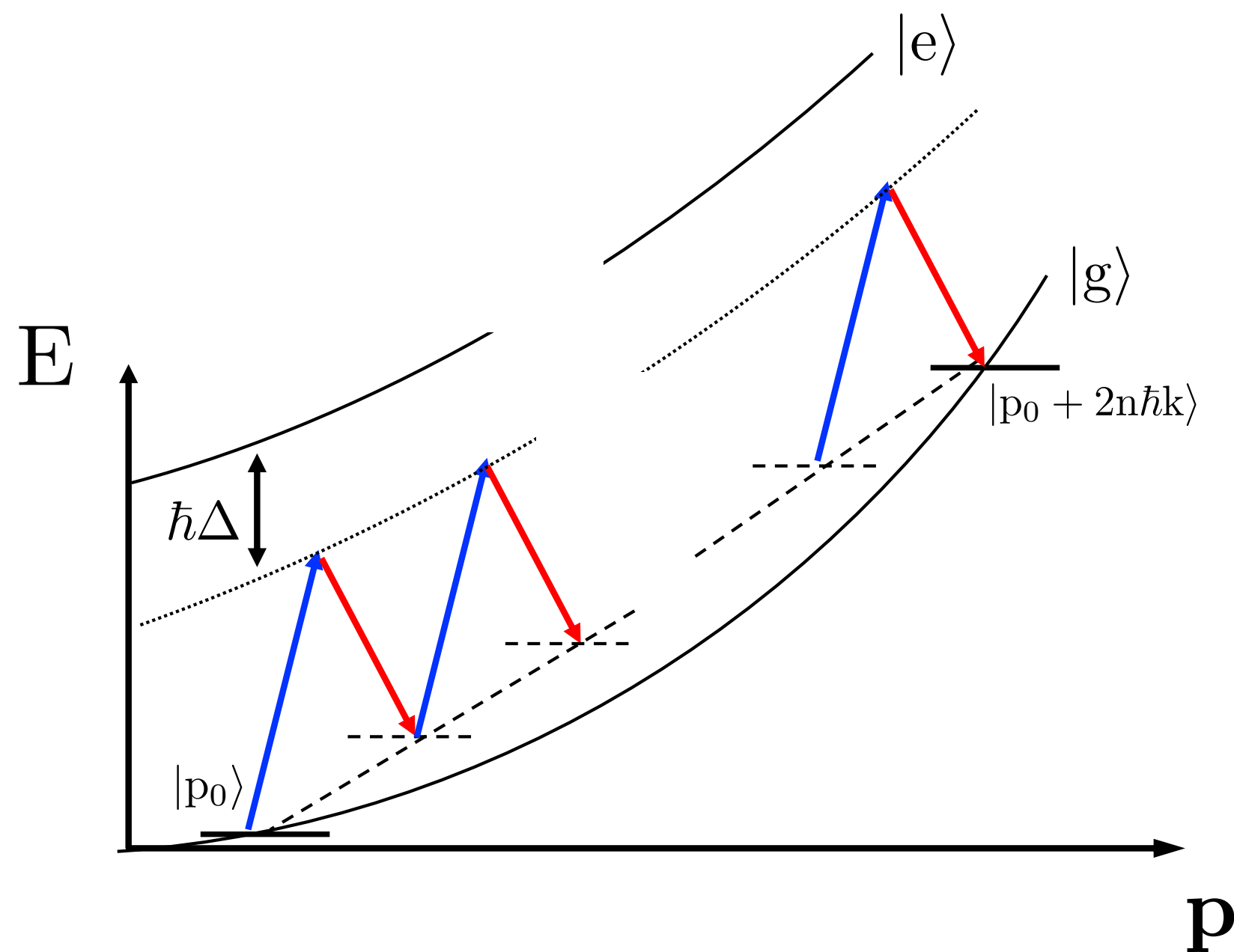
Resonance condition : $\delta = (\omega_1 - \omega_2) - 2kv - 4n\omega_R$

Bragg regime: $\Omega \ll 16\omega_R$

Effective coupling order n is weak

$$\tau_{\text{int}} \gg \frac{2^n (n-1)!^2}{8(n-1)^n}$$

$$n = 3 \rightarrow \tau > 200 \mu\text{s}$$



Difficult in the Bragg regime

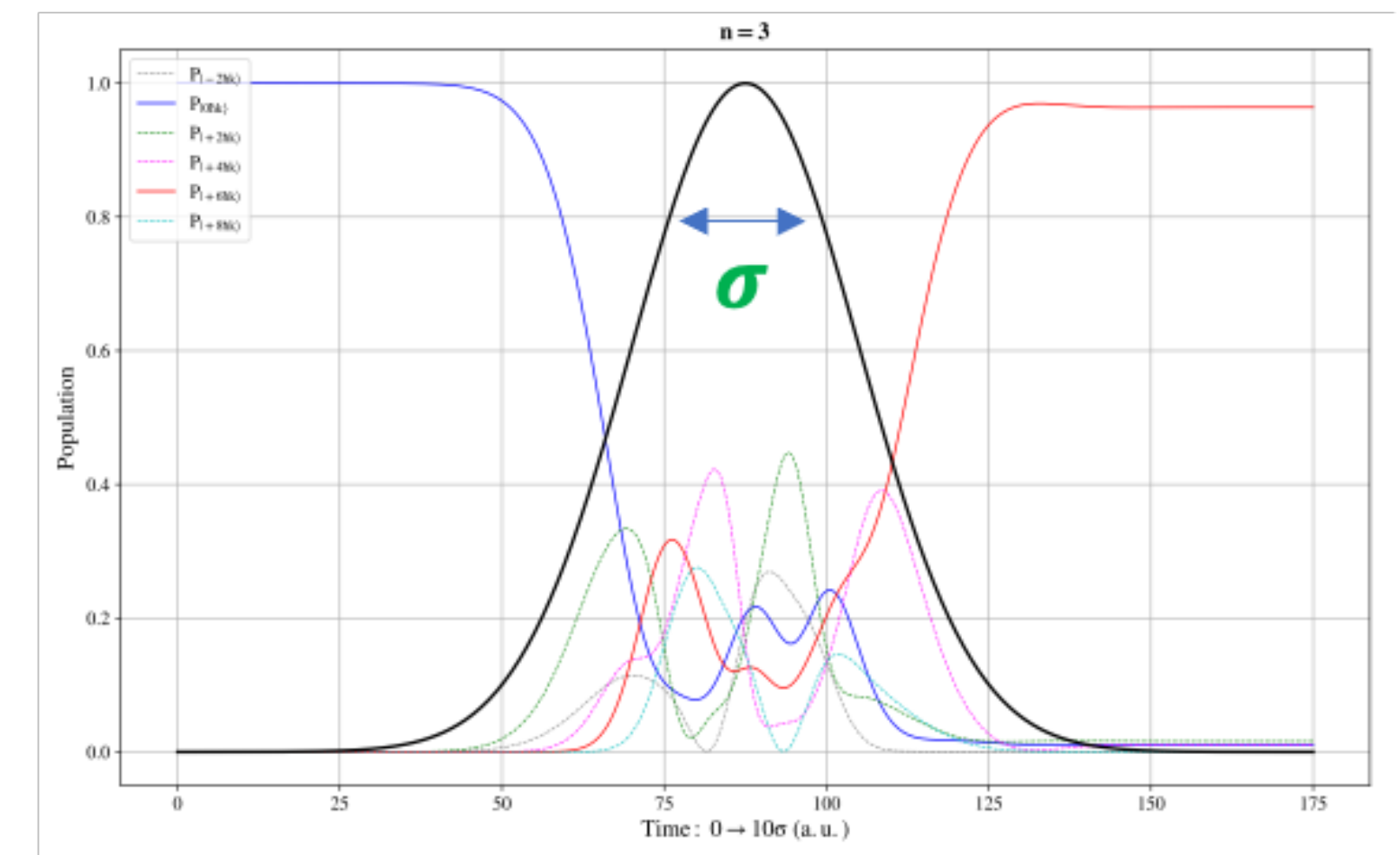
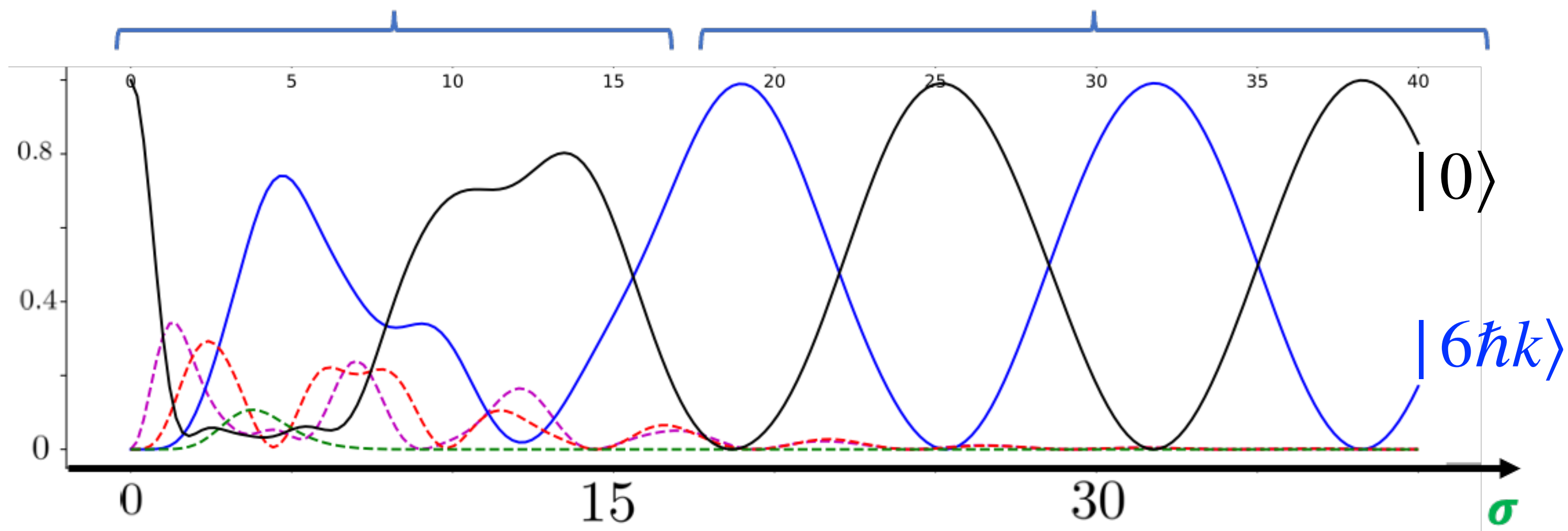
Quasi-Bragg transitions

Smooth evolution of the pulse amplitude

Short pulse duration.
Complex dynamics.

Long pulse duration.
« Rabi » type evolution.

Gaussian pulse.

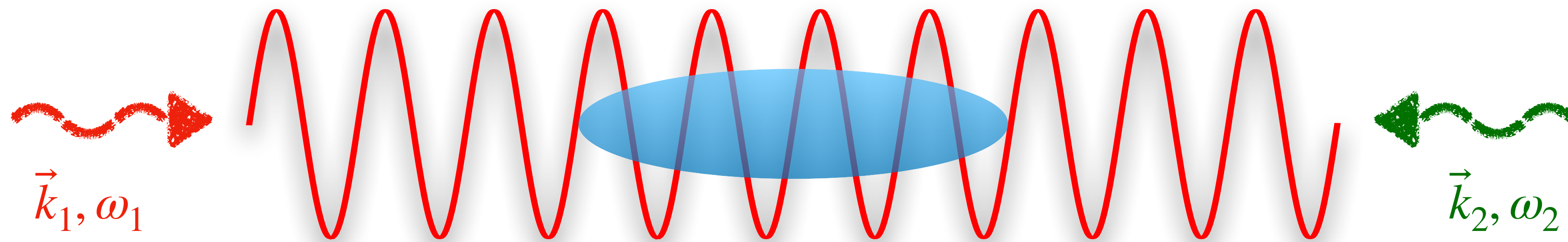


The two-level model is not readily applicable

Quasi-Bragg transitions

Optical lattice = periodic potential created by light

Counter-propagating laser beams = Spatially periodic intensity pattern



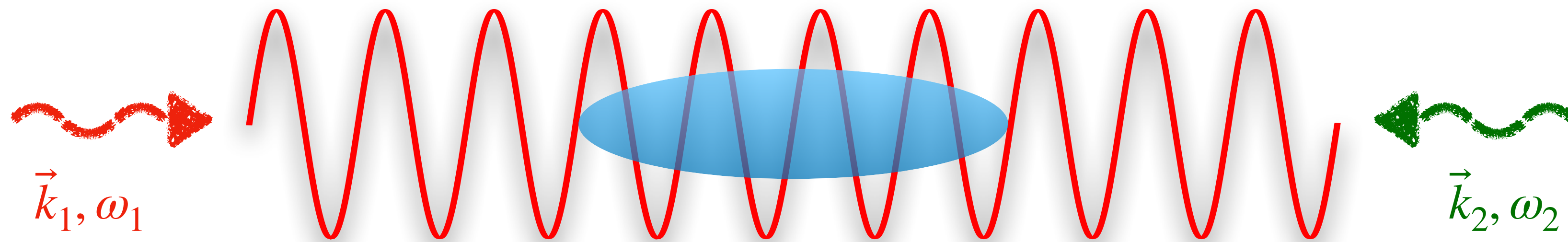
Periodicity: $\frac{\lambda}{2} = \frac{\pi}{k}$

Lattice motion : $v = \frac{\omega_2 - \omega_1}{2k}$

Quasi-Bragg transitions

Optical lattice = periodic potential created by light

Counter-propagating laser beams = Spatially periodic intensity pattern



$$\text{Periodicity: } \frac{\lambda}{2} = \frac{\pi}{k}$$

$$\text{Lattice motion : } v = \frac{\omega_2 - \omega_1}{2k}$$

Light shift creates periodic potential for atoms

$$\hat{H} = \frac{\hat{p}^2}{2m} - 2\hbar\Omega \cos^2(k\hat{z} - \phi(t)) = \frac{\hat{p}^2}{2m} - \hbar\Omega (e^{2ik\hat{z}} + e^{-2ik\hat{z}})$$

$\rightarrow e^{-2ik\hat{z}} |p\rangle = |p + 2\hbar k\rangle$
couple atomic states differing by $2\hbar k$

Quasi-Bragg transitions

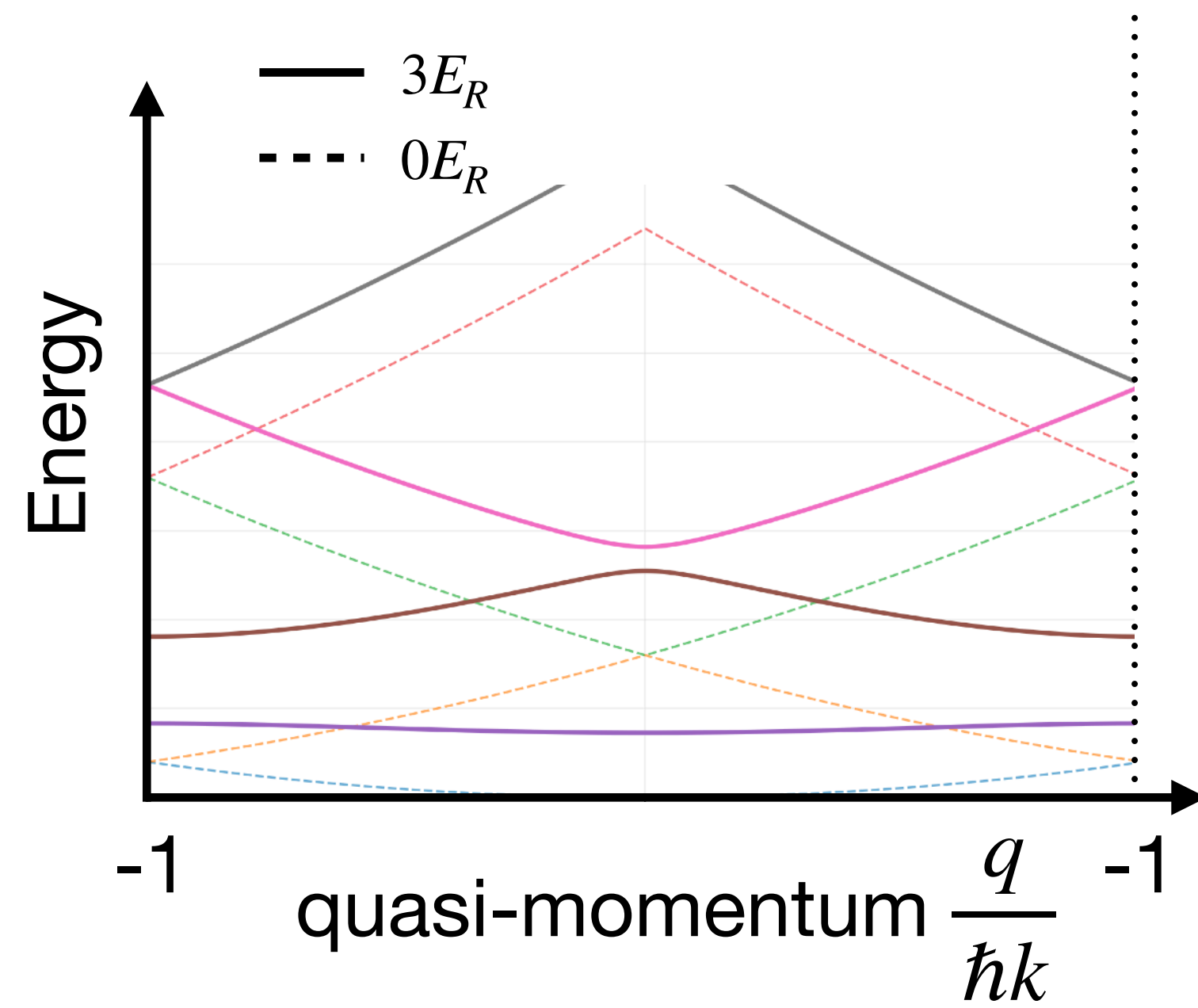
Optical lattice eigenstates

Eigenstates = Bloch states $|\Psi(n, q)\rangle = \sum_j c_j^{(n)} |p + 2j\hbar k\rangle$

● Tridagonal matrix: $\langle q + 2j\hbar k | H | q + 2l'\hbar k \rangle = \frac{(q + 2j\hbar k)^2}{2M} \delta_{j,j'} + \frac{\hbar\Omega}{2} \delta_{l',l\pm 1}$

● Diagonalization of H leads to Band Energy diagram

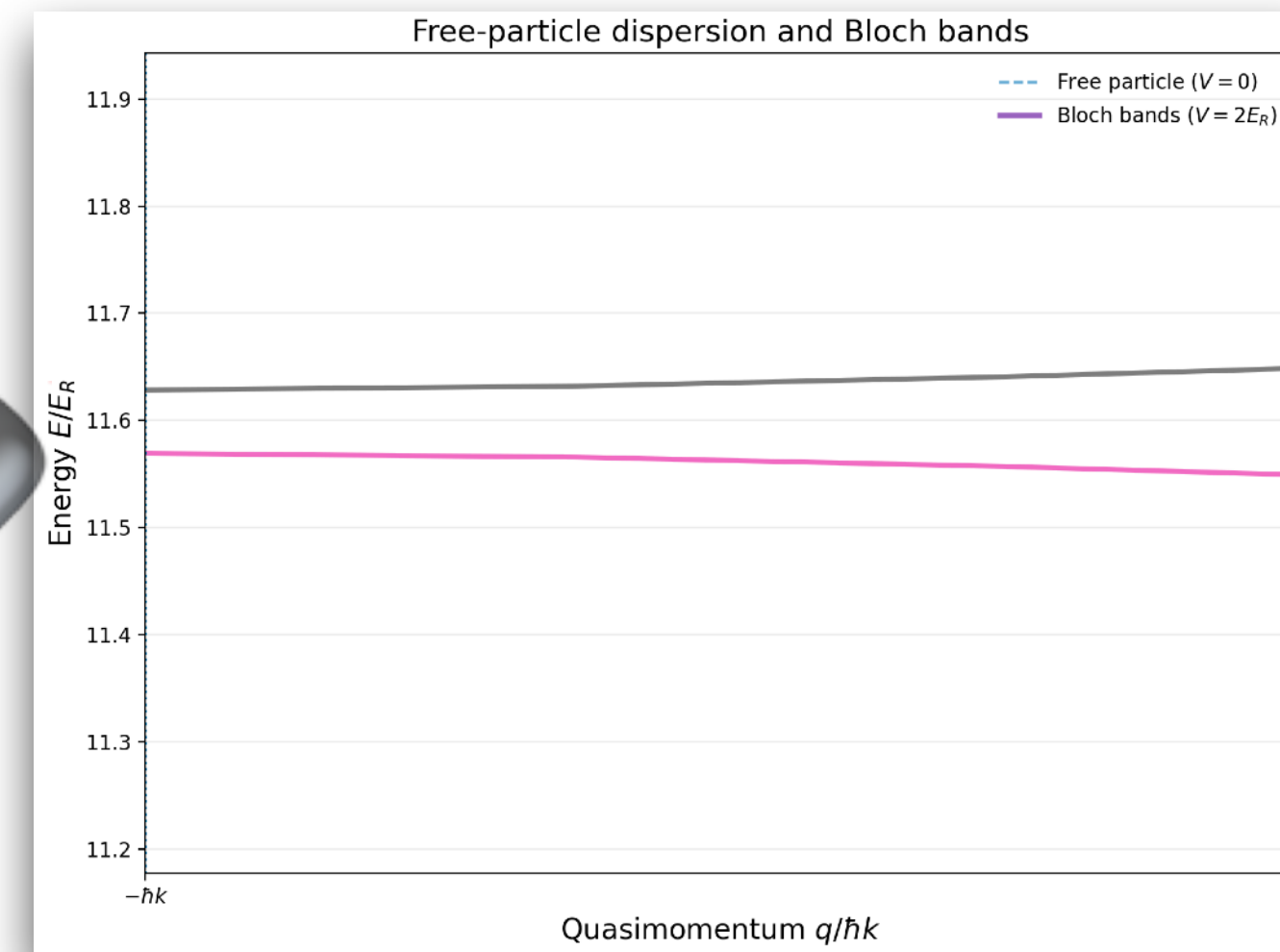
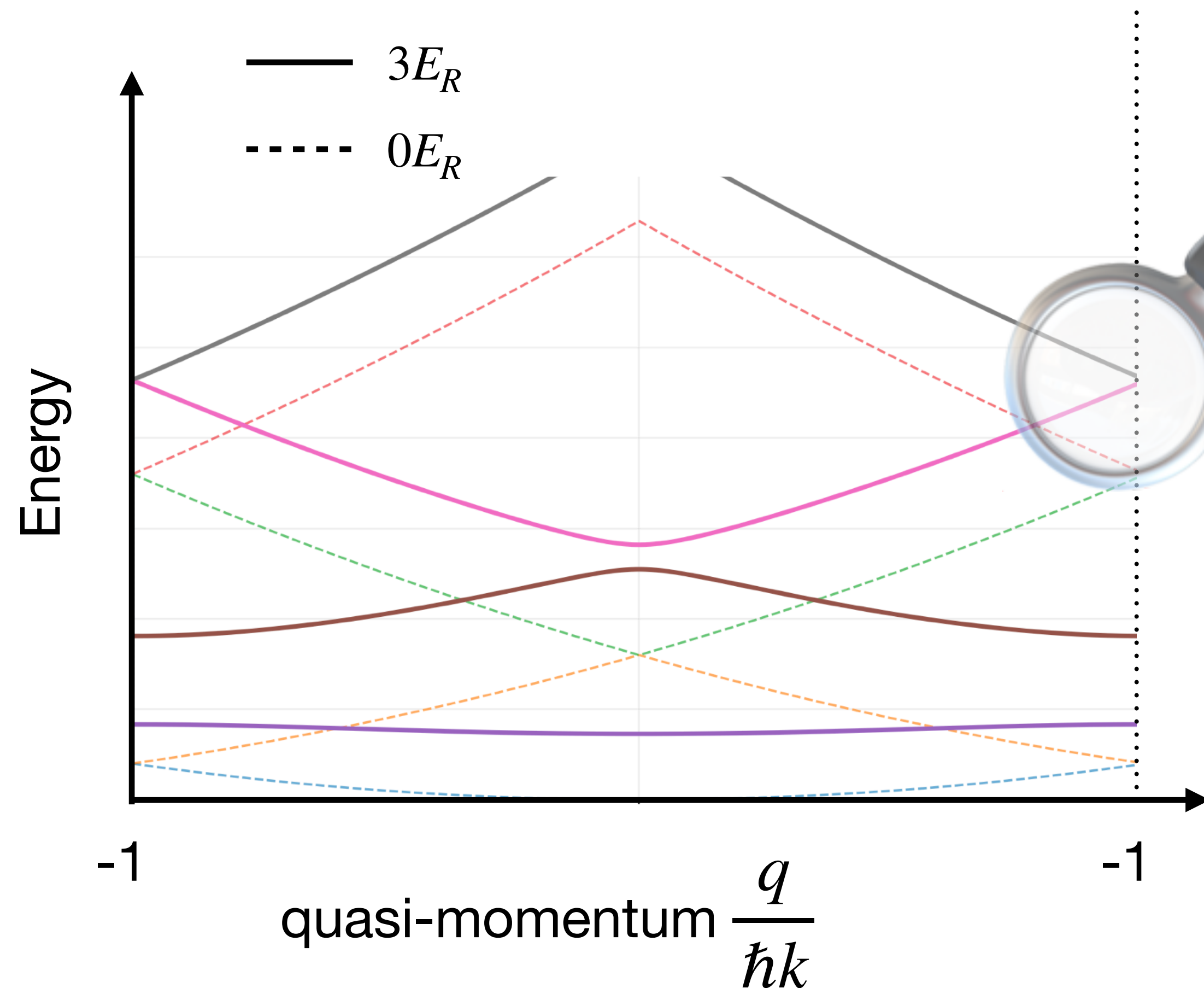
$$4\hbar\omega_r \begin{pmatrix} \delta_{-m} & \gamma & 0 & \dots & \dots & \dots & 0 \\ \gamma^* & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \delta_n & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \gamma \\ 0 & \dots & \dots & \dots & \dots & \gamma^* & \delta_{n+m} \end{pmatrix}$$



- The parabola intersects at a depth of $0E_R$
- Gaps open at the avoided crossing
- Quasi-momentum q is conserved

Quasi-Bragg transitions

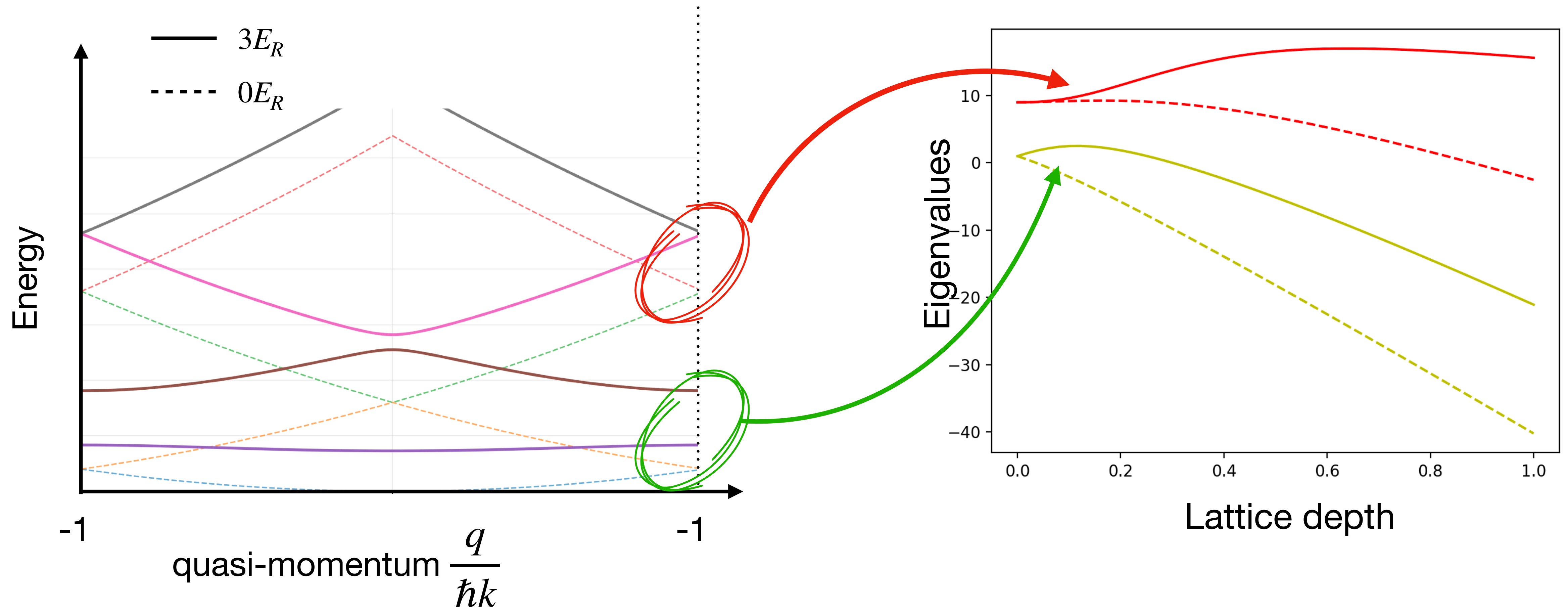
Optical lattice Band energy diagram



- The parabola intersects at a depth of $0E_R$
- Gaps open at the avoided crossing
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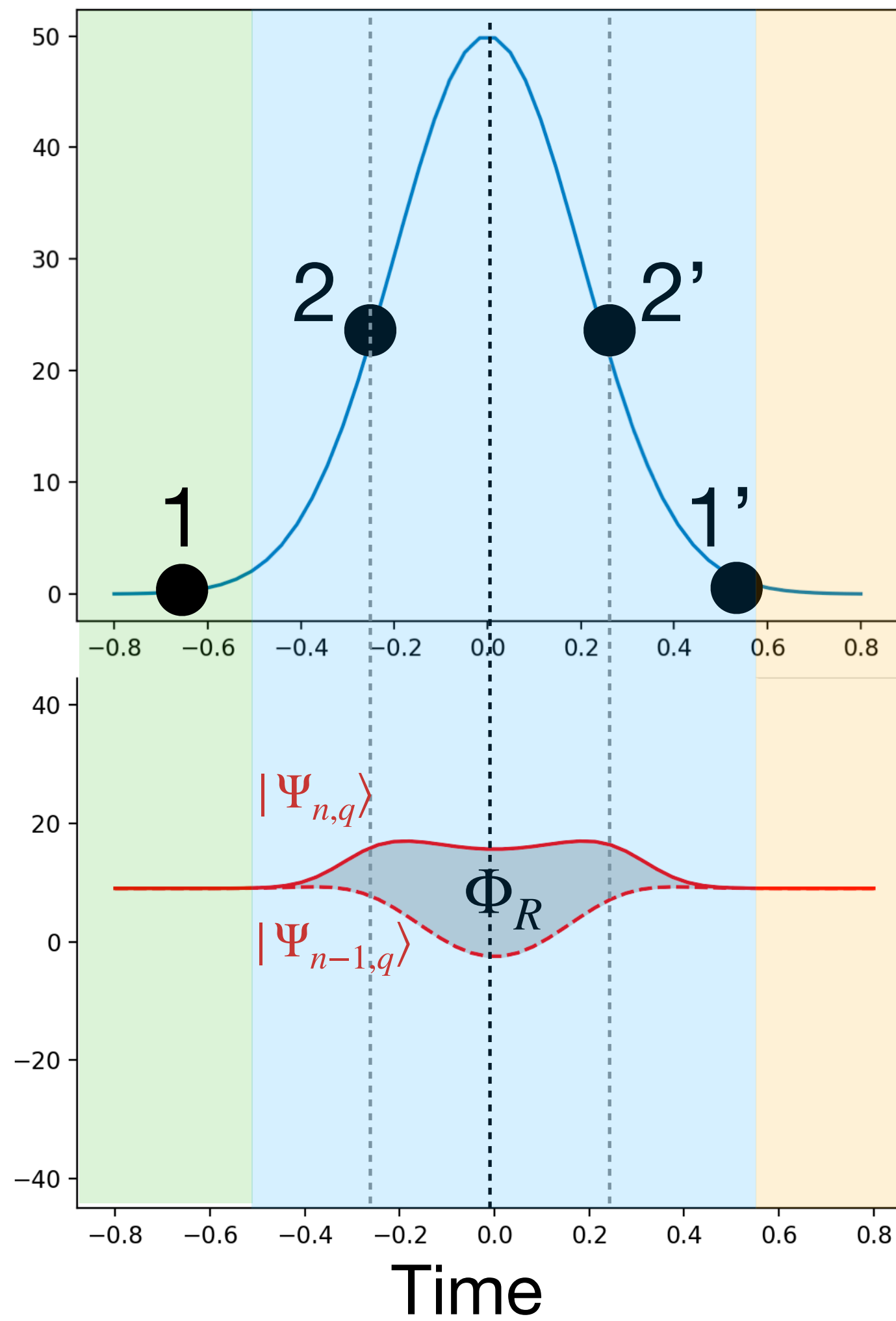
Quasi-Bragg transitions

Optical lattice Band gap vs lattice depth



Quasi-Bragg transitions

Long pulse

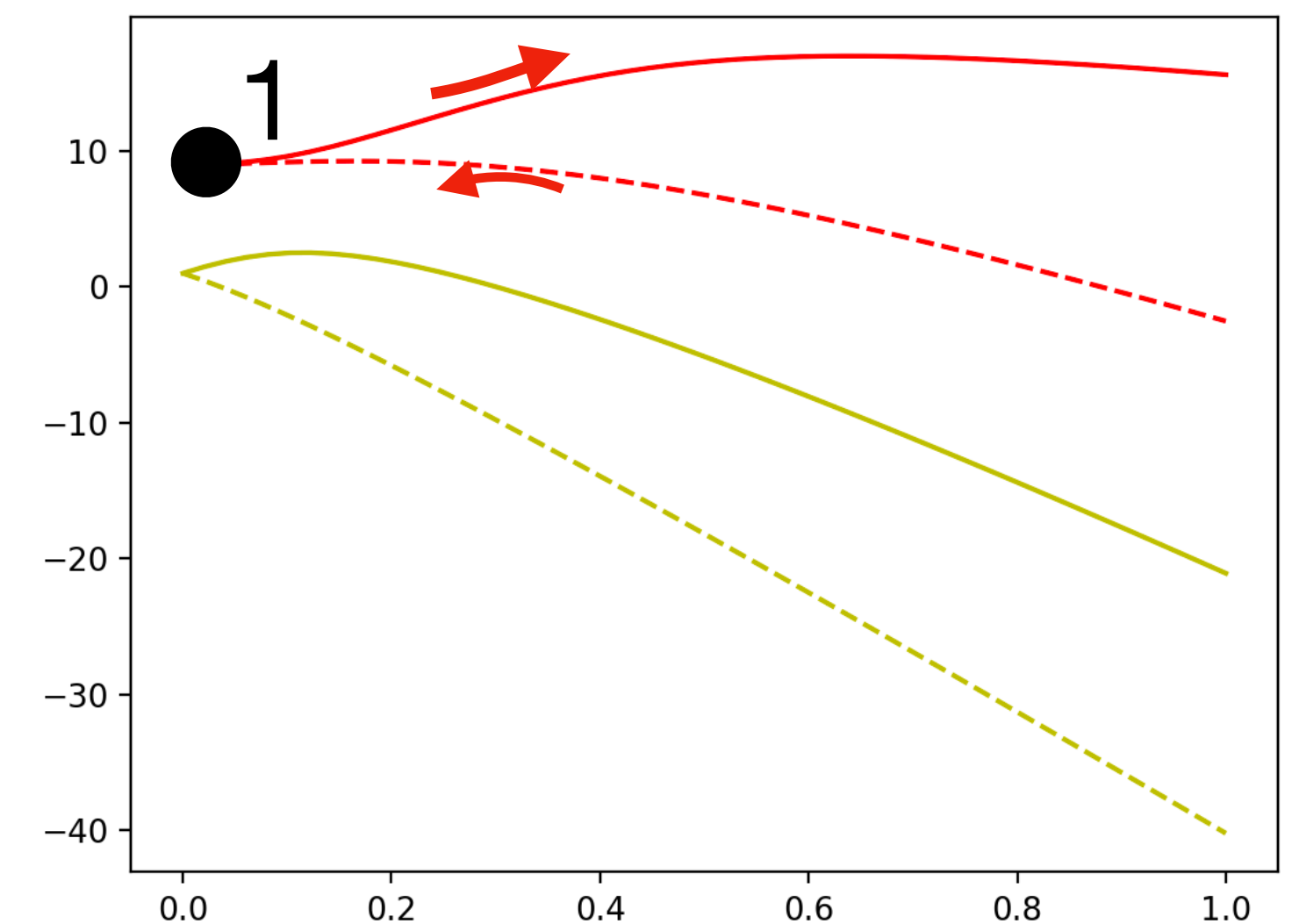


1- Initial state $t=0$ $| -n \rangle = \frac{1}{\sqrt{2}} \left(|\Psi_{n,q}\rangle + |\Psi_{n-1,q}\rangle \right)$

2 - Adiabatic following $\frac{1}{\sqrt{2}} \left(|\Psi_{n,q}\rangle + |\Psi_{n-1,q}\rangle e^{i\Phi_R} \right)$

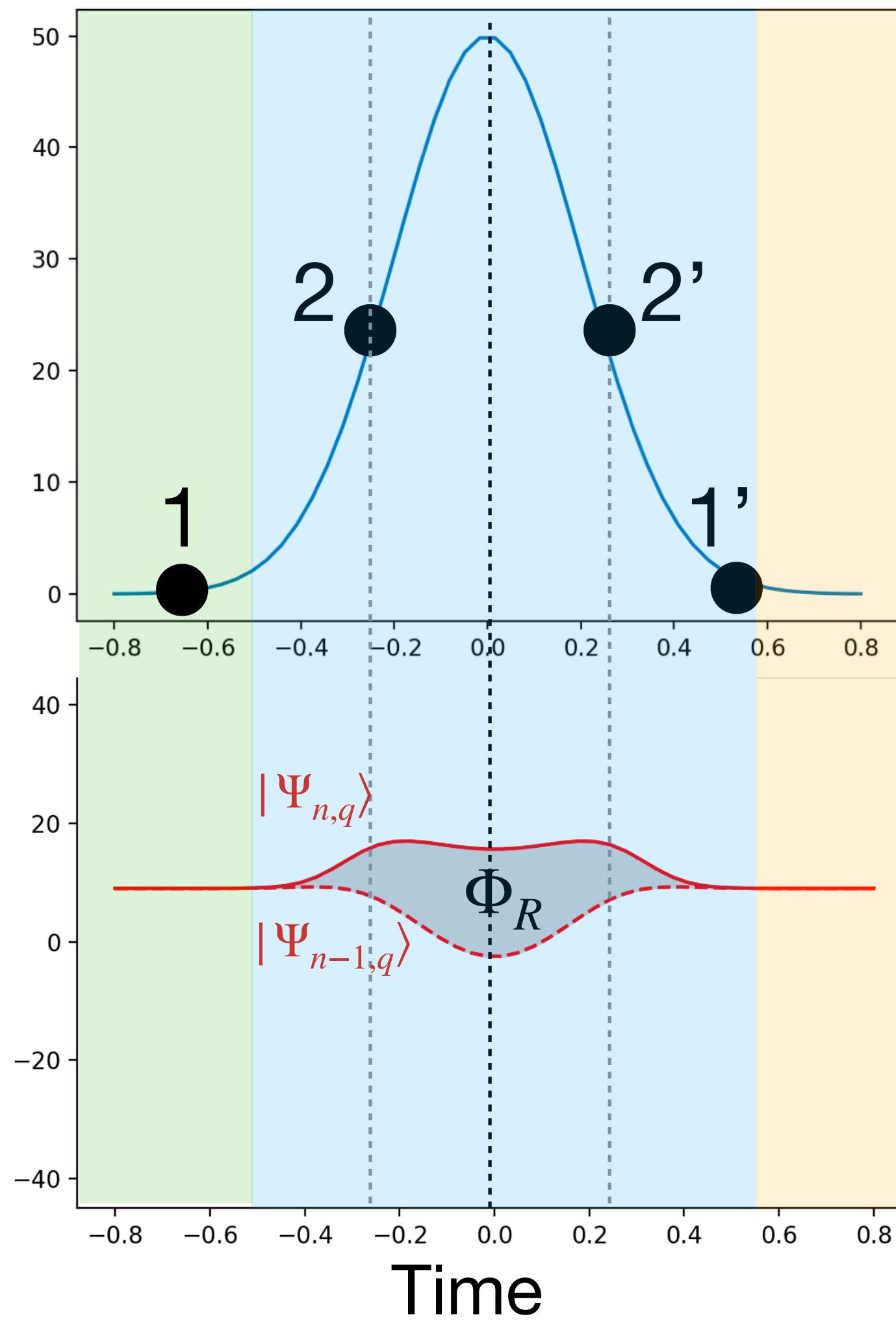
$$\text{Rabi phase } \Phi_R = \int \frac{E_+ - E_-}{\hbar} dt$$

3 - Measurement $|m\hbar k\rangle \langle m\hbar k|$



Quasi-Bragg transitions

Long pulse

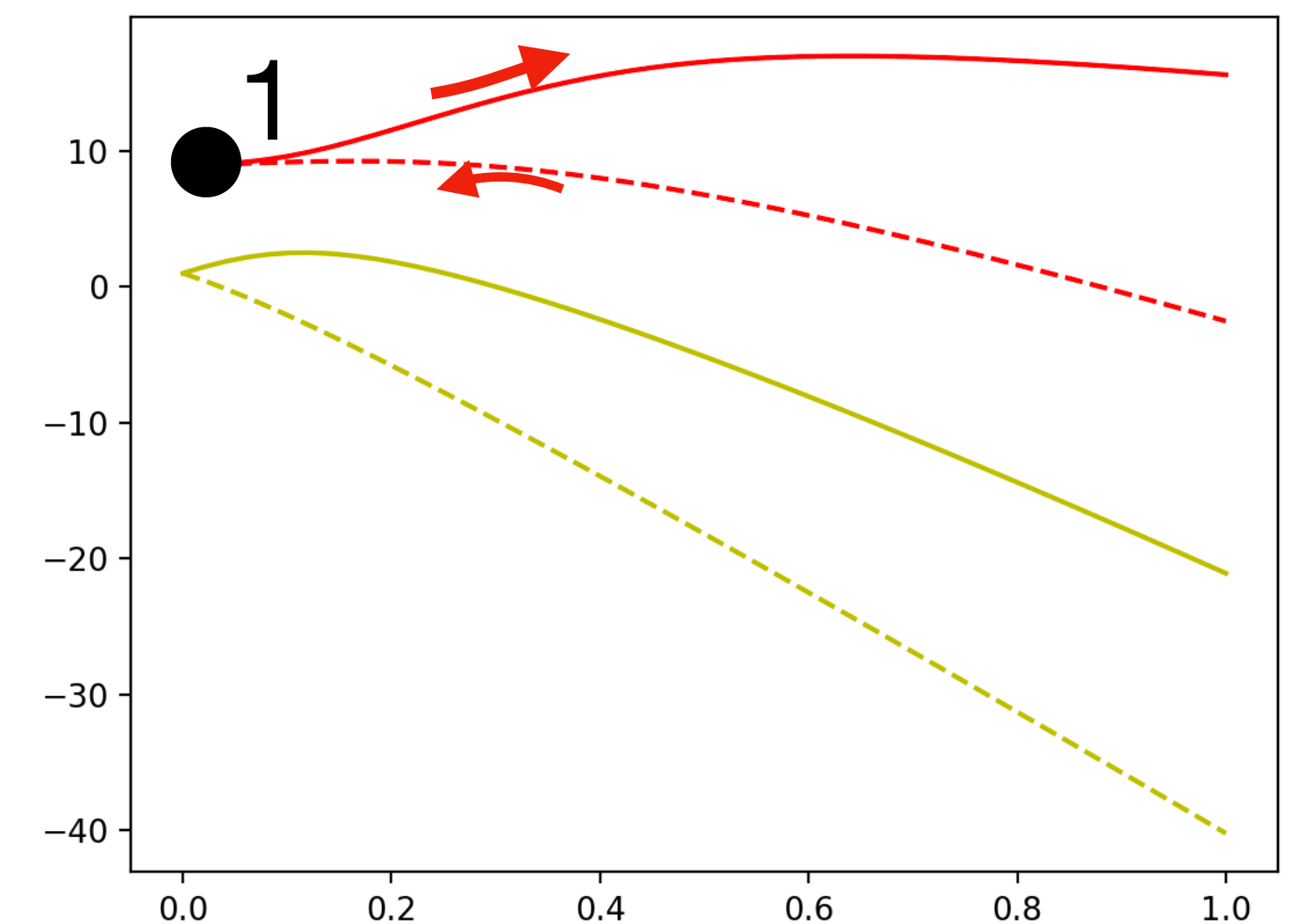
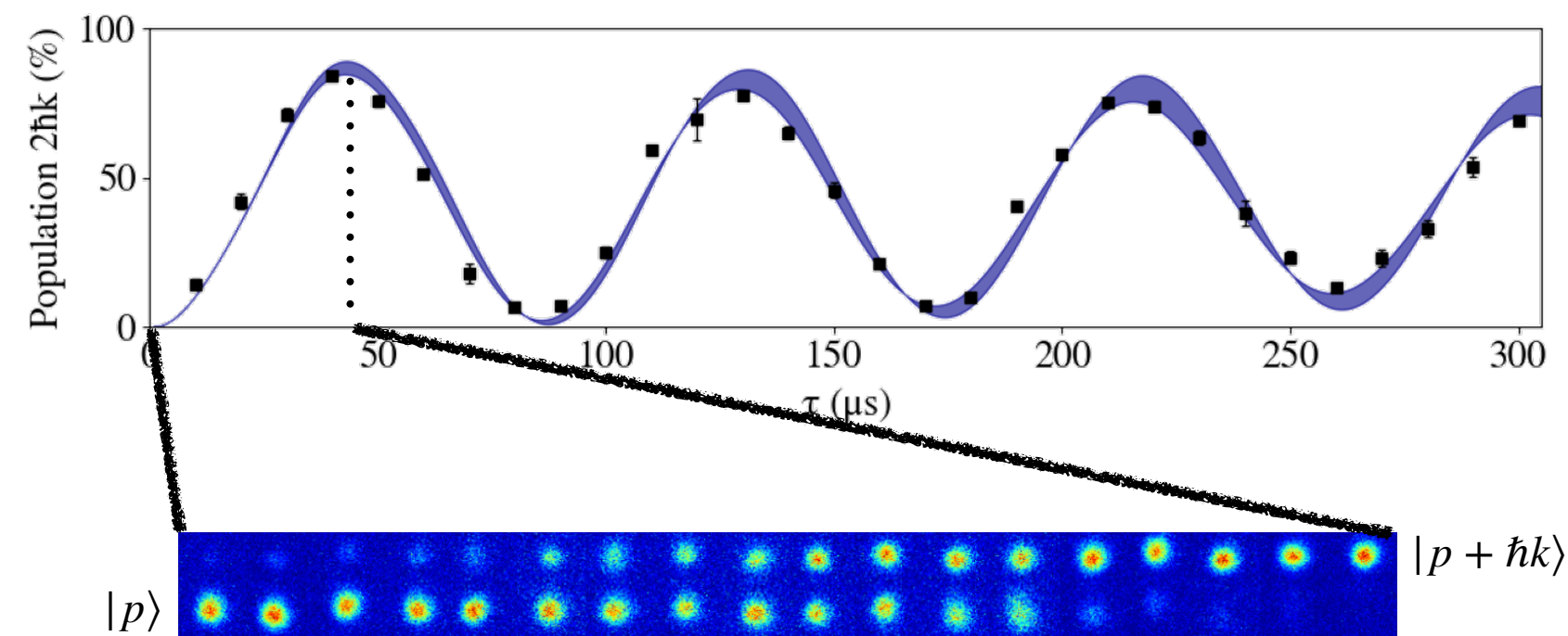


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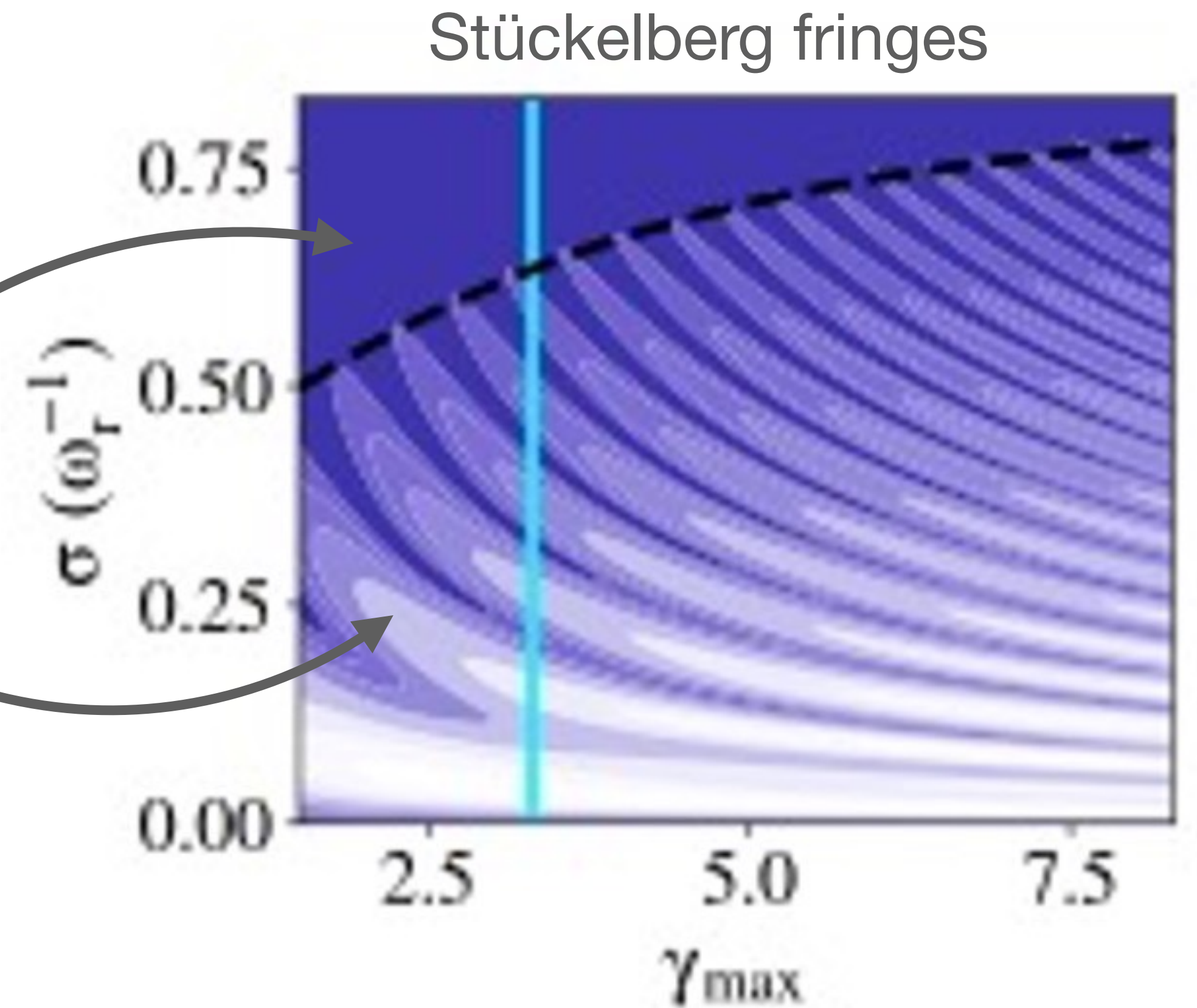
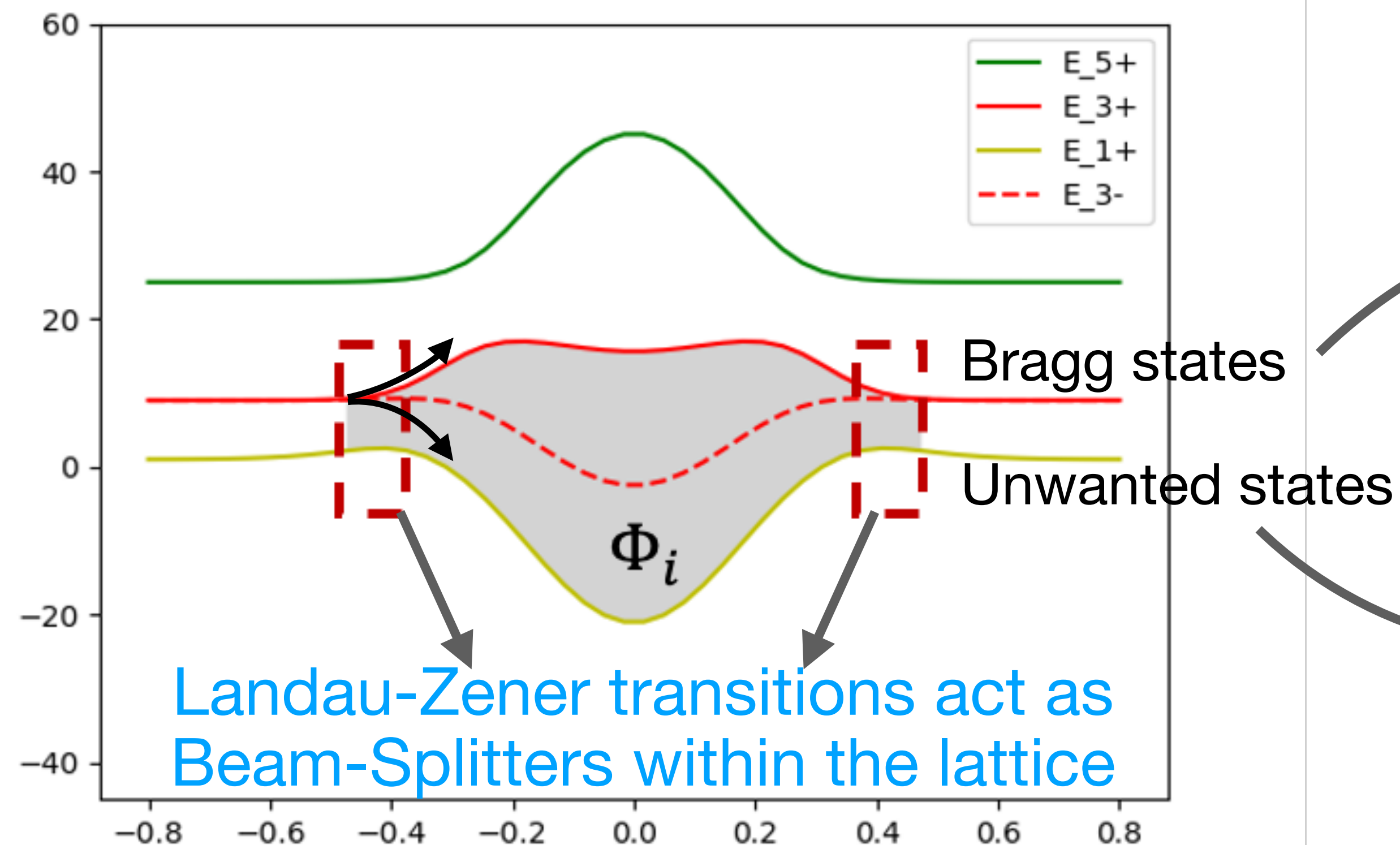
$$\text{Rabi phase } \Phi_R = \int \frac{E_+ - E_-}{\hbar} dt$$

3 - Measurement $|m\hbar k\rangle \langle m\hbar k|$



Quasi-Bragg transitions

Short pulse

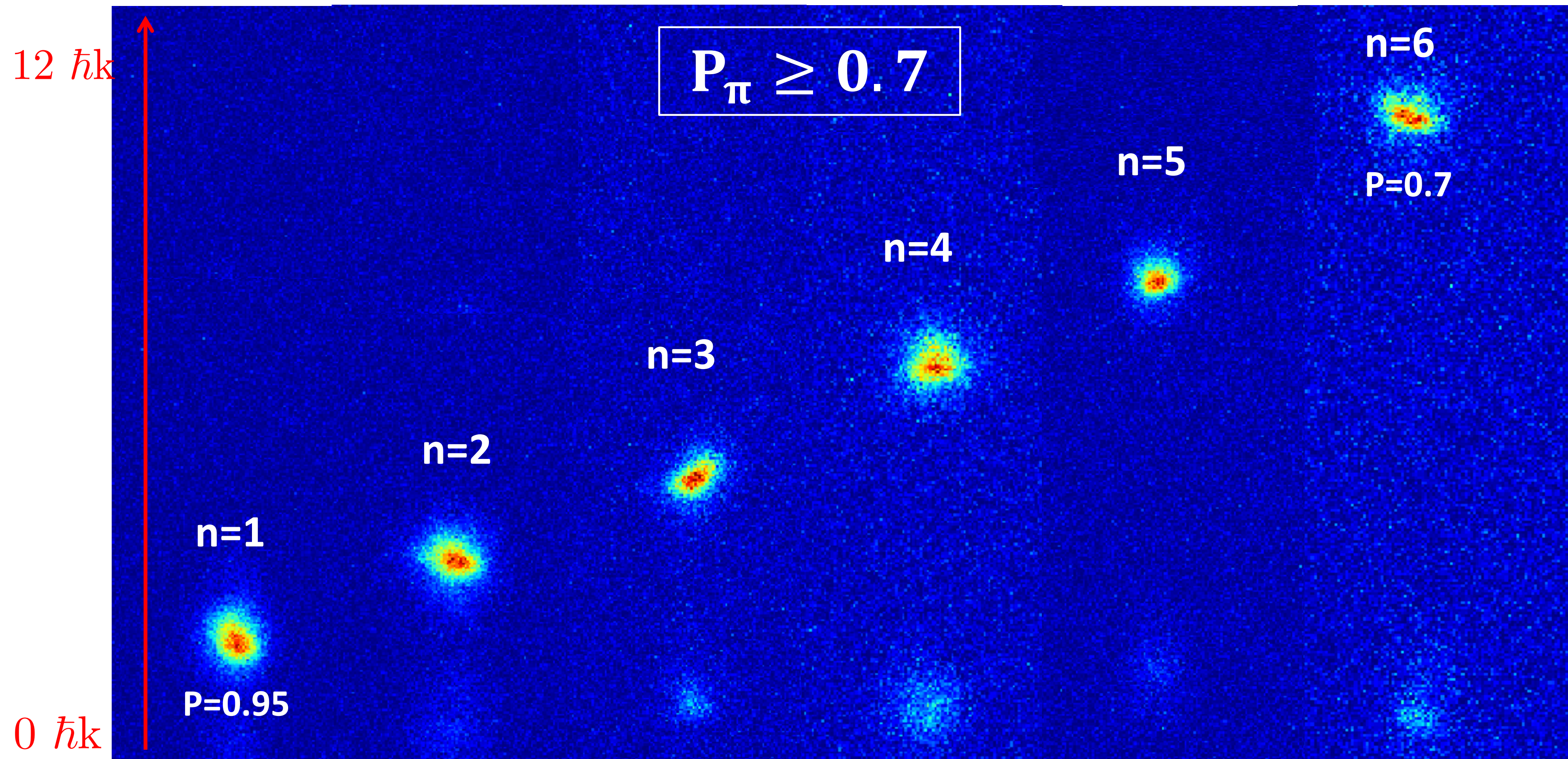


Quasi-Bragg pulse itself can be seen as a (Stückelberg) interferometer.

Lattice diffraction is multi- $\hbar k$ by nature !!

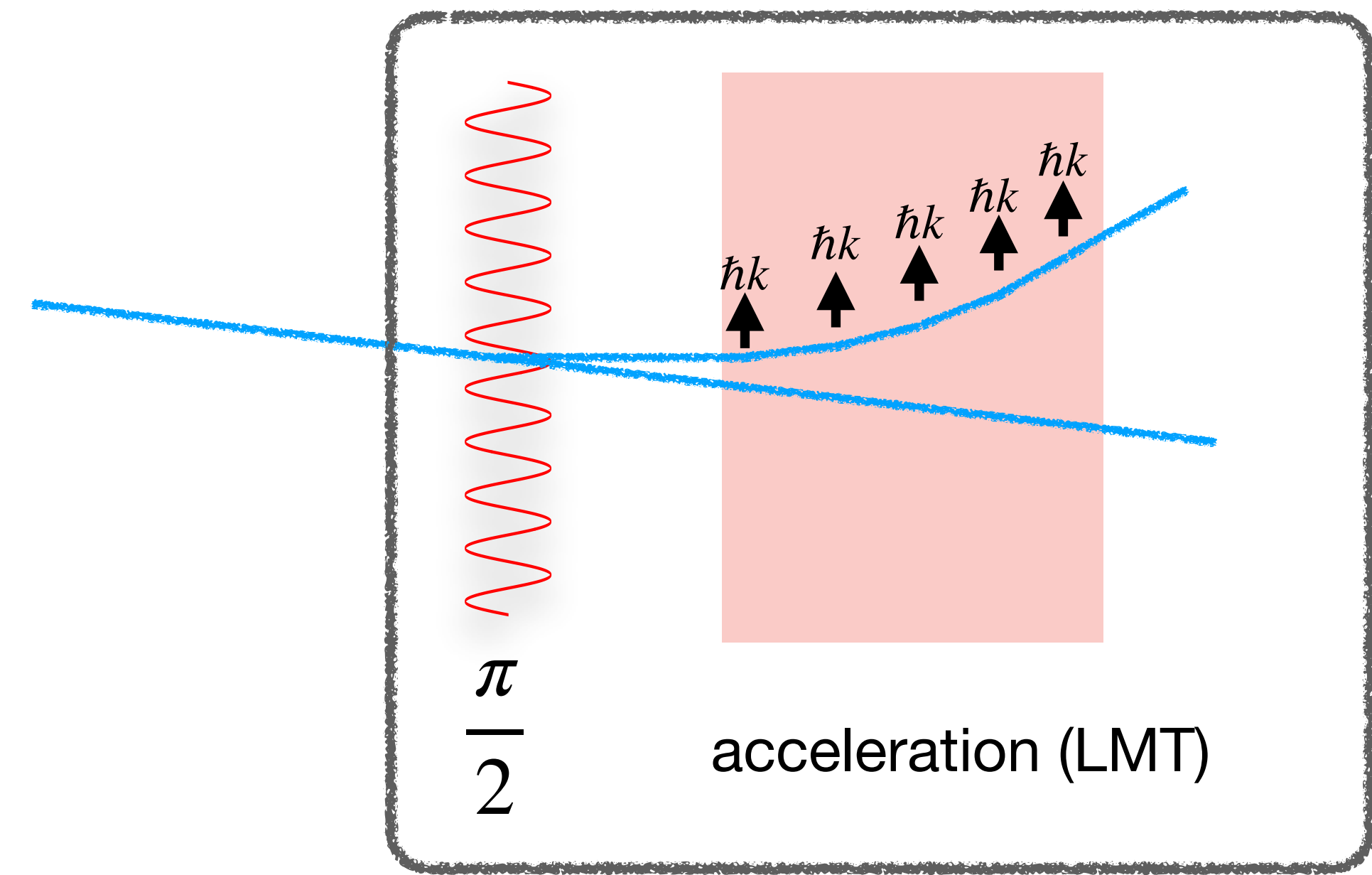
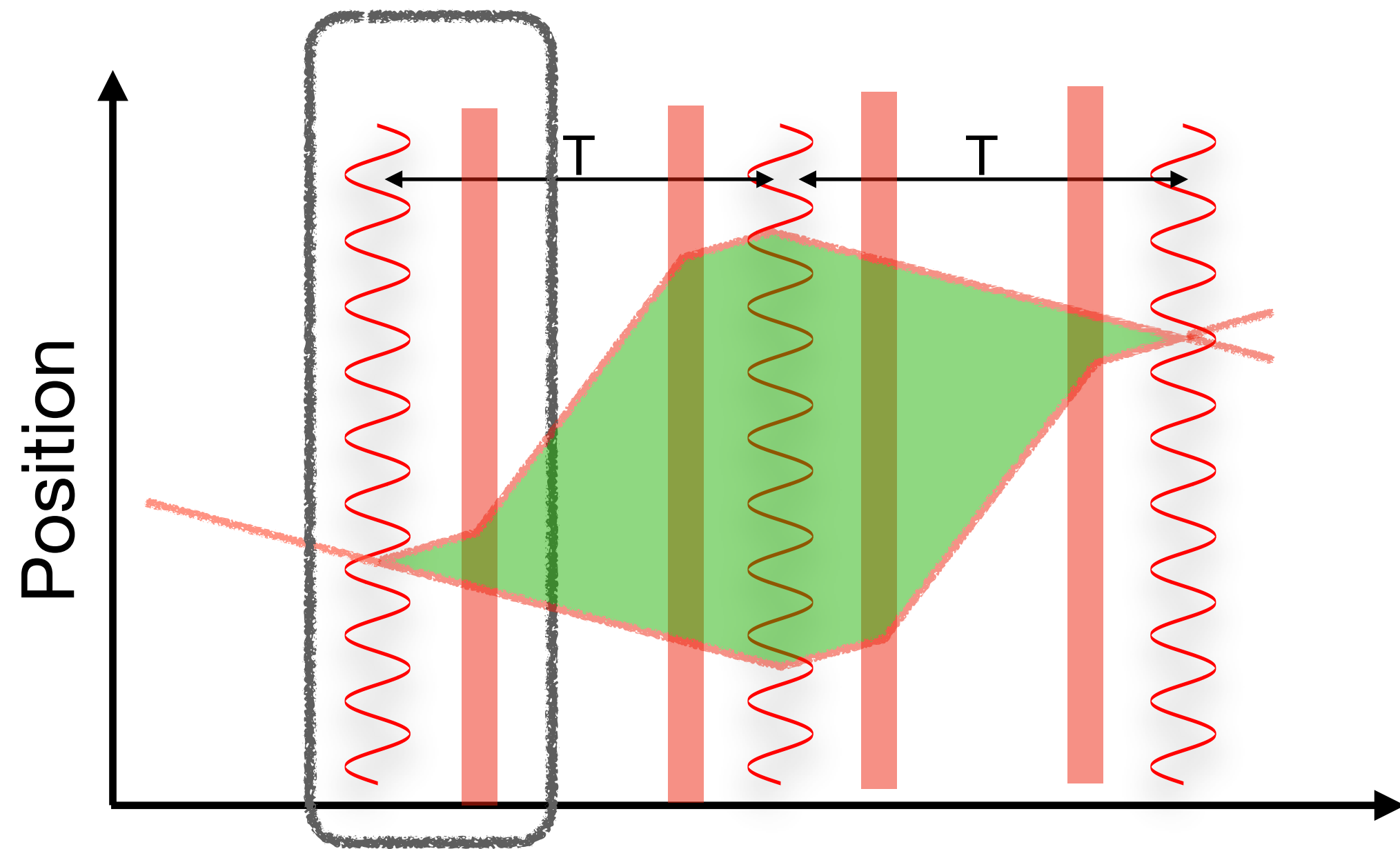
High order Quasi-Bragg

Achieving diffraction orders greater than a few tens is difficult.



Large Momentum Transfer Atom Optics

LMT based on a beam splitter followed by coherent acceleration



Proposal for $3\hbar k$ using 3 successive π -pulses

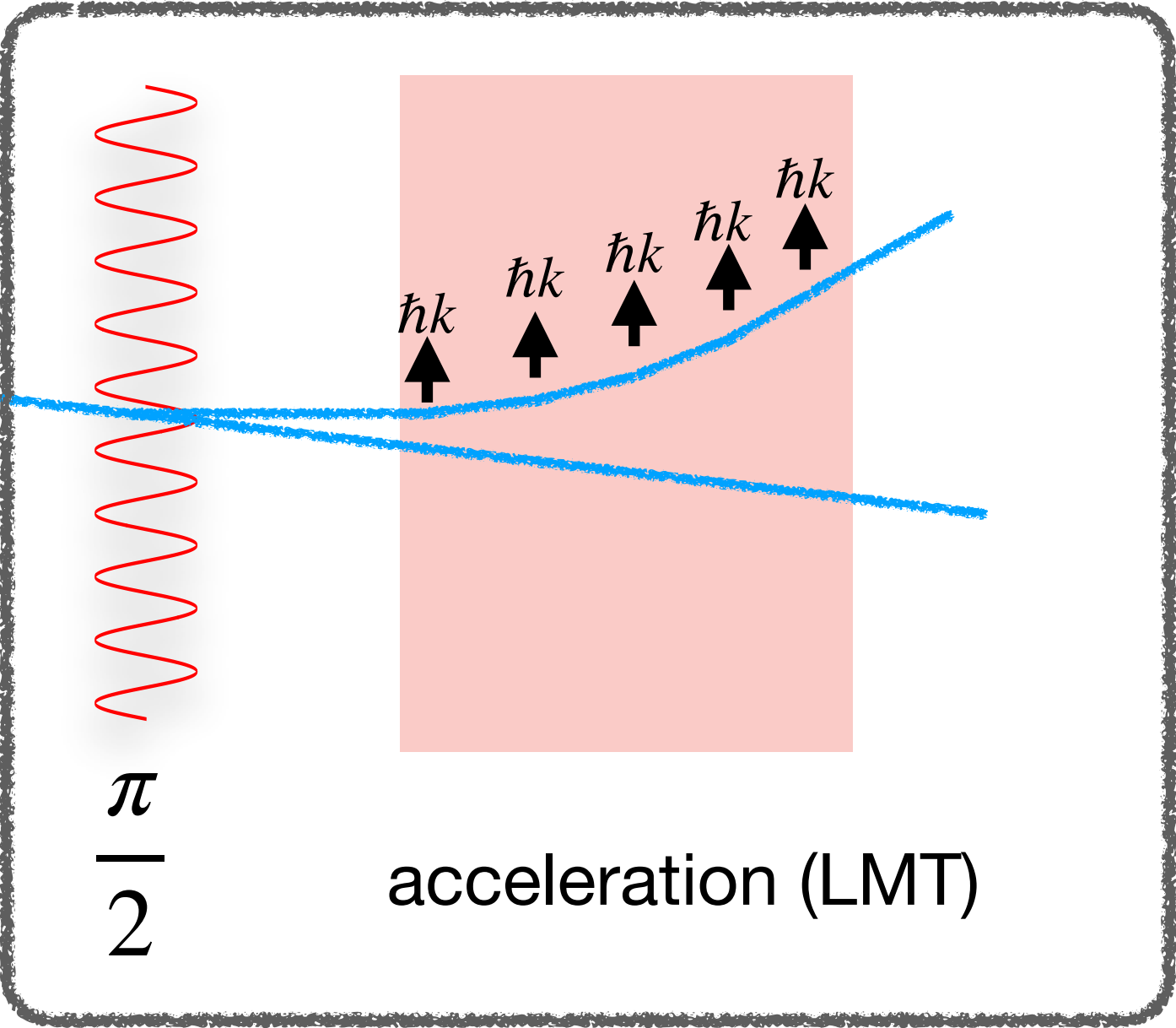
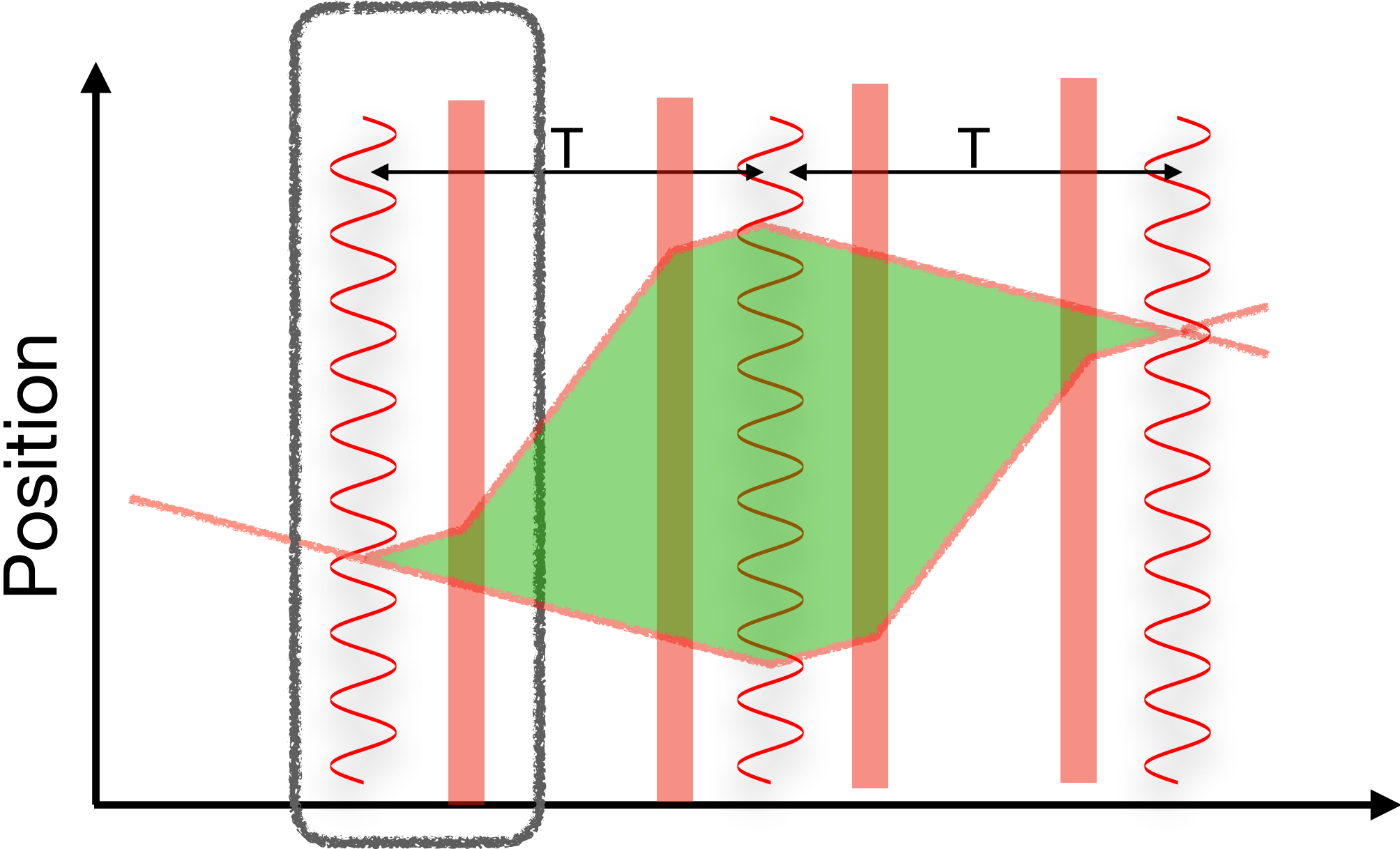
C.J. Bordé et al. AIP Proceeding (1993)

« We believe that there are no serious barriers to realization of momentum transfers greater than $10\hbar k$. »

J.M. McGuirk et al. Large Area Light-Pulse Atom interferometry PRL (2000)

Large Momentum Transfer Atom Optics

LMT based on a beam splitter followed by coherent acceleration



Proposal for $3\hbar k$ using 3 successive π -pulses
C.J. Bordé et al. AIP Proceeding (1993)

TVLBAI (GW detector) - roadmap:

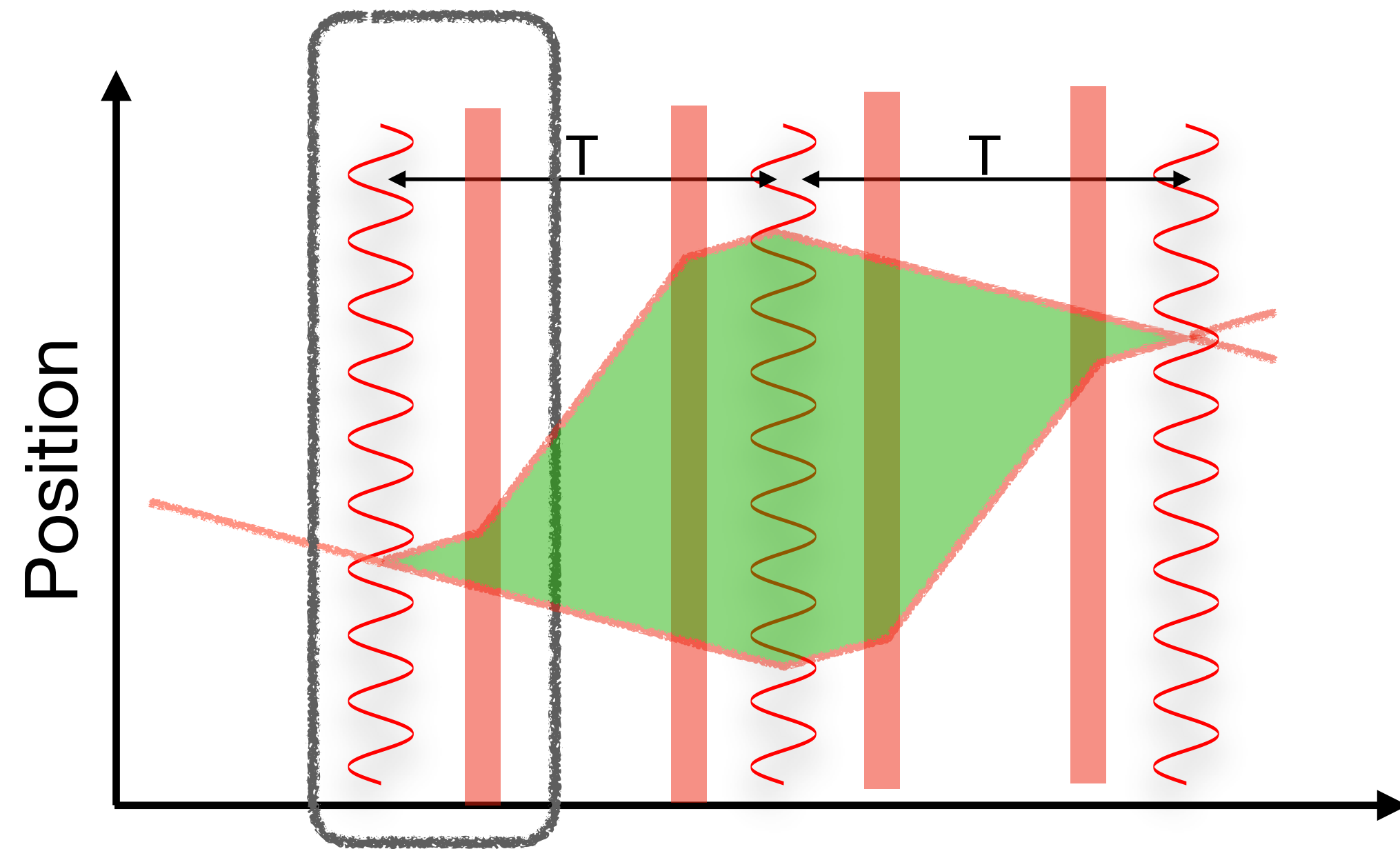
Sensor Technology	State-of-the-art	Target	Enhancement
LMT atom optics	$10^2 \hbar k$	$10^4 \hbar k$	100
Matter-wave lensing	50 pK	5 pK	—
Laser Power	10 W	100 W	—
Spin squeezing	20 dB (Rb), 0 dB (Sr)	20 dB (Sr)	10
Atom flux	10^5 atoms/s (Rb)	10^7 atoms/s (Sr)	10
Baseline length	10 m	1000 m	100

« We believe that there are no serious barriers to realization of momentum transfers greater than $10\hbar k$. »

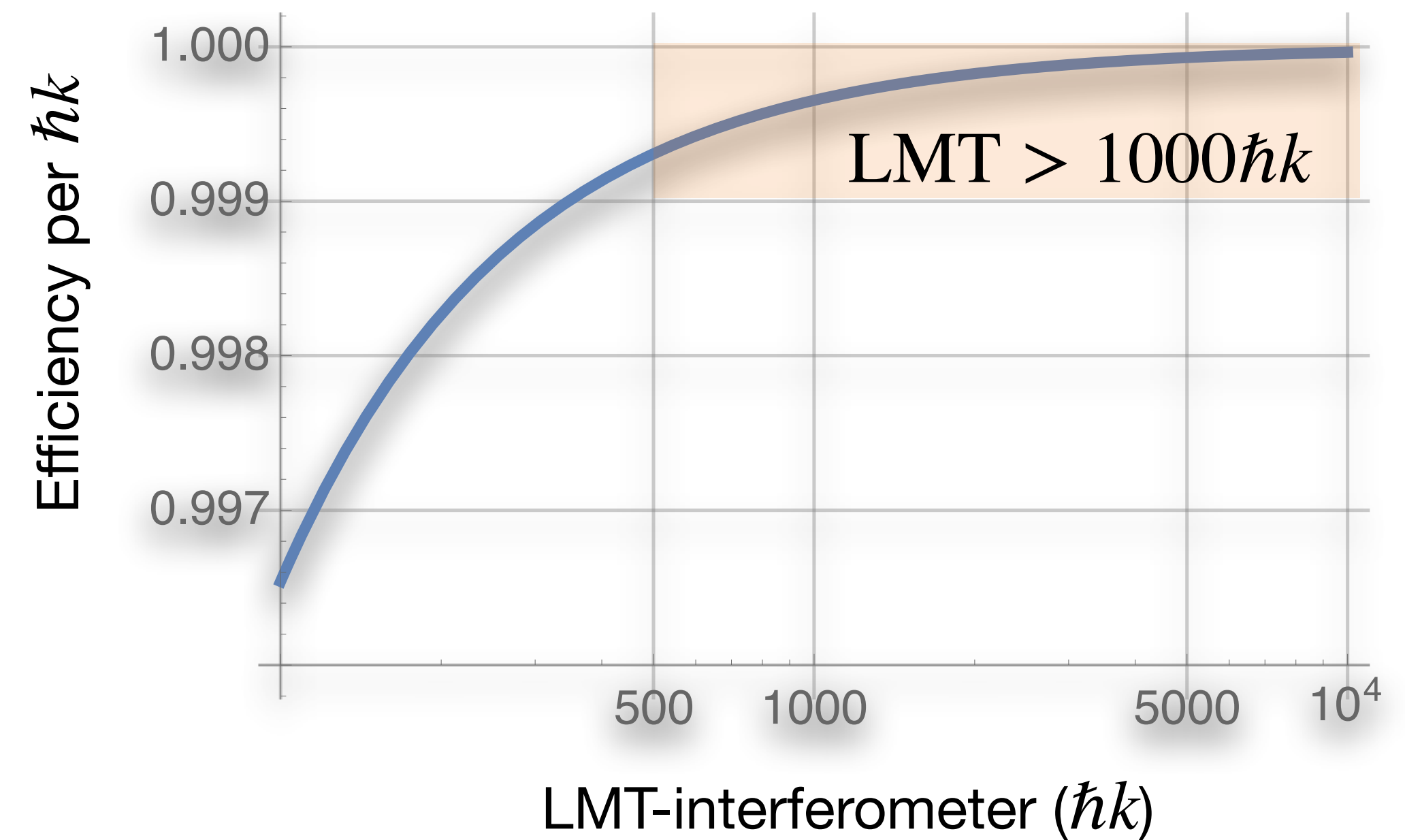
J.M. McGuirk et al. Large Area Light-Pulse Atom interferometry PRL (2000)

Large Momentum Transfer Atom Optics

Principle & State of the Art



Reaching $> 1000\hbar k$?



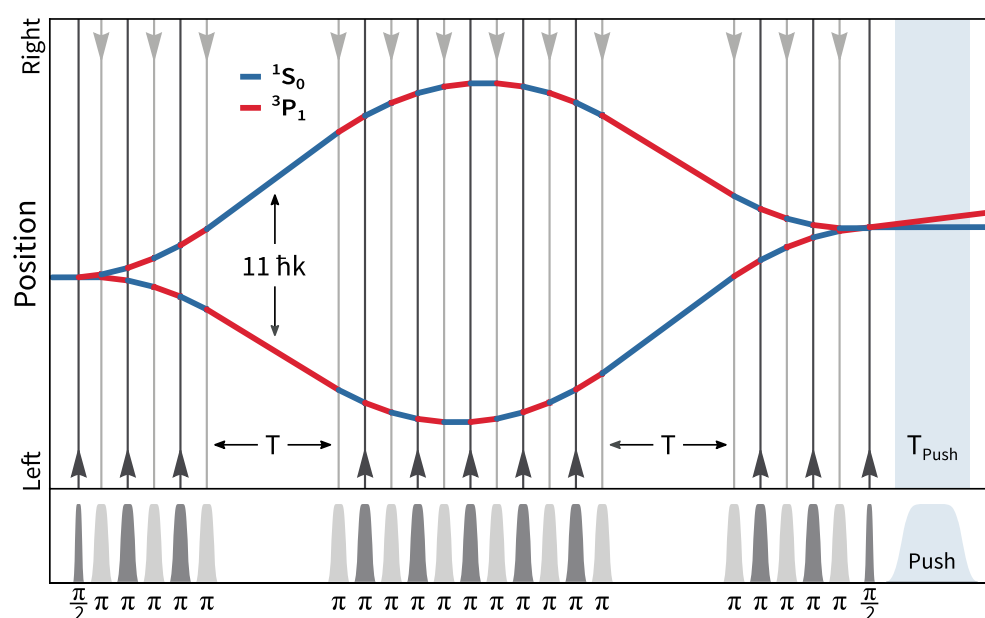
High efficiency & High rate

Light-pulse Atom Interferometry

Large Momentum Transfer

Inelastic

Single photon transition

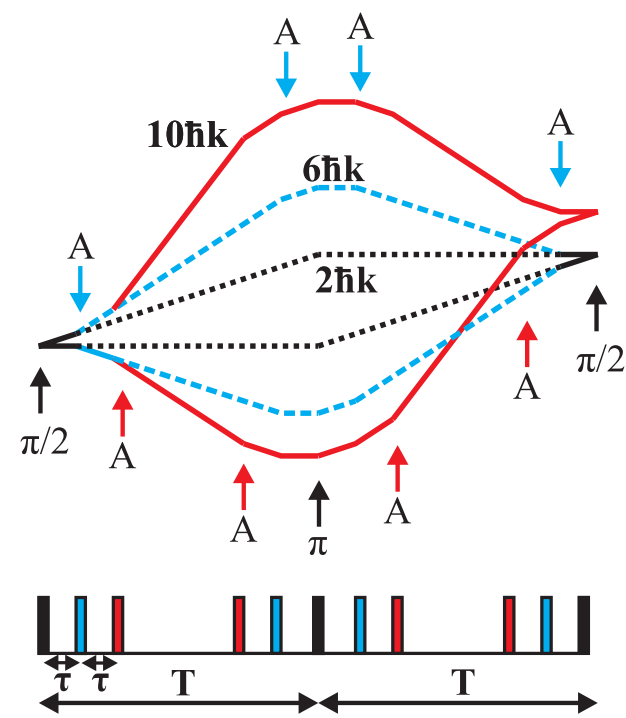


Rudolph et al., Phys. Rev. Lett. **124**, 083604 (2020)

- 2-level system optical transition
- Metastable excited state
- Resonant transition $\rightarrow$ Spont. Emission
- Rabi frequency $\Omega \sim \text{MHz}$
- Demonstrated $400\hbar k$
- Very high rate transfer: $\sim 0.1 \mu\text{s}/\hbar k$

Wilkason et al., Phys. Rev. Lett., **129**, 183202 (2022)

Two-photon Raman transition



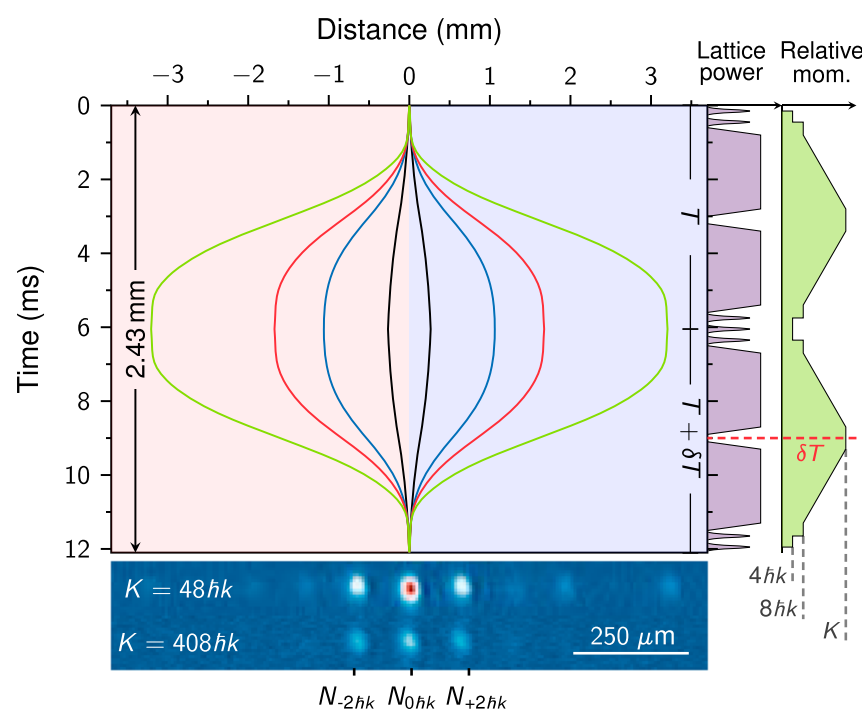
- Effective 2-level system, 2 photons transition
- Ground states
- Rabi frequency $\Omega \sim 100\text{kHz}$
- Demonstrated $30\hbar k$
- High rate transfer: $\sim 2.5 \mu\text{s}/\hbar k$

McGuirk et al., Phys. Rev. Lett., **85**, 4 4981(2015)

Kotru et al., Phys. Rev. Lett., **115**, 103001(2015)

Elastic

Optical lattice Bloch-Type acceleration



Gebbe et al., Nat Commun, **12**, 2544 (2021)

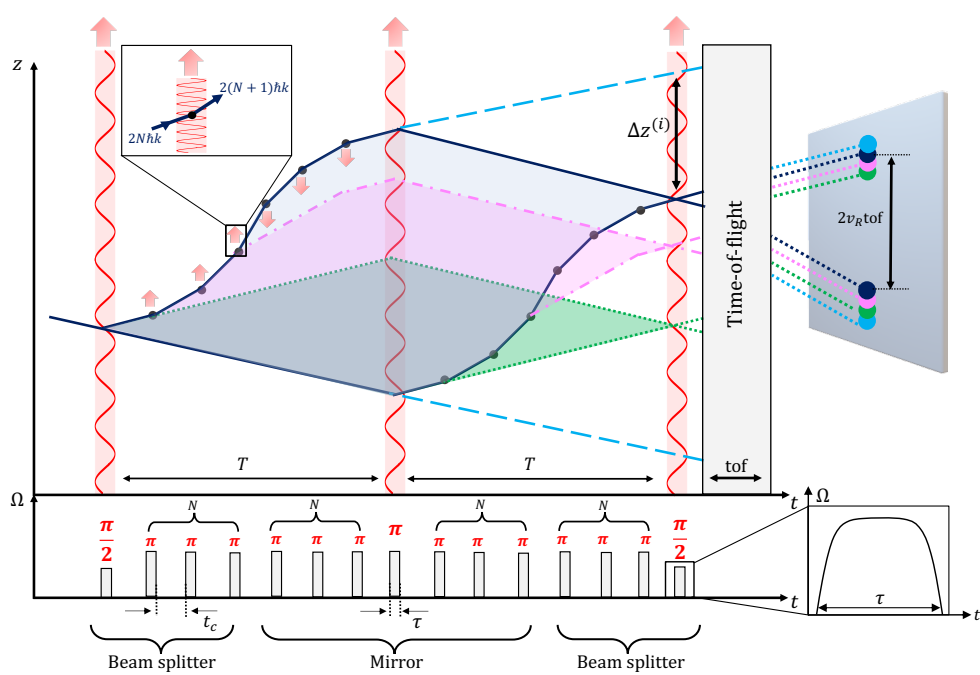
- multi-photon transitions - lattice states
- Ground state
- Rabi frequency $\Omega \sim 10\text{kHz}$
- Demonstrated $> 400\hbar k$
- Rate transfer: $\sim 6 \mu\text{s}/\hbar k$

Cladé et al., Phys. Rev. Lett. **102**, 240402 (2009)

McDonald et al., Phys. Rev. A **88**, 053620 (2013)

Pagel et al., Phys. Rev. A **102** 053312 (2020)

Optical lattice Sequence of Bragg pulses



Béguin et al., Phys. Rev. Lett., **131**, 143401 (2023)

- multi-photon transitions - lattice states
- Ground state
- Rabi frequency $\Omega \sim 100\text{kHz}$
- Demonstrated $600\hbar k$
- High rate transfer: $\sim 2.5 \mu\text{s}/\hbar k$

Chiu et al., Phys. Rev. Lett., **107**, 130403 (2011)

Plotkin-Swing et al., Phys. Rev. Lett., **121**, 133201 (2018)

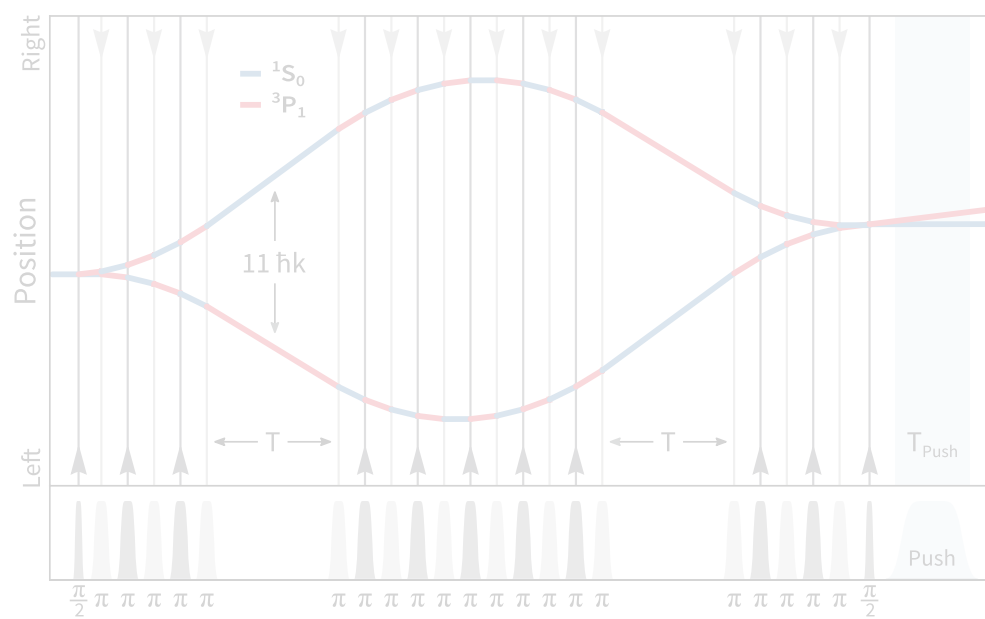
Rodzinka et al. Nat Comm **15**, 10281 (2024)

Light-pulse Atom Interferometry

Large Momentum Transfer

Inelastic

Single photon transition

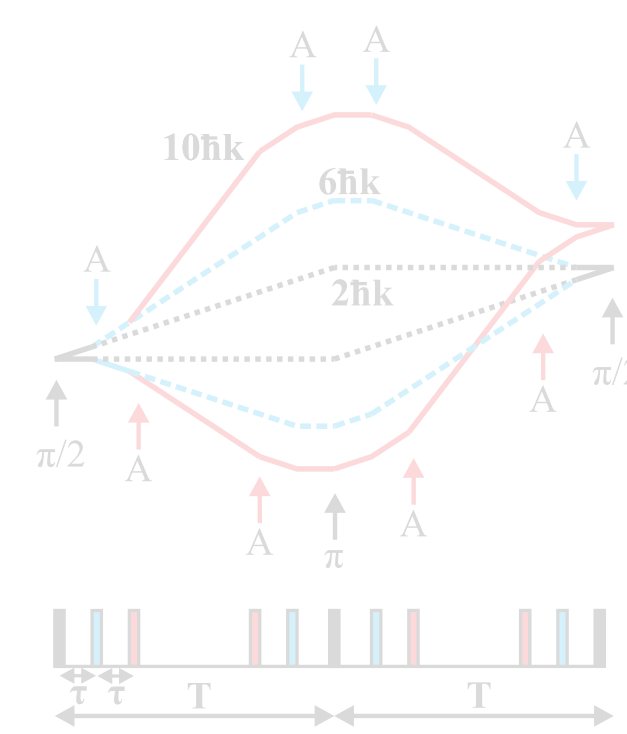


Rudolph et al., Phys. Rev. Lett. **124**, 083604 (2020)

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Wilkason et al., Phys. Rev. Lett., **129**, 183202 (2022)

Two-photon Raman transition



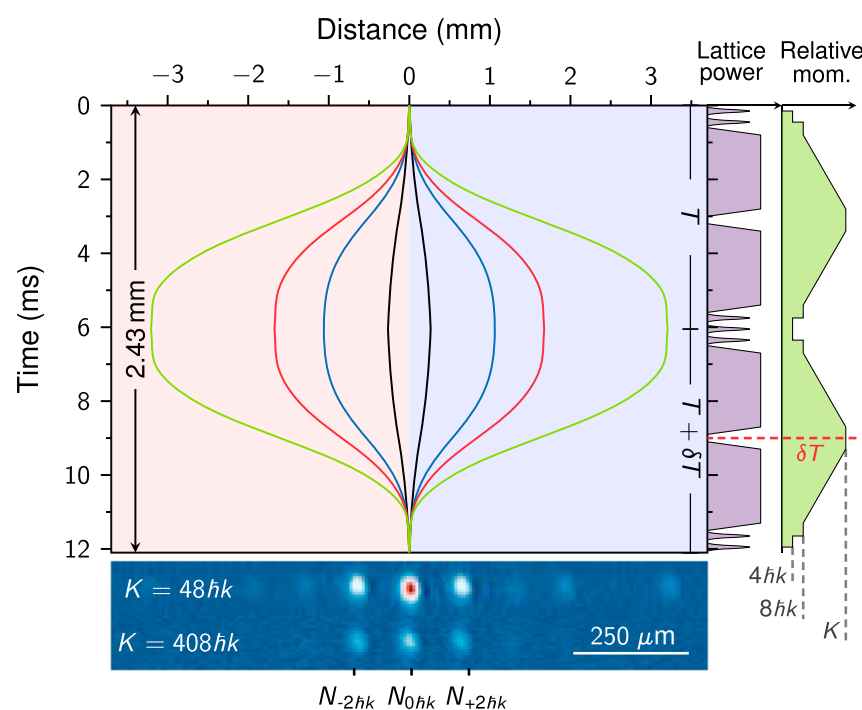
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Optical lattice Bloch-Type acceleration



Gebbe et al., Nat Commun, **12**, 2544 (2021)

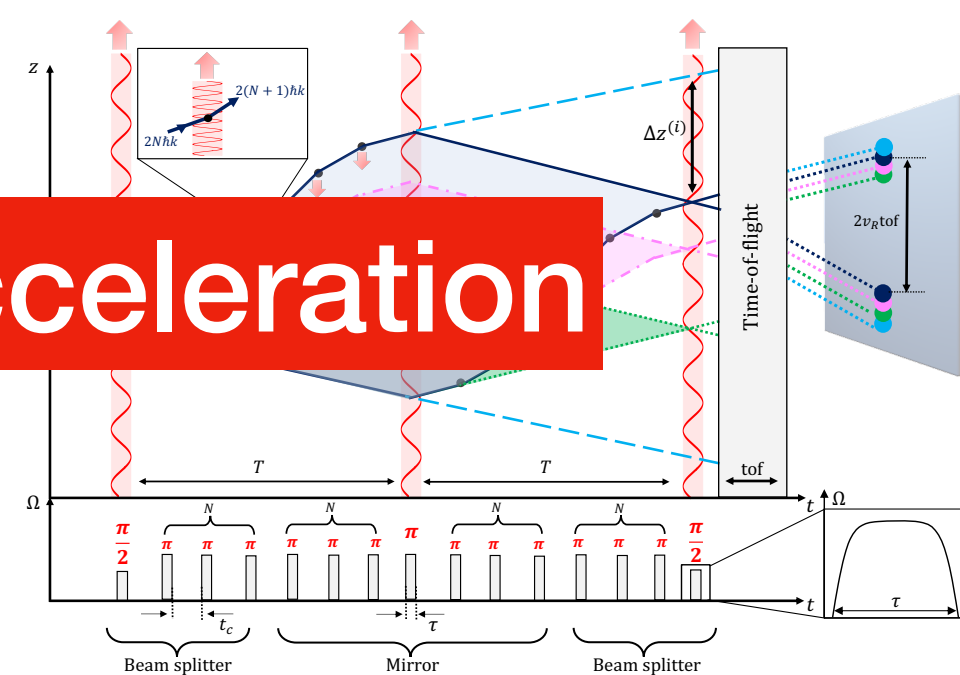
- multi-photon transitions - lattice states
- Ground state
- Rabi frequency $\Omega \sim 100\text{kHz}$
- Demonstrated $100\hbar k$
- Rate transfer: $\sim 6 \mu\text{s}/\hbar k$

Cladé et al., Phys. Rev. Lett. **102**, 240402 (2009)

McDonald et al., Phys. Rev. A **88**, 053620 (2013)

Pagel et al., Phys. Rev. A **102** 053312 (2020)

Optical lattice Sequence of Bragg pulses



Béguin et al., Phys. Rev. Lett., **131**, 143401 (2023)

- multi-photon transitions - lattice states
- Ground state
- Rabi frequency $\Omega \sim 100\text{kHz}$
- Demonstrated $600\hbar k$
- High rate transfer: $\sim 2.5 \mu\text{s}/\hbar k$

Chiu et al., Phys. Rev. Lett., **107**, 130403 (2011)

Plotkin-Swing et al., Phys. Rev. Lett., **121**, 133201 (2018)

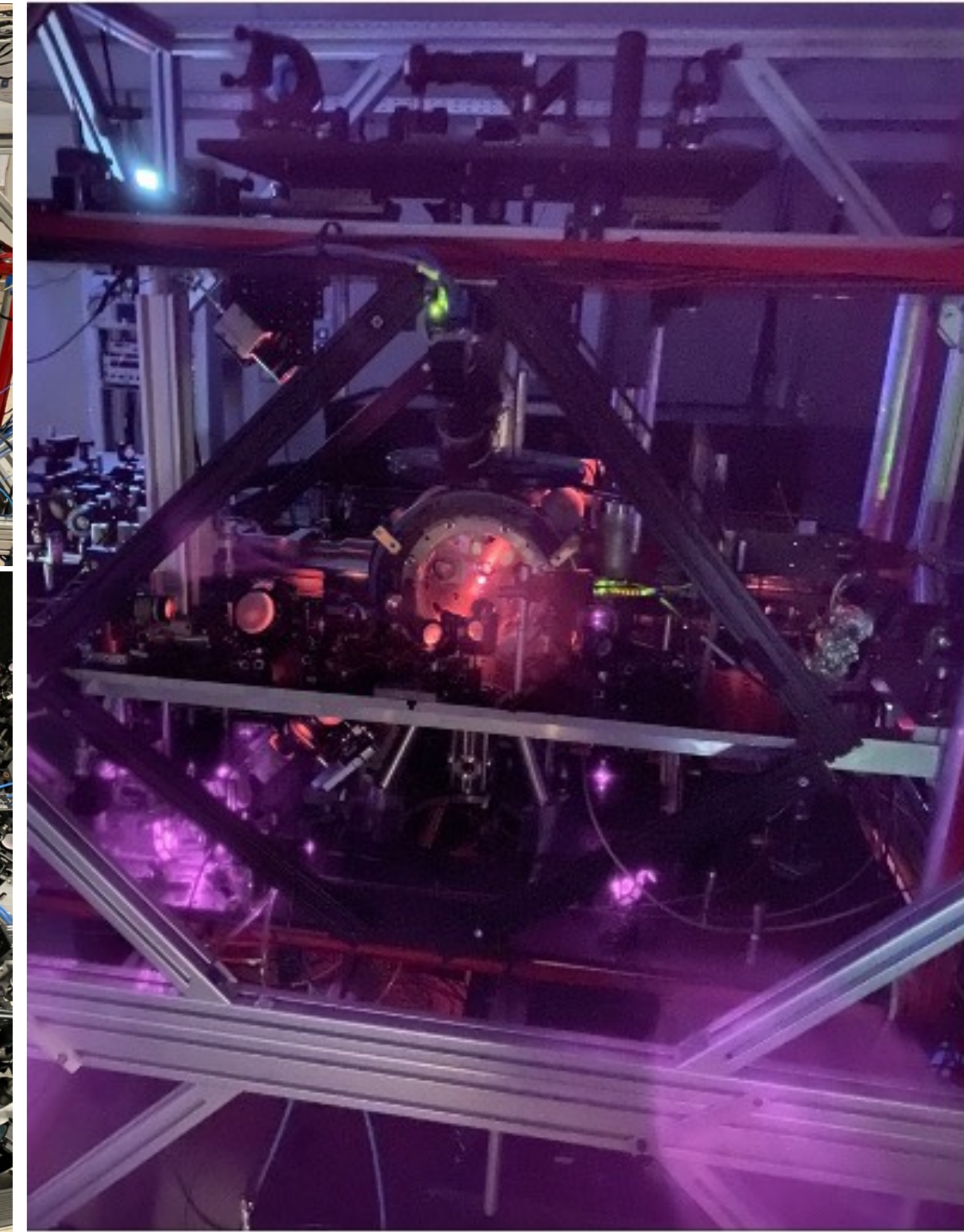
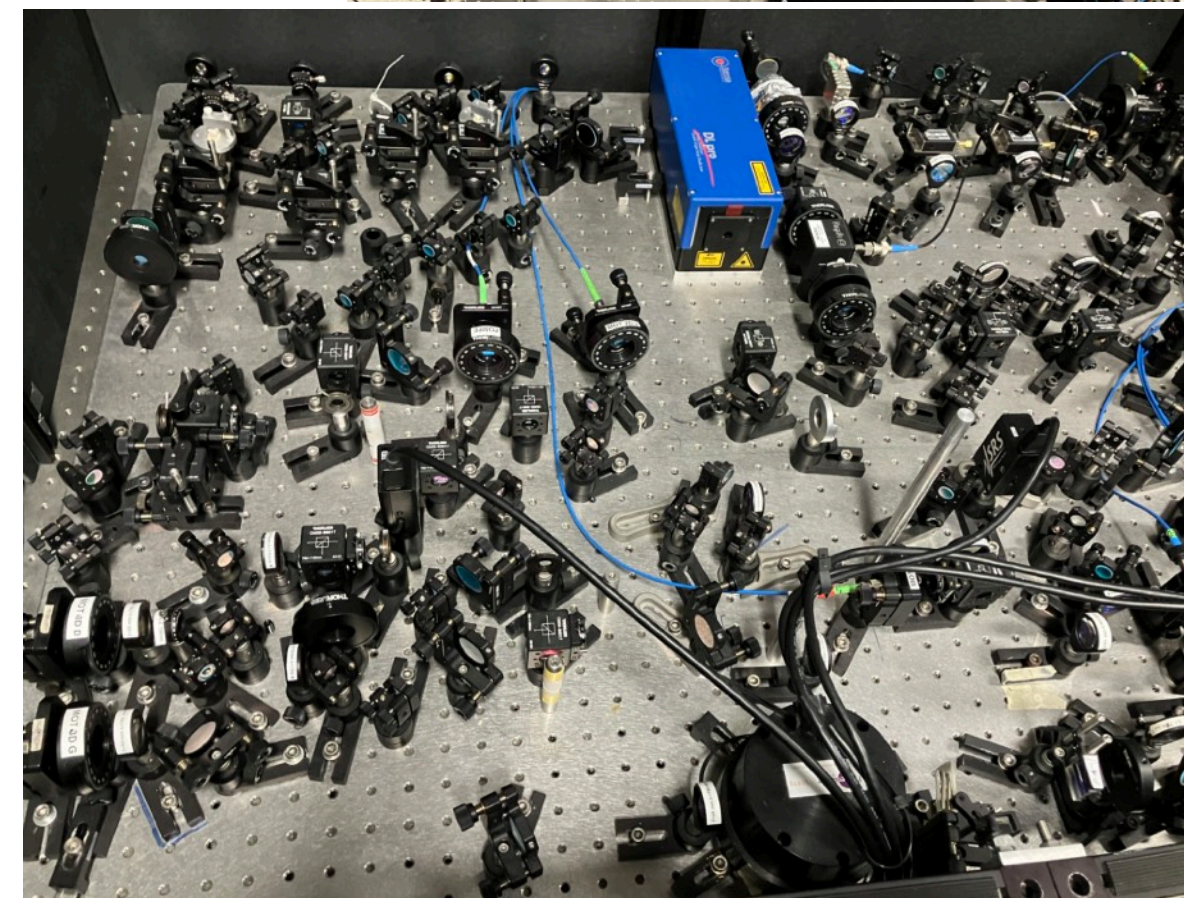
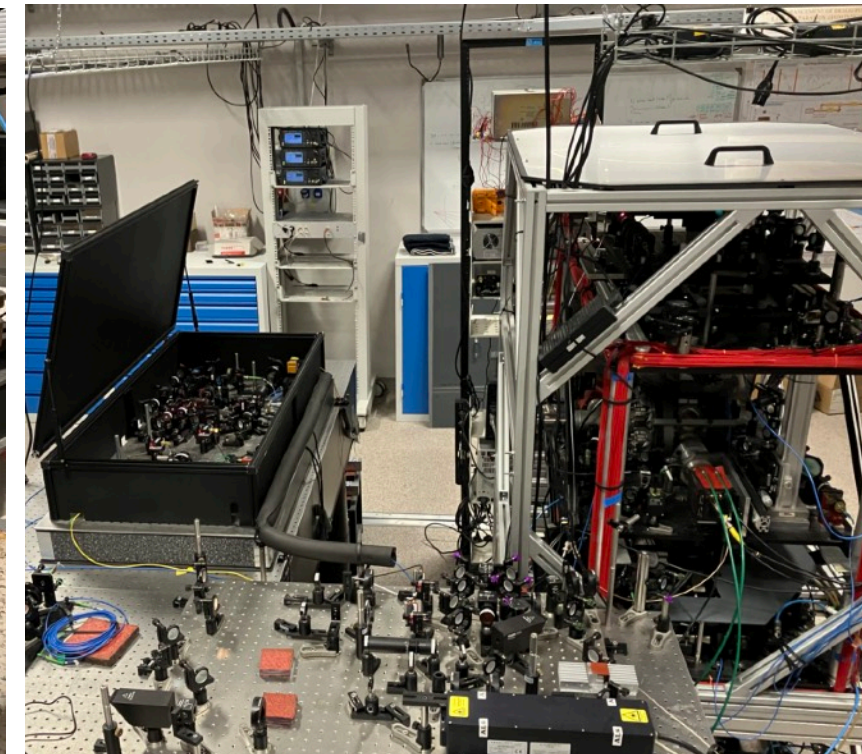
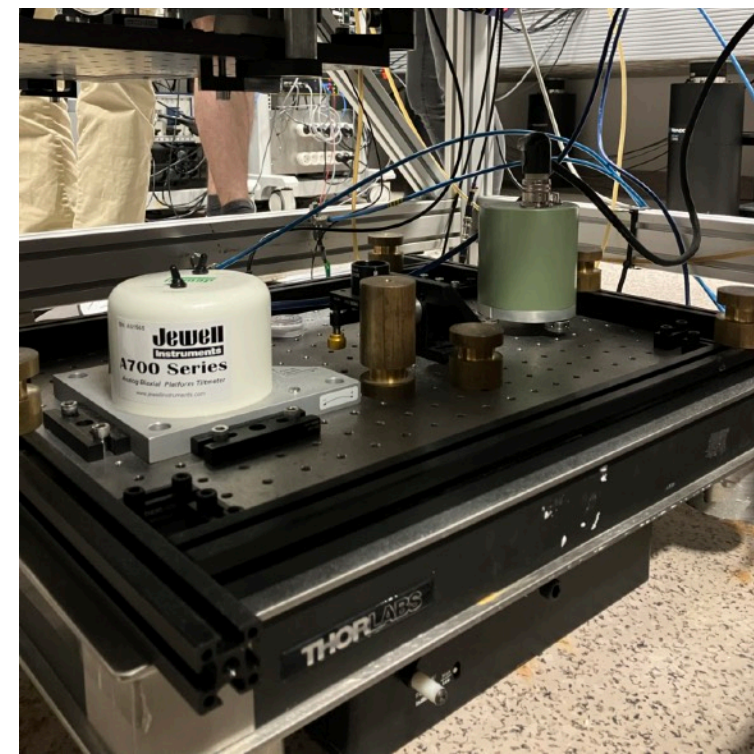
Rodzinka et al. Nat Comm **15**, 10281 (2024)

Optical Lattice Acceleration

Experimental setup

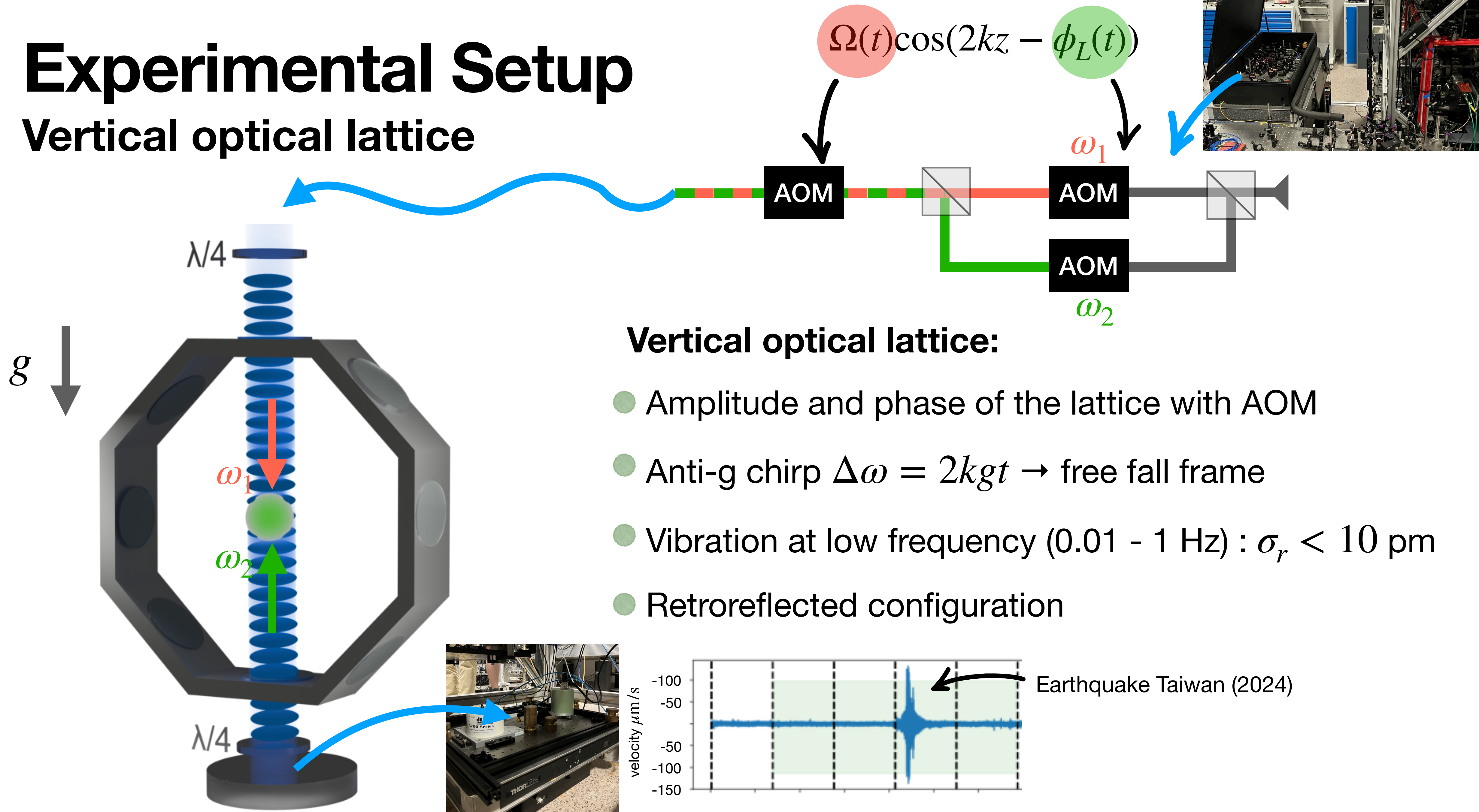
Message for our theorist friends: Easy on Paper, Hard in the Lab

- Critical Technical Challenges
- Designed and Built in the Lab
- UHV chamber 10^{-11} mbar
- > 15 laser beams on the atoms
- Lasers (x6) frequency locks
- Vibration mitigation



Experimental Setup

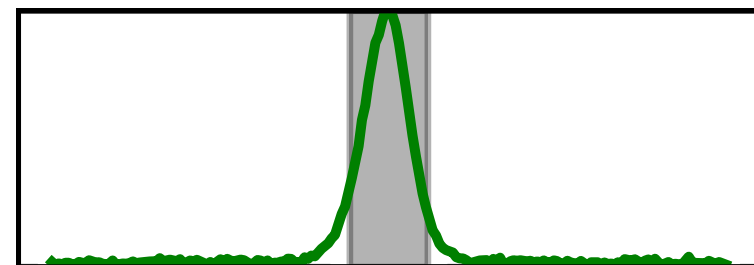
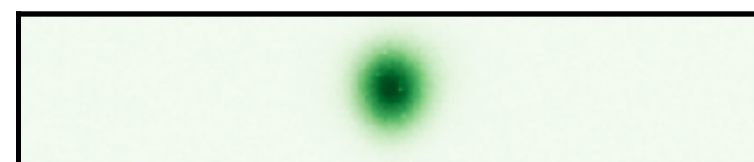
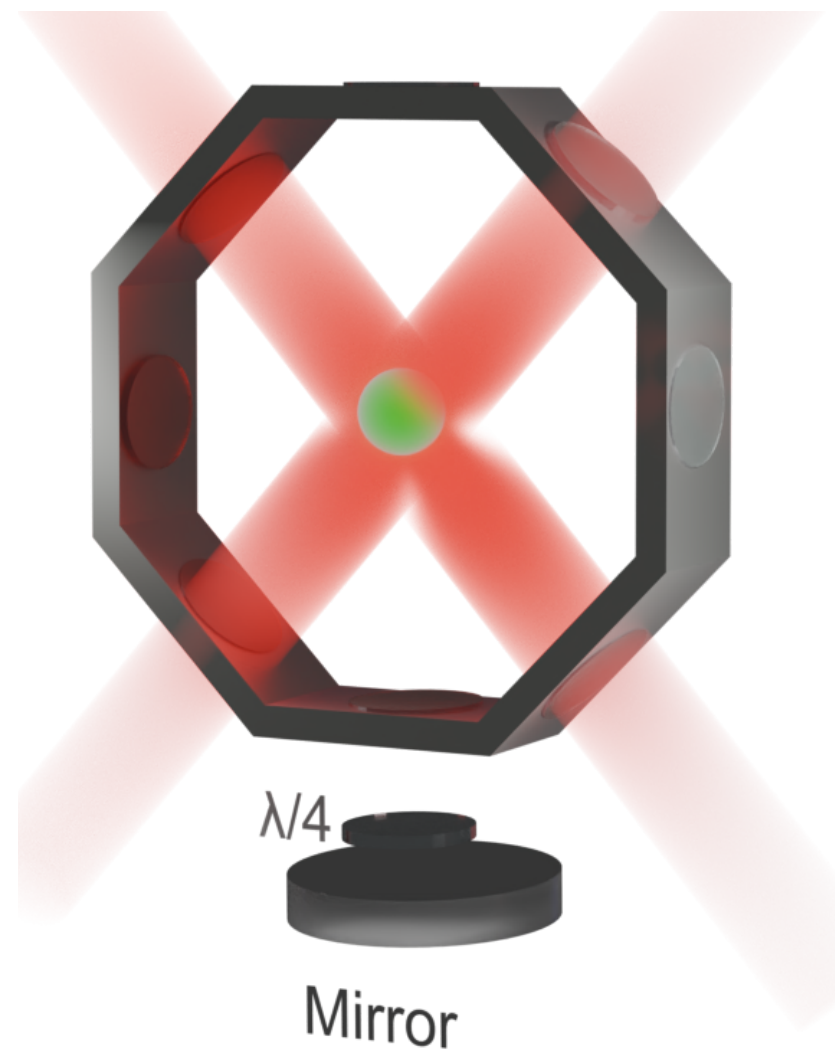
Vertical optical lattice



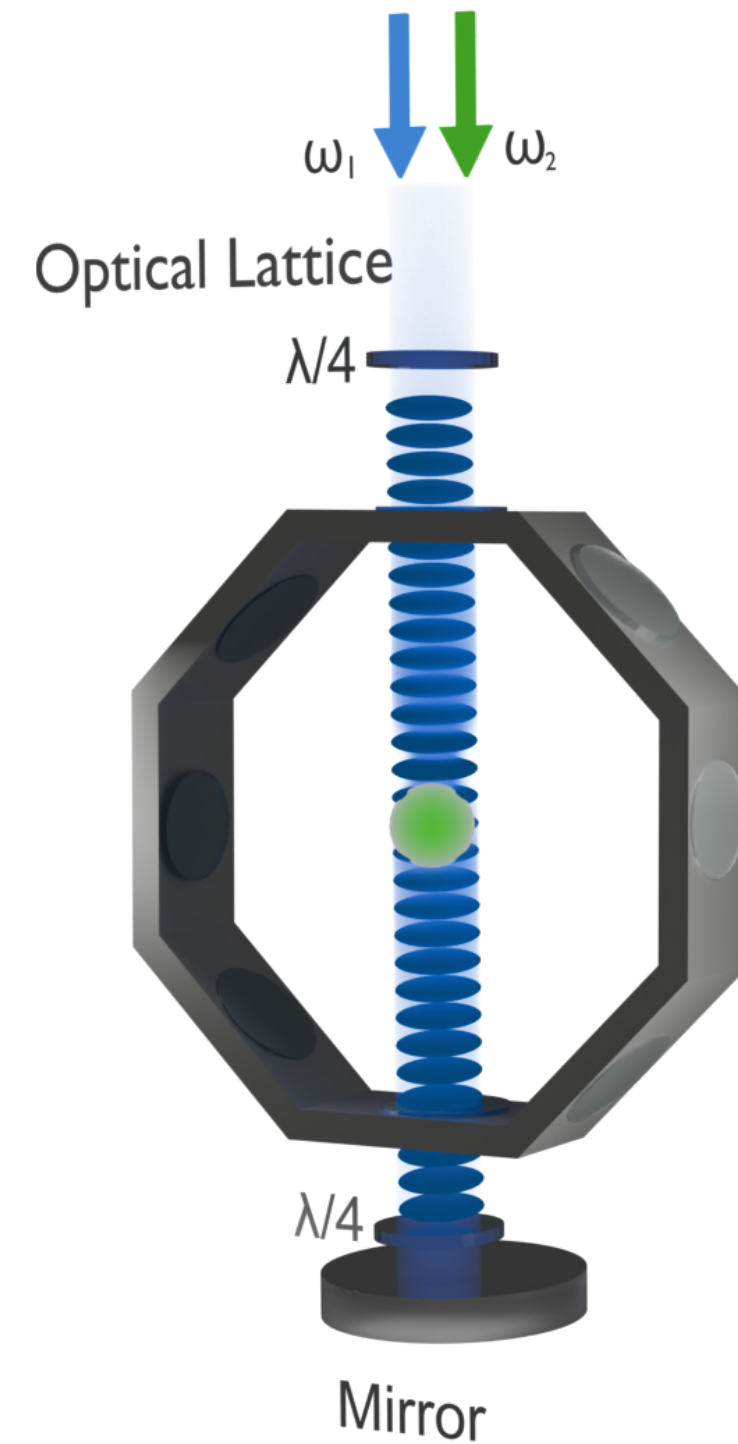
Experimental Sequence

BEC Source

3×10^4 atoms, 30 nK

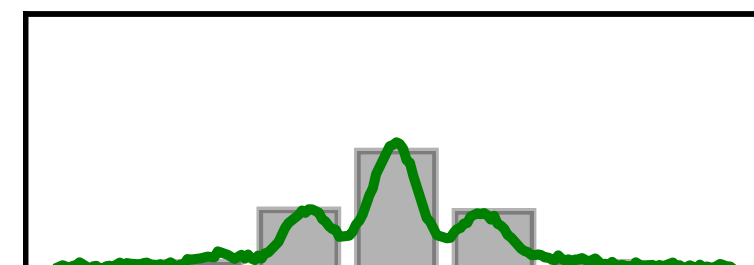


Optical Lattice pulses



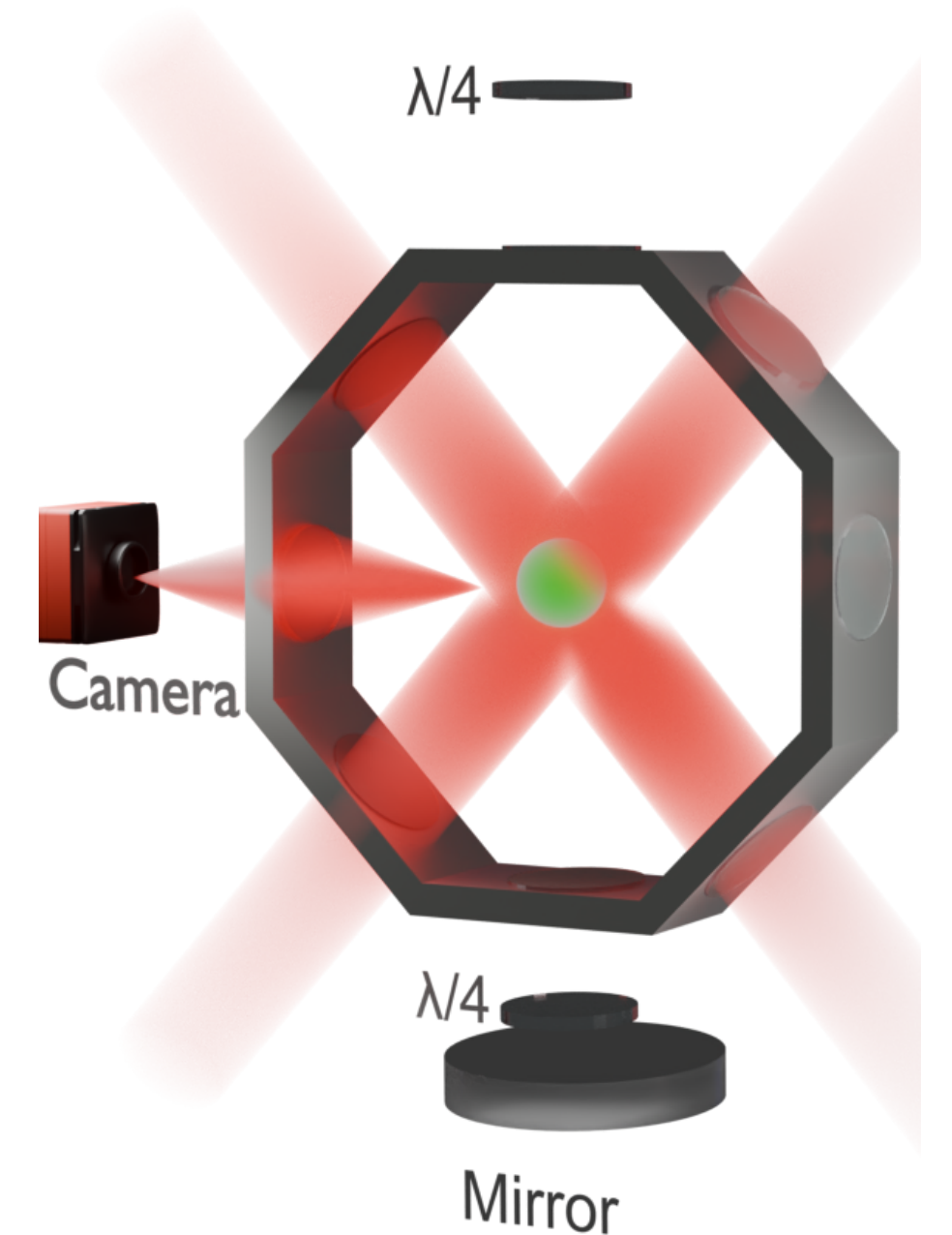
Momentum space

$$|\Psi\rangle = \sum c_n |p + 2n\hbar k\rangle$$



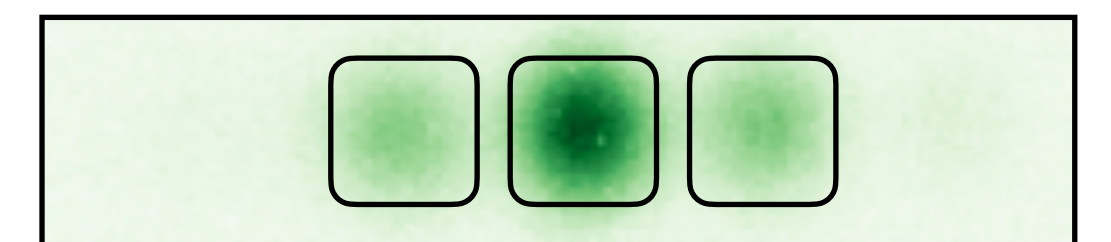
Time of Flight

Fluorescence detection



Position space image

-1 1 1

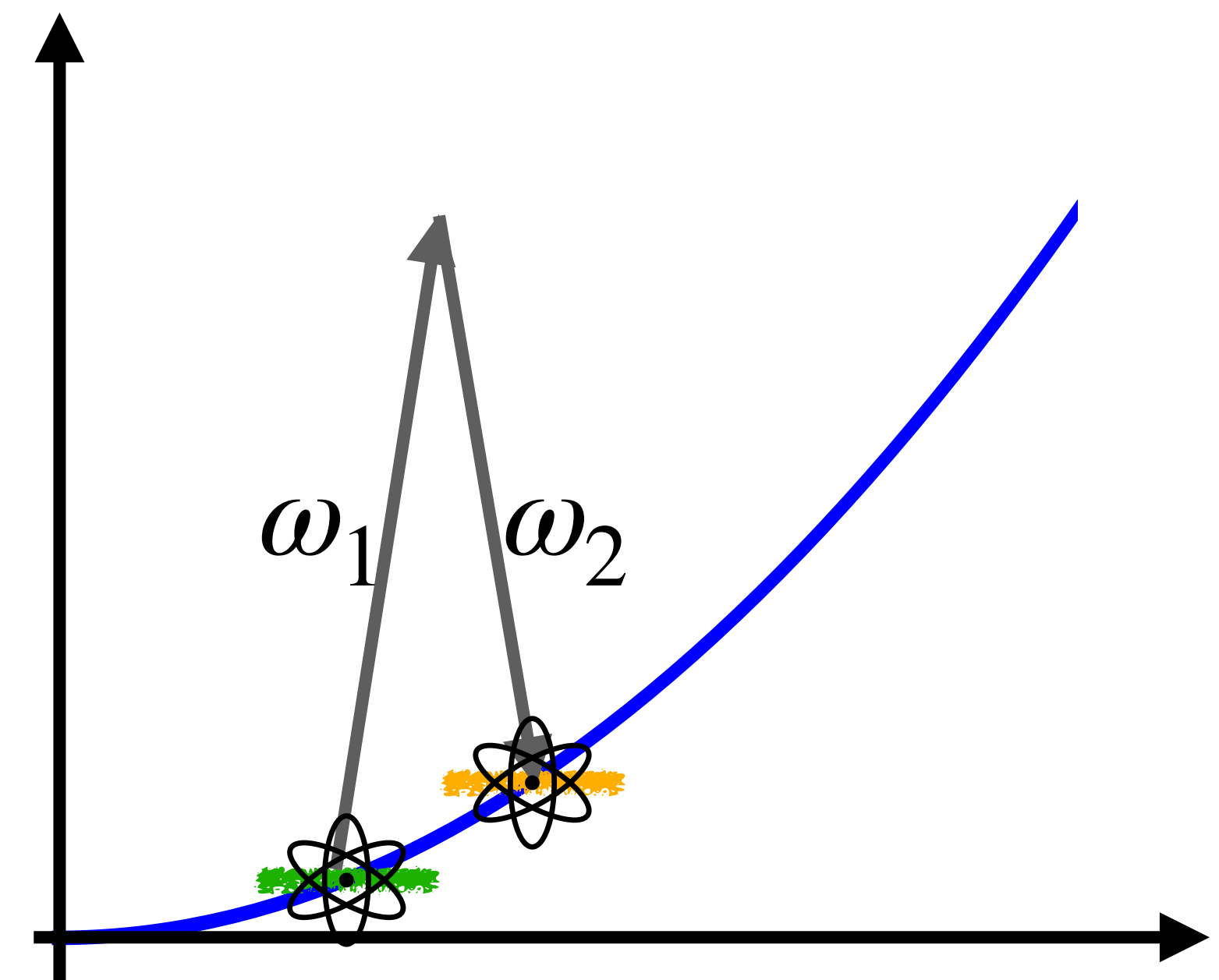
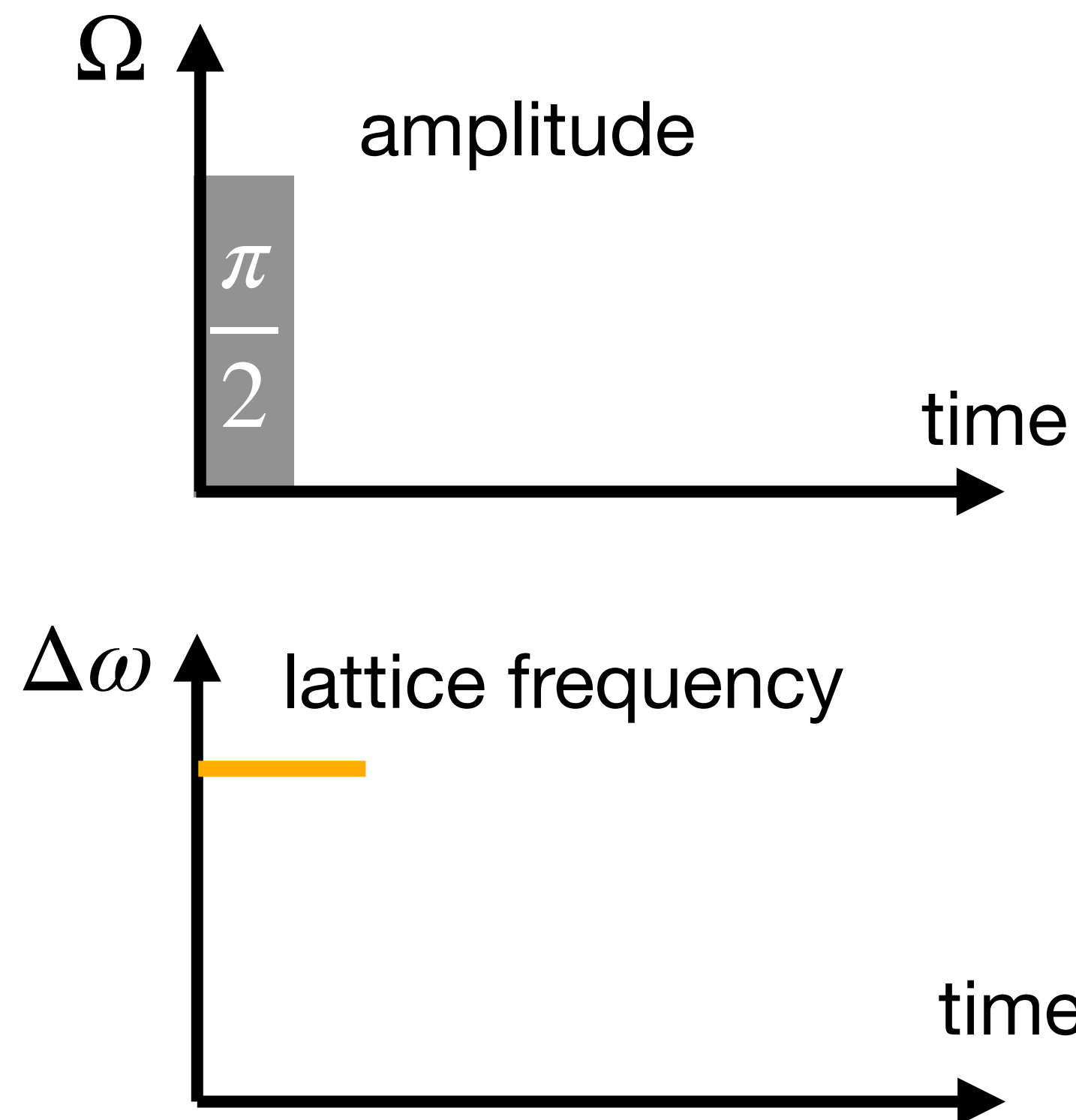
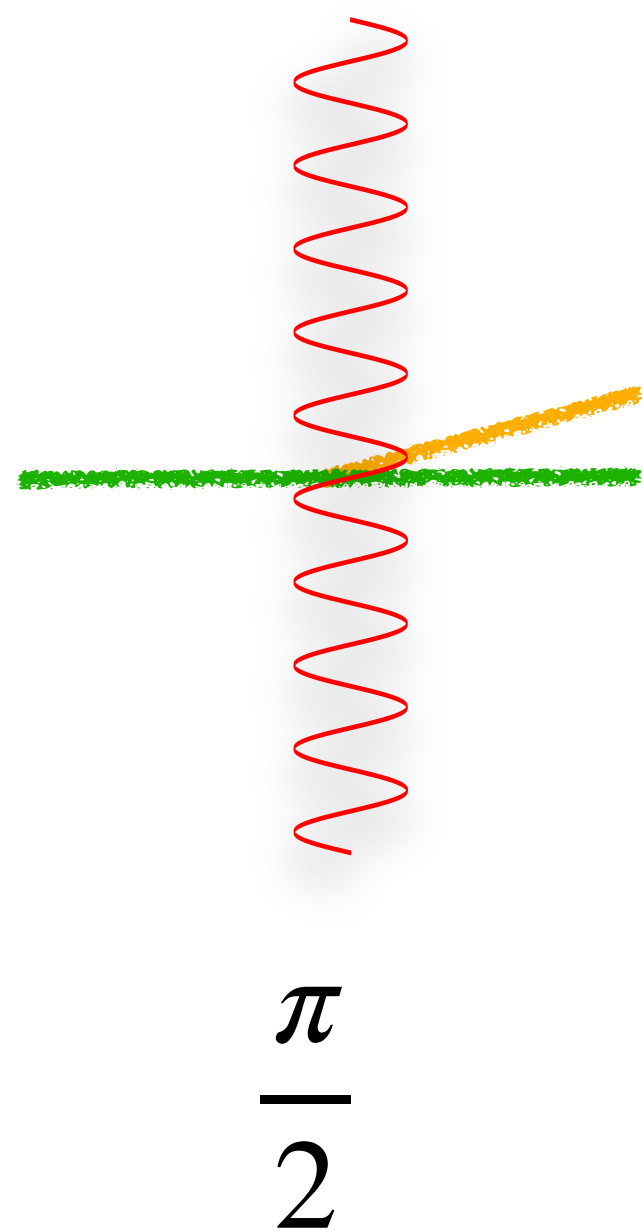


Large Momentum Transfer: 1st idea

Sequence of independant π -pulses

Beam splitter made with multiple π -pulses.

Constant detuning for each pulse



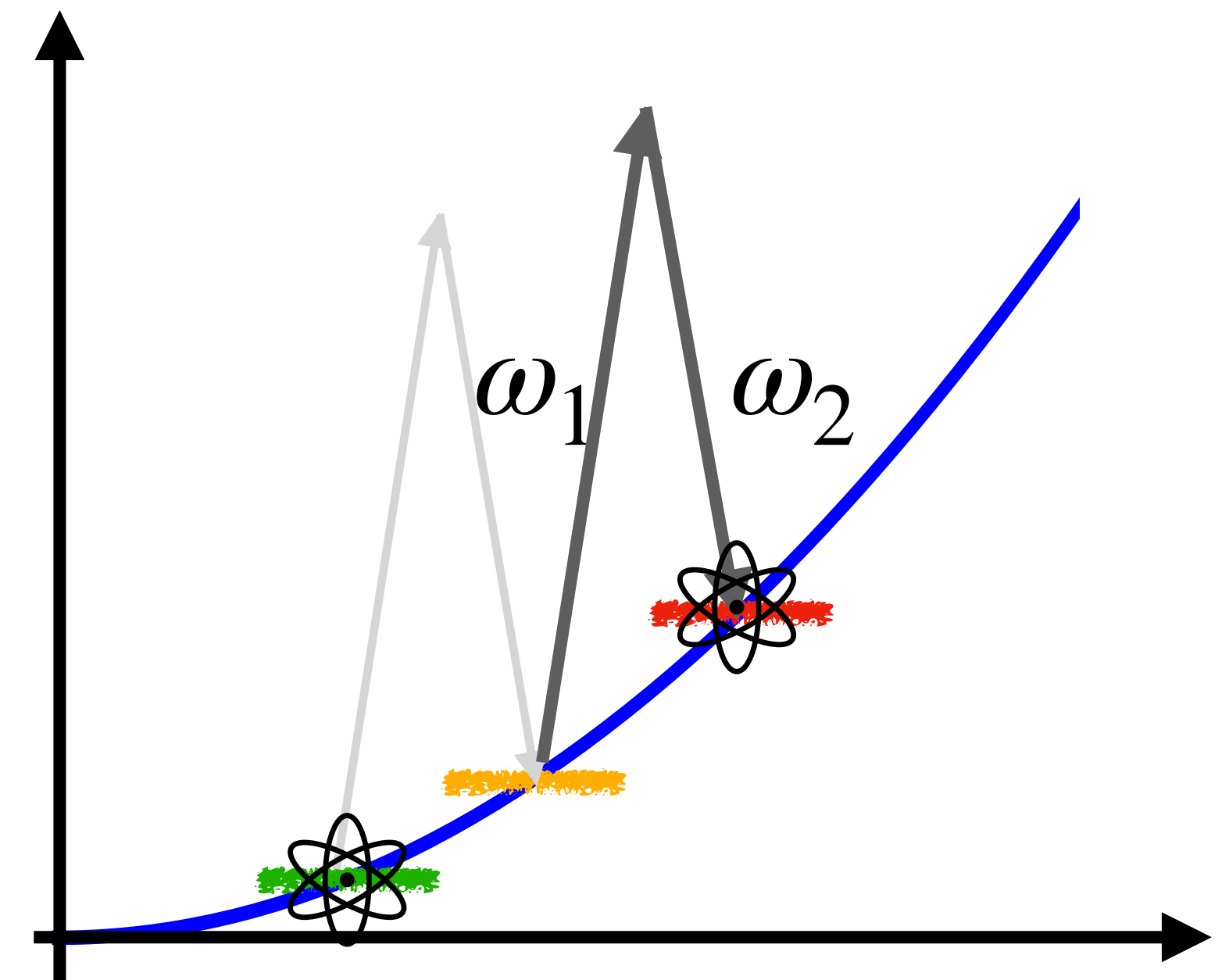
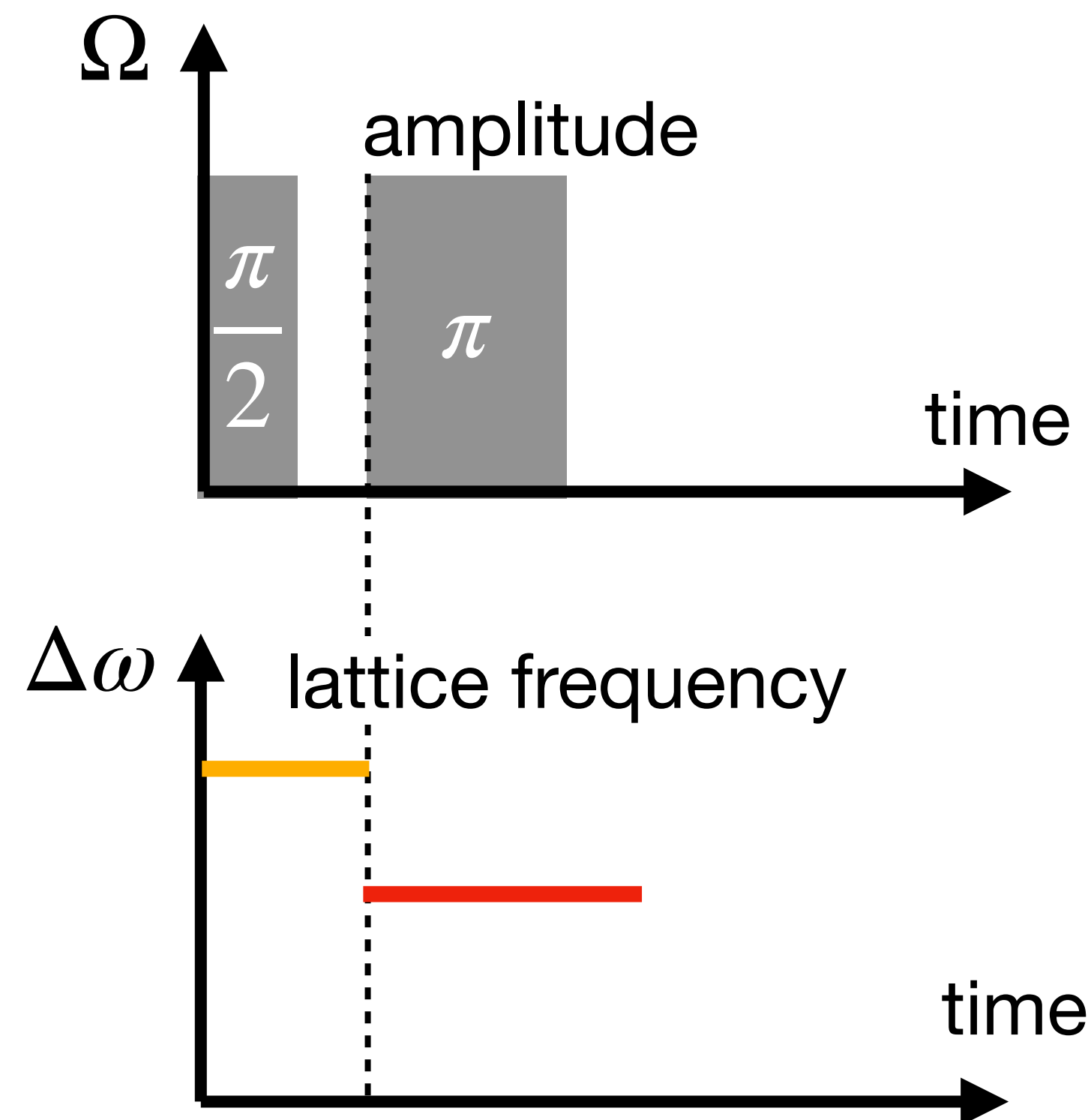
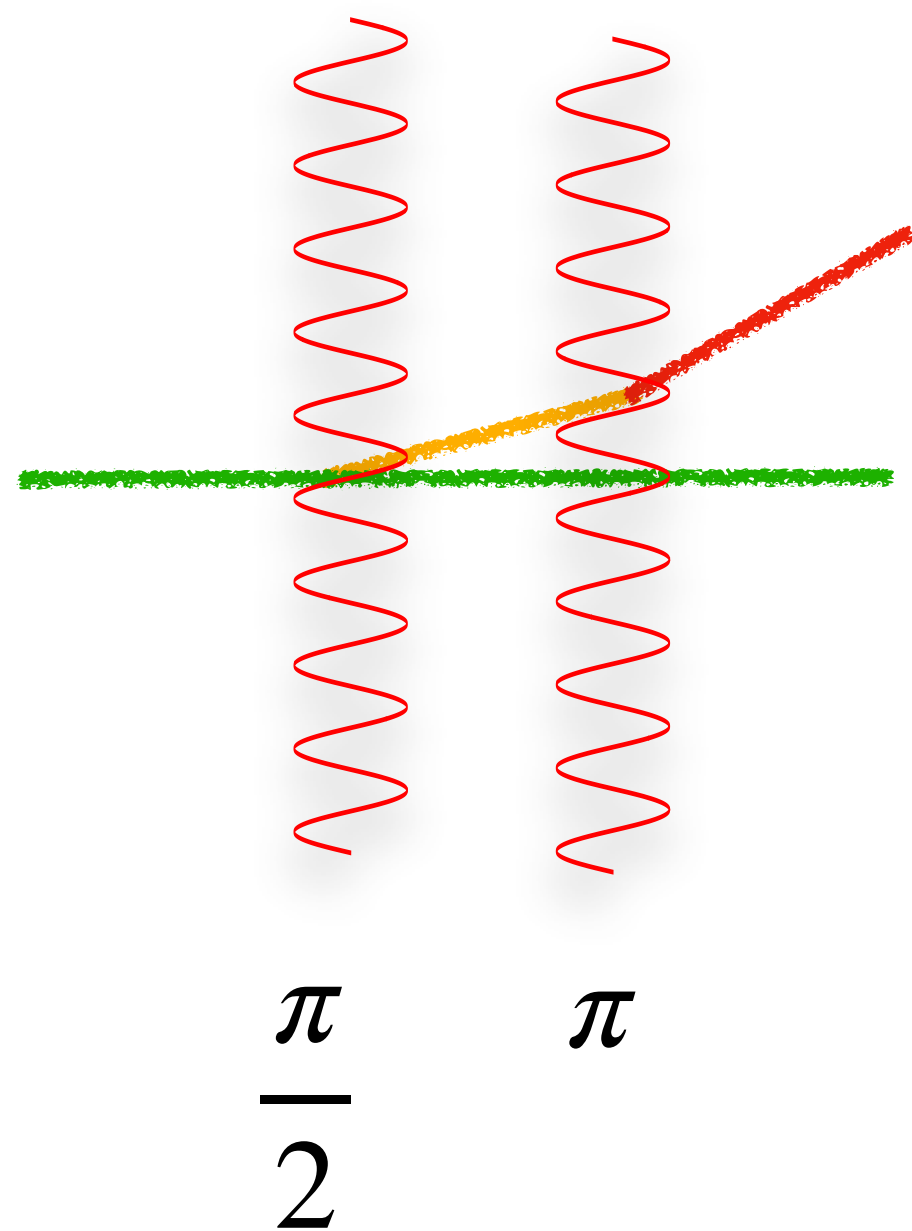
Long pulses to keep $\hbar\Omega \ll \Delta E$

Large Momentum Transfer: 1st idea

1st idea: Sequence of independant π -pulses

Beam splitter made with multiple π -pulses.

Constant detuning for each pulse



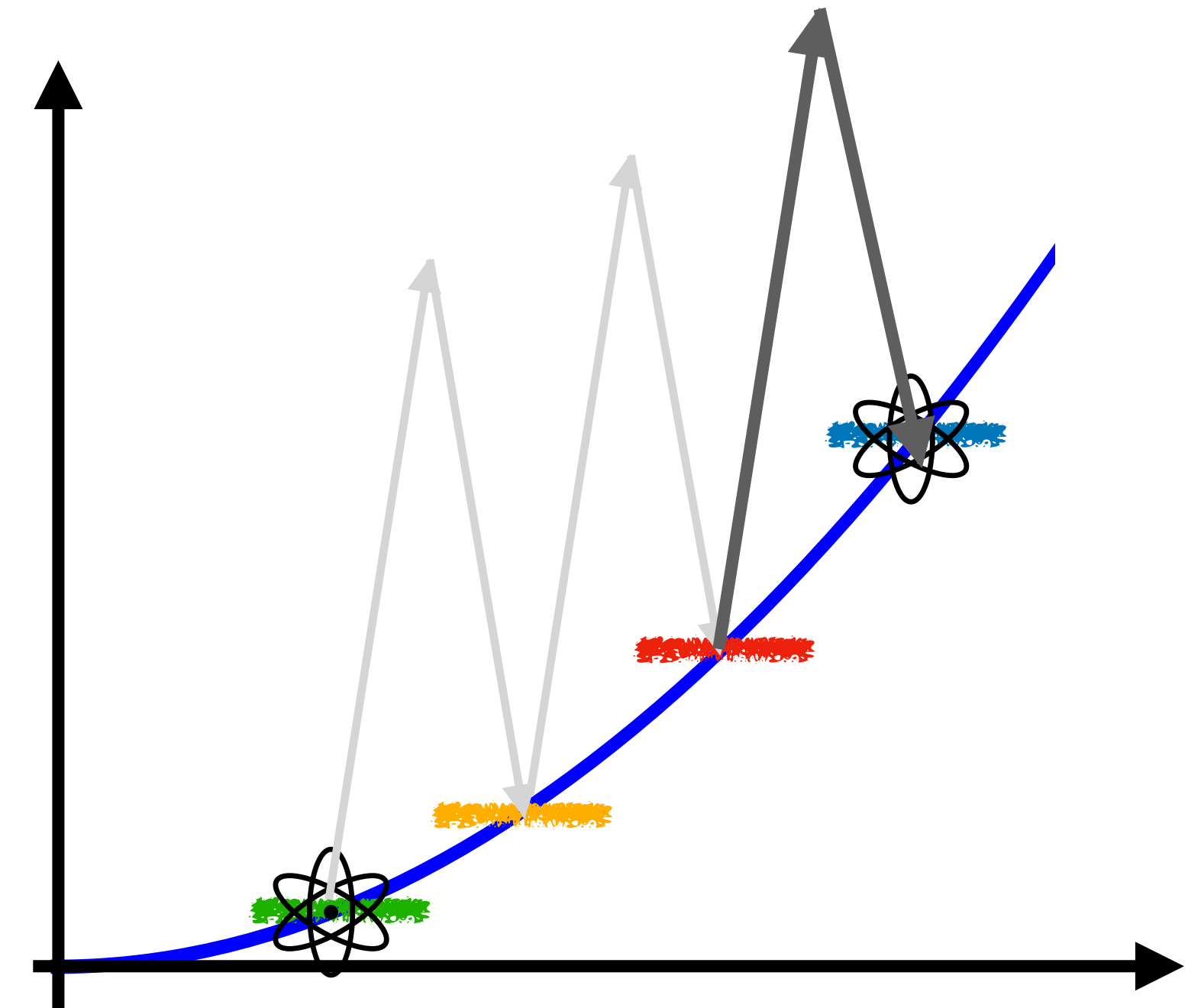
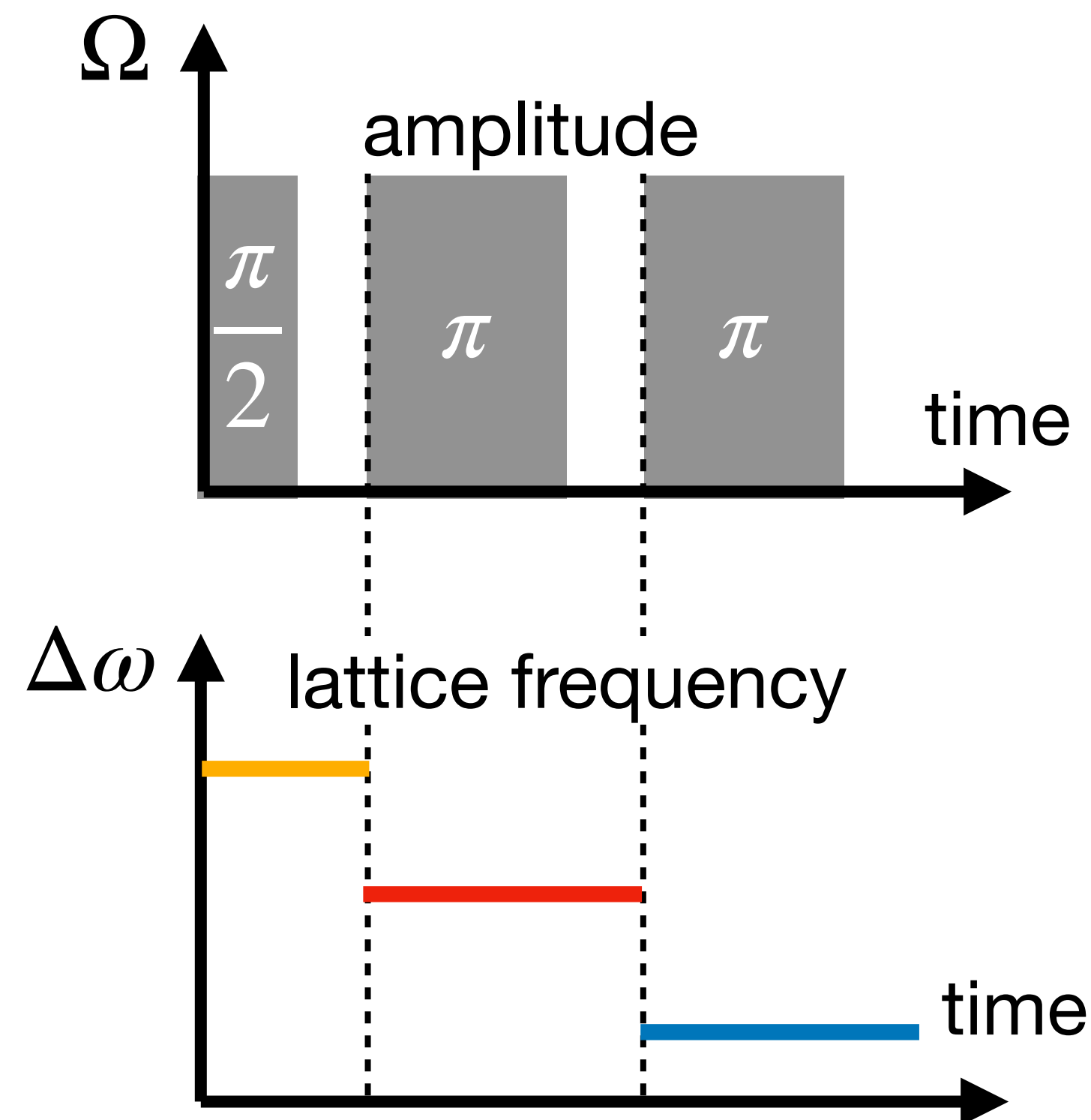
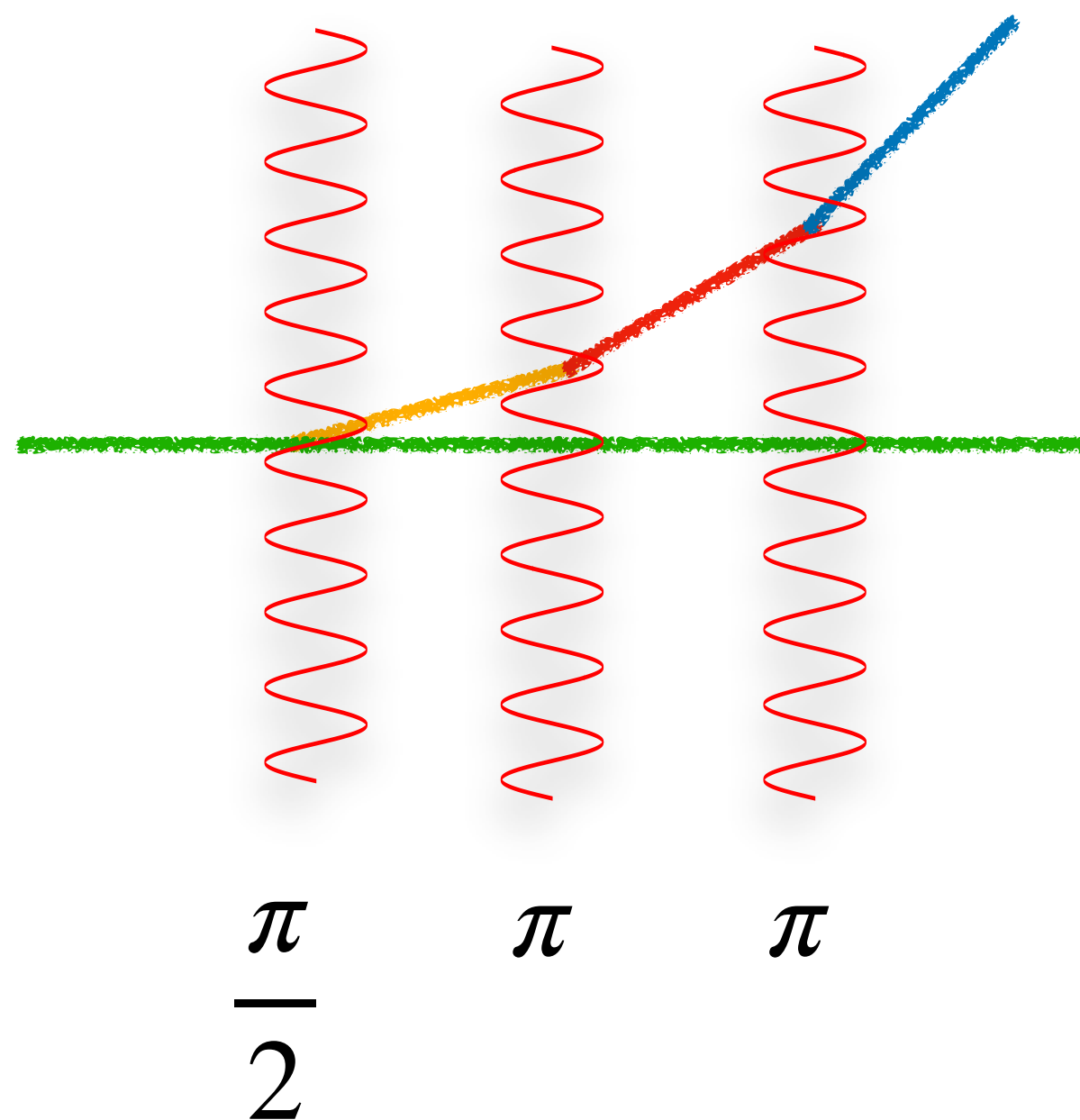
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Large Momentum Transfer: 1st idea

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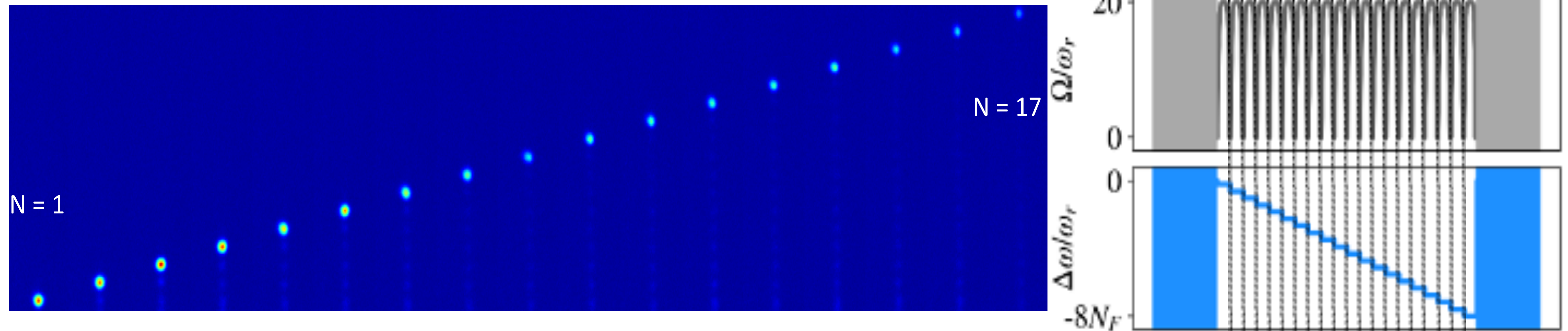
Long pulses to keep $\hbar\Omega \ll \Delta E$

Large Momentum Transfer: 1st idea

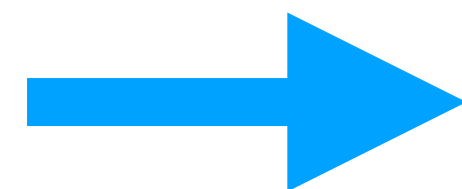
1st idea: Sequence of independant π -pulses

Beam splitter made with multiple π -pulses.

Constant detuning for each pulse



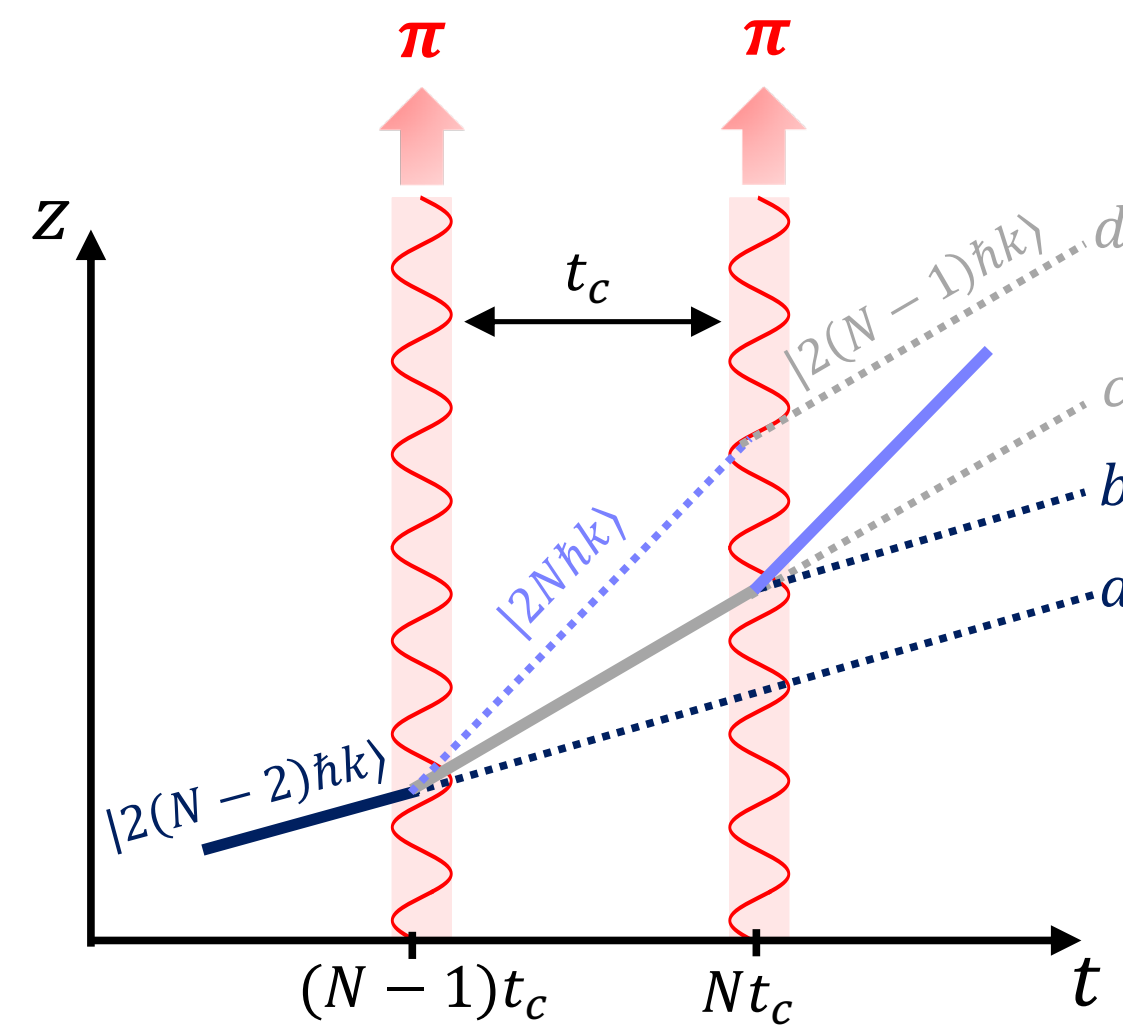
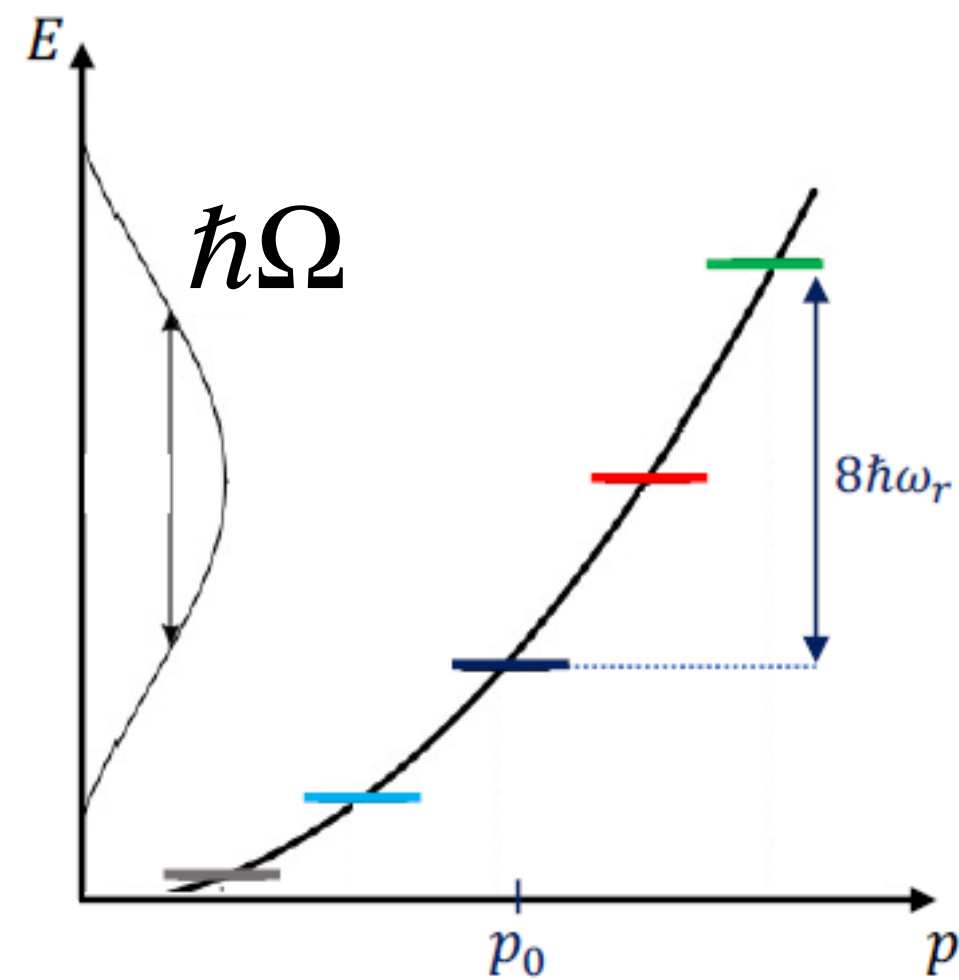
Long pulses to keep
 $\hbar\Omega \ll \Delta E$



- ▶ Velocity selection: requires ultra-cold source (pK)
- ▶ Low acceleration (typ. $1\text{ms}/\hbar k$)

Large Momentum Transfer: 2nd idea

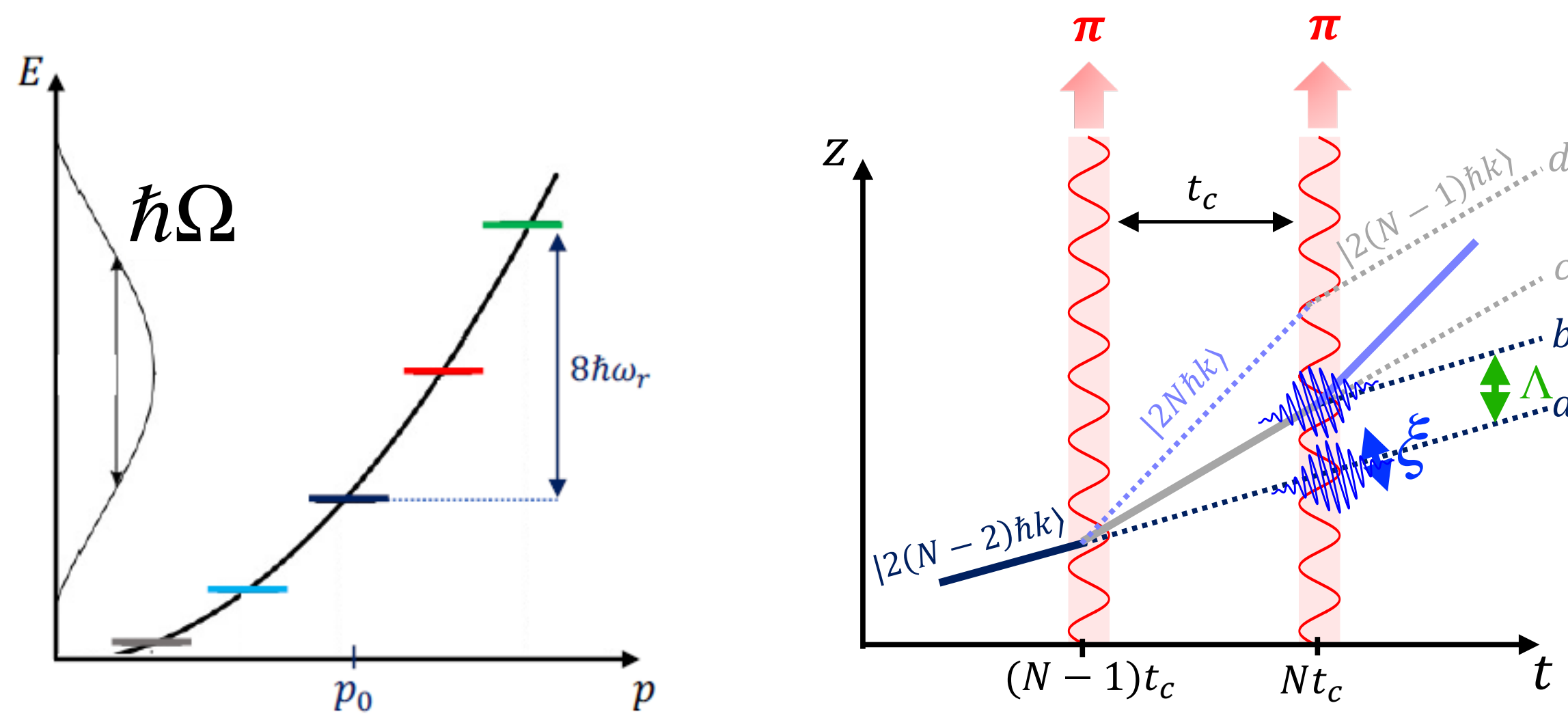
Short pulses: How can we manage coupling to unwanted states ?



Short pulses
Non-adiabatic losses = coherent losses

Large Momentum Transfer: 2nd idea

Short pulses: How can we manage coupling to unwanted states ?



Short pulses
Non-adiabatic losses = coherent losses

+

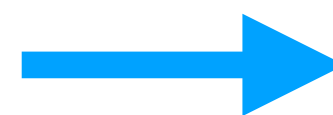
Fast train pulses

Small spatial separation $\Lambda = 2v_R t_c \ll \xi = \frac{\hbar}{m\sigma_v}$

Toy model [Béguin et al. Phys. Rev. Lett., 131, 143401, \(2023\)](#)

Losses interfere destructively:

$$P_{|N-2\rangle} \approx 2\epsilon^2 [1 + \cos(\pi + 4\omega_R t_c)]$$

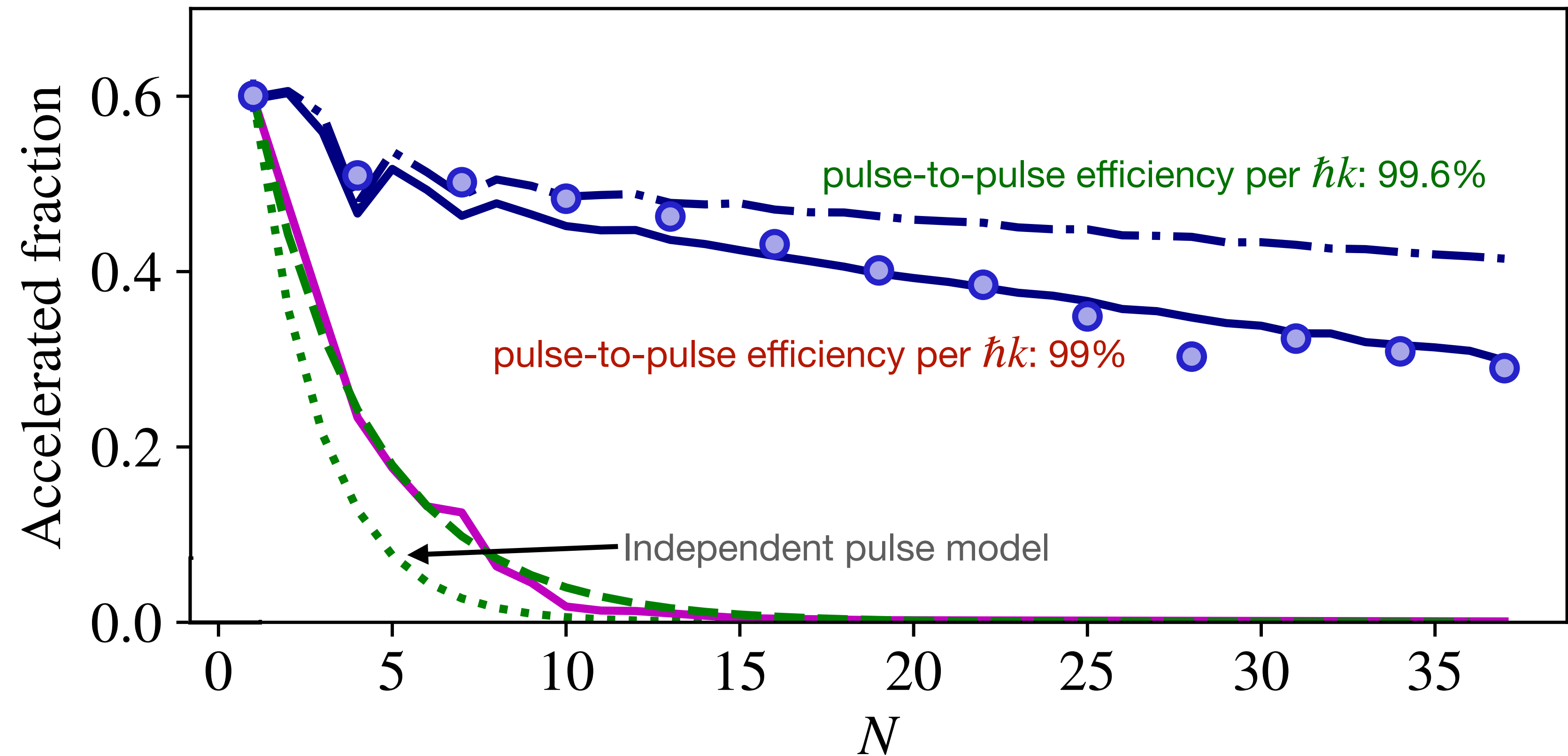
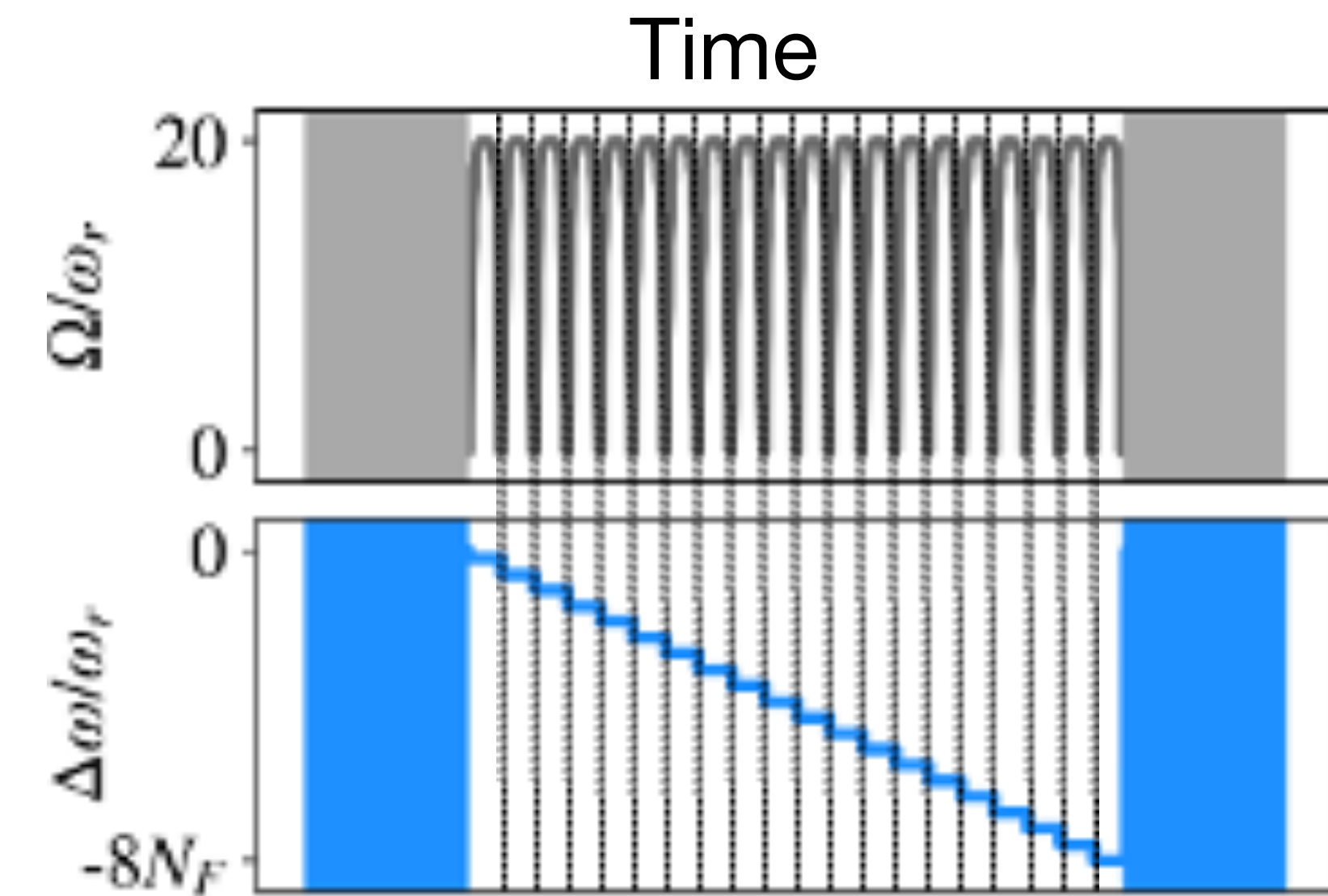


Coherent Enhancement of Bragg Sequences

Large Momentum Transfer: 2nd idea

Coherent Enhancement of Bragg Sequence: results

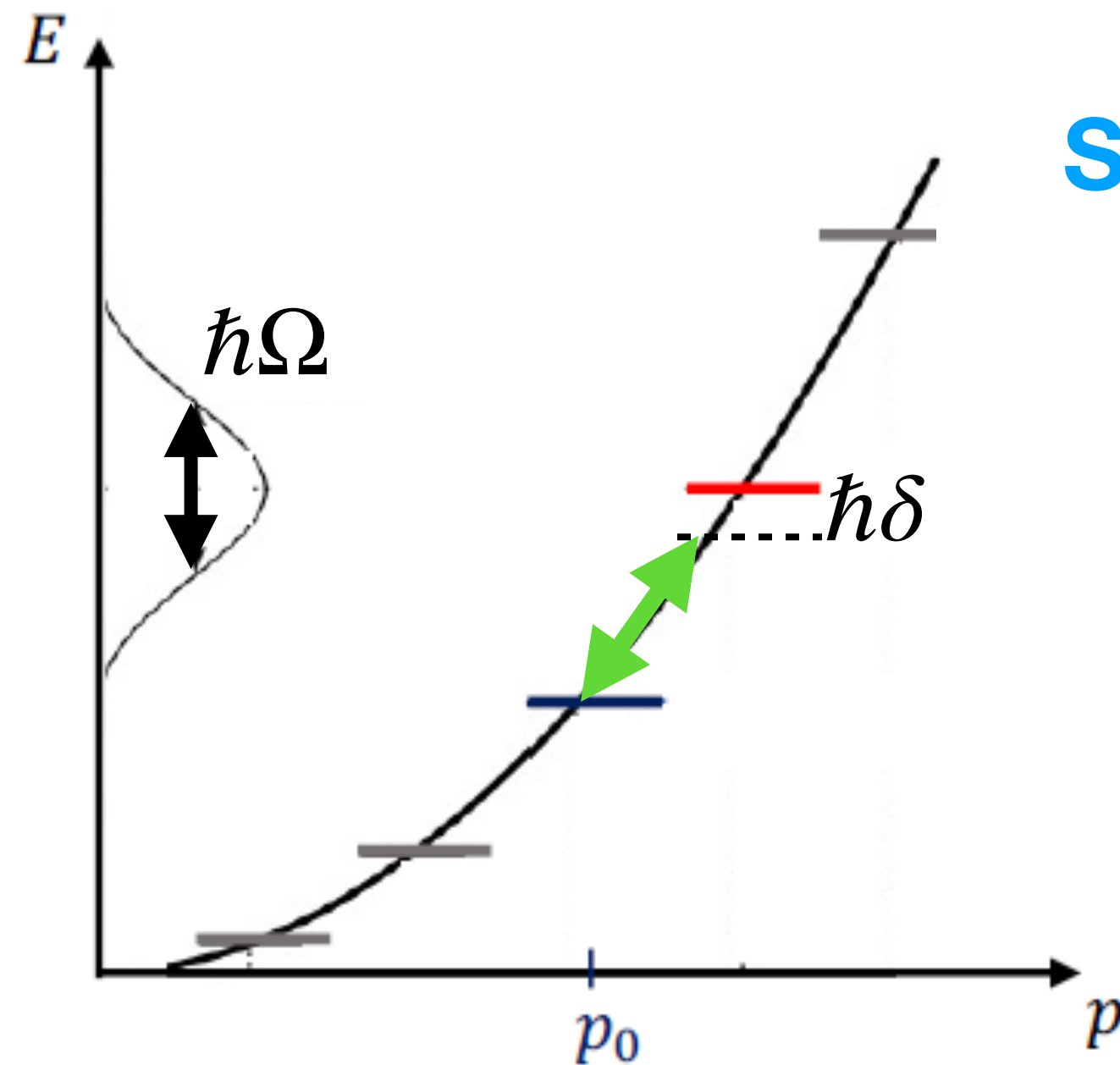
Béguin et al. Phys. Rev. Lett., **131**, 143401, (2023)



Higher efficiency & Faster transfer ?

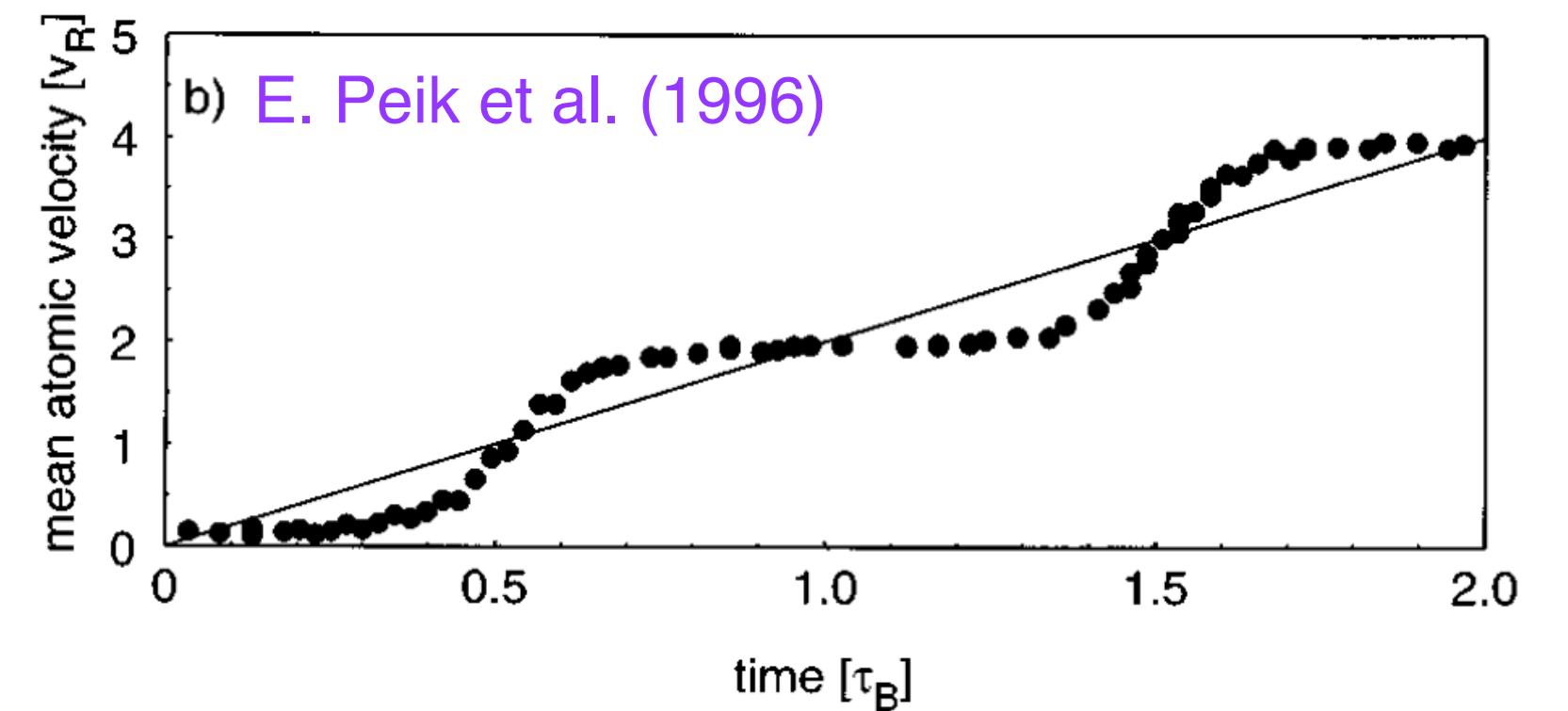
Optical lattice accelerations

Bloch acceleration = linear change of the lattice frequency



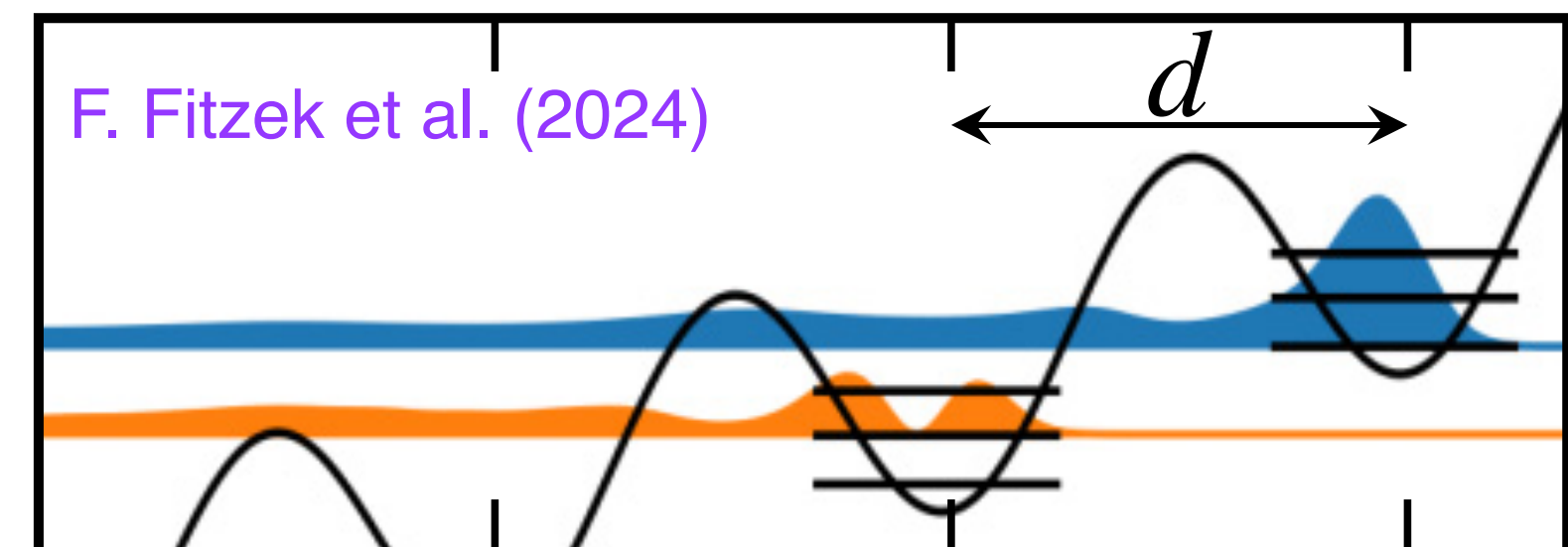
Shallow lattice: Adiabatic Rapid Passage between $|p + 2n\hbar k\rangle$

$$P_{n+1} = \frac{1}{2} + \frac{\delta}{\sqrt{\delta^2 + \Omega^2}}$$



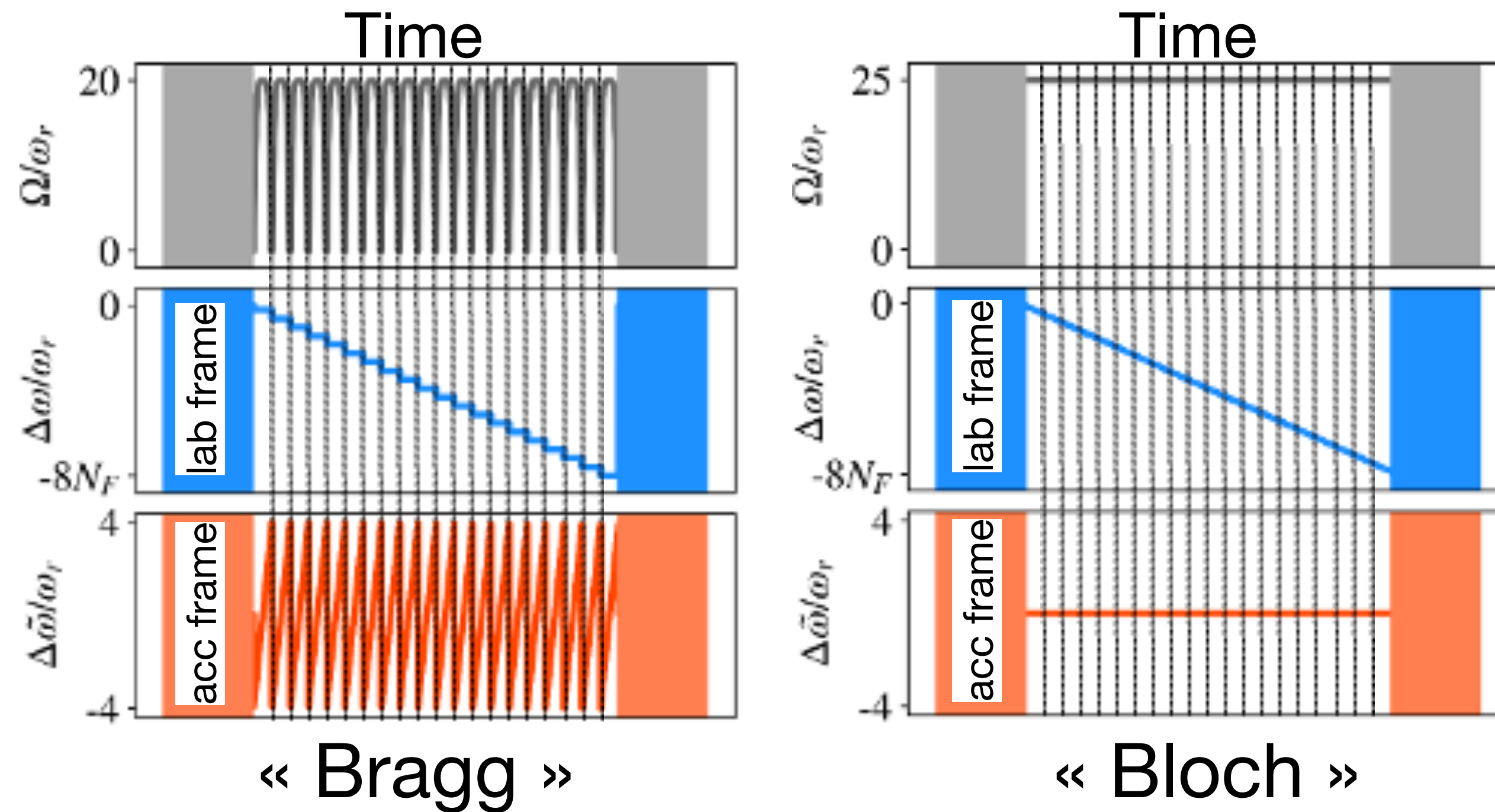
Deep Lattice: more than 2 levels ...

→ Wannier states



Optical lattice accelerations

Can we have unify the Bloch and Bragg type approaches ?



Lattice shape is periodic in the accelerated frame



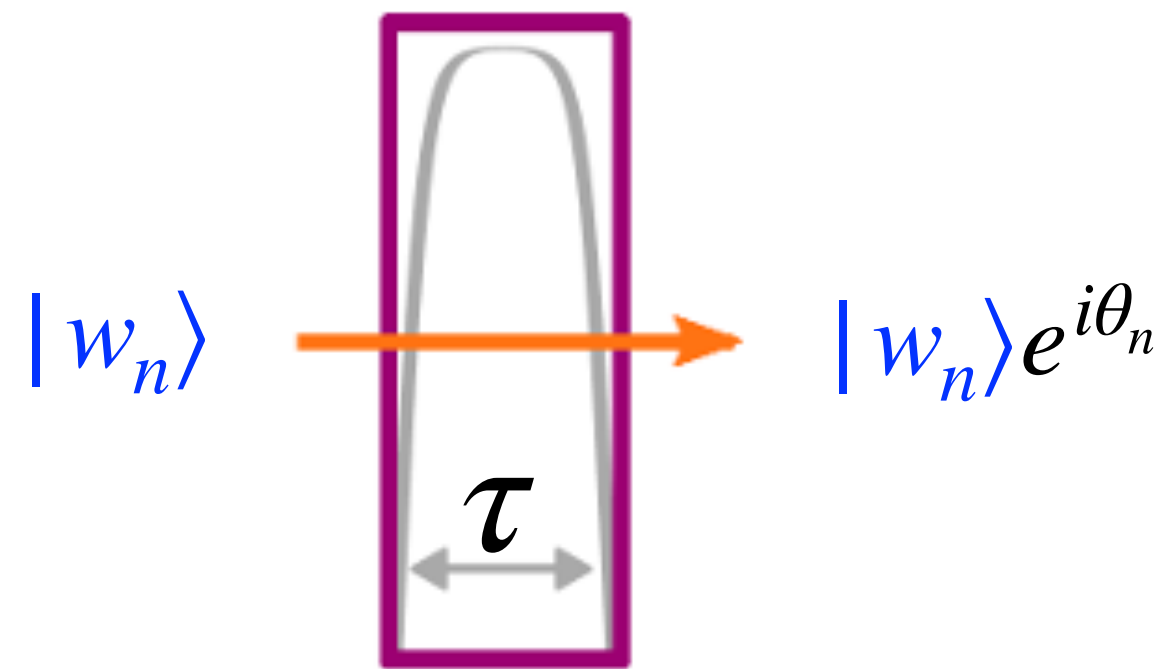
Periodic hamiltonian $H(t_0) = H(t_0 + \tau)$

$$\tilde{H}(t) = \frac{1}{2m} \left(\hat{p} - \frac{2m}{2k} + 2\beta \right) + \hbar \Omega \cos(2k\hat{z}) + 2\dot{\beta}/\hbar$$

Is there a state that follows this periodicity and is therefore stationary in an accelerated frame?

Large Momentum Transfer: à la Floquet

Stabilization in the accelerated frame

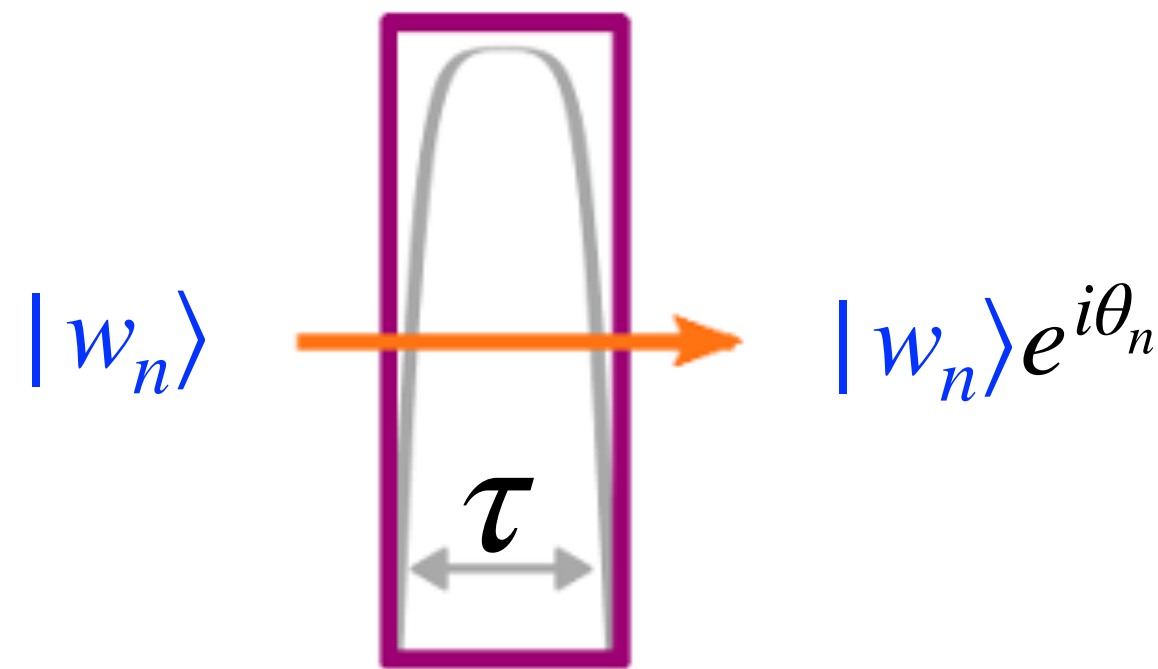


$$i\hbar \frac{d|\Psi\rangle}{dt} = \hat{H}(t) |\Psi\rangle \rightarrow \text{Propagator } \hat{U}(\tau, t_0)$$

Spectral decomposition: $\hat{U}(\tau, t_0) |w_n\rangle = e^{-i\epsilon_n(\tau-t_0)} \boxed{|w_n\rangle}$ Floquet state

Large Momentum Transfer: à la Floquet

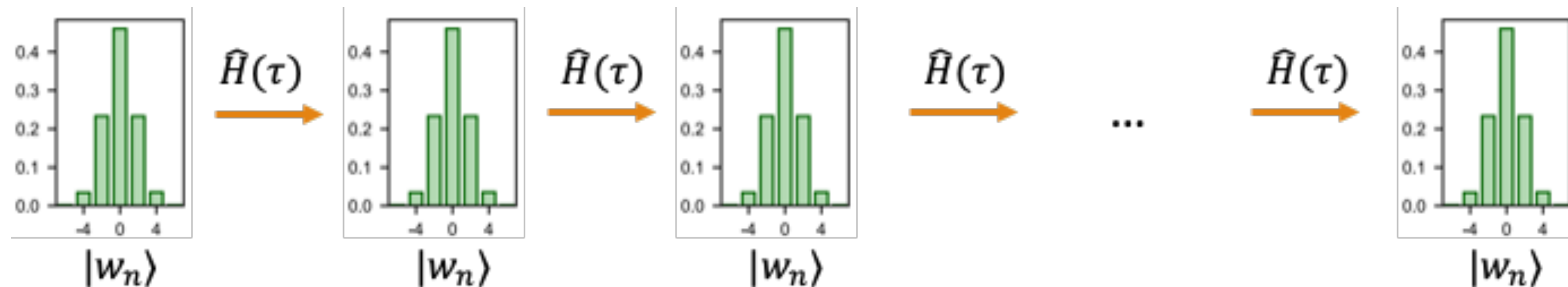
Stabilization in the accelerated frame



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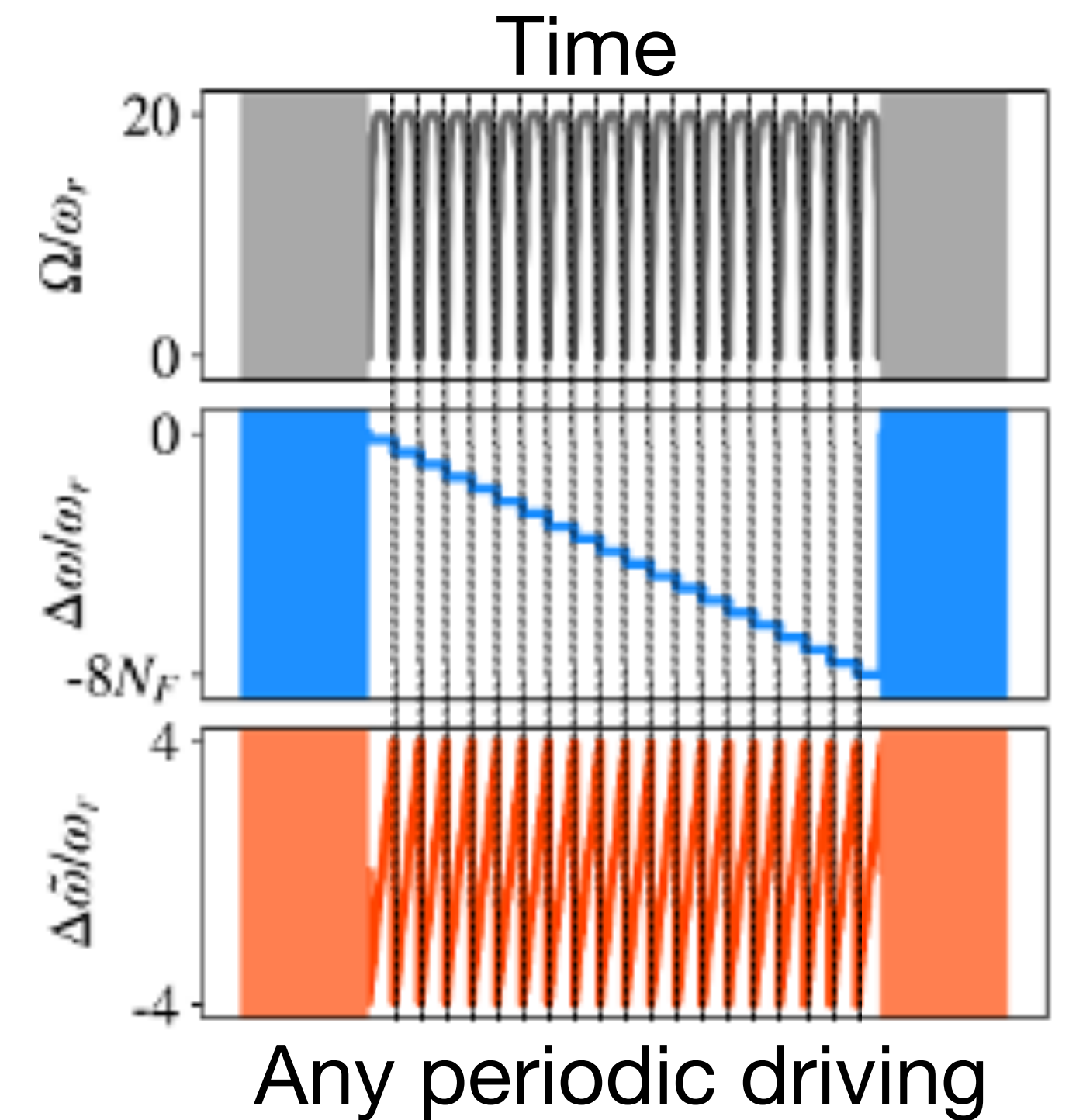
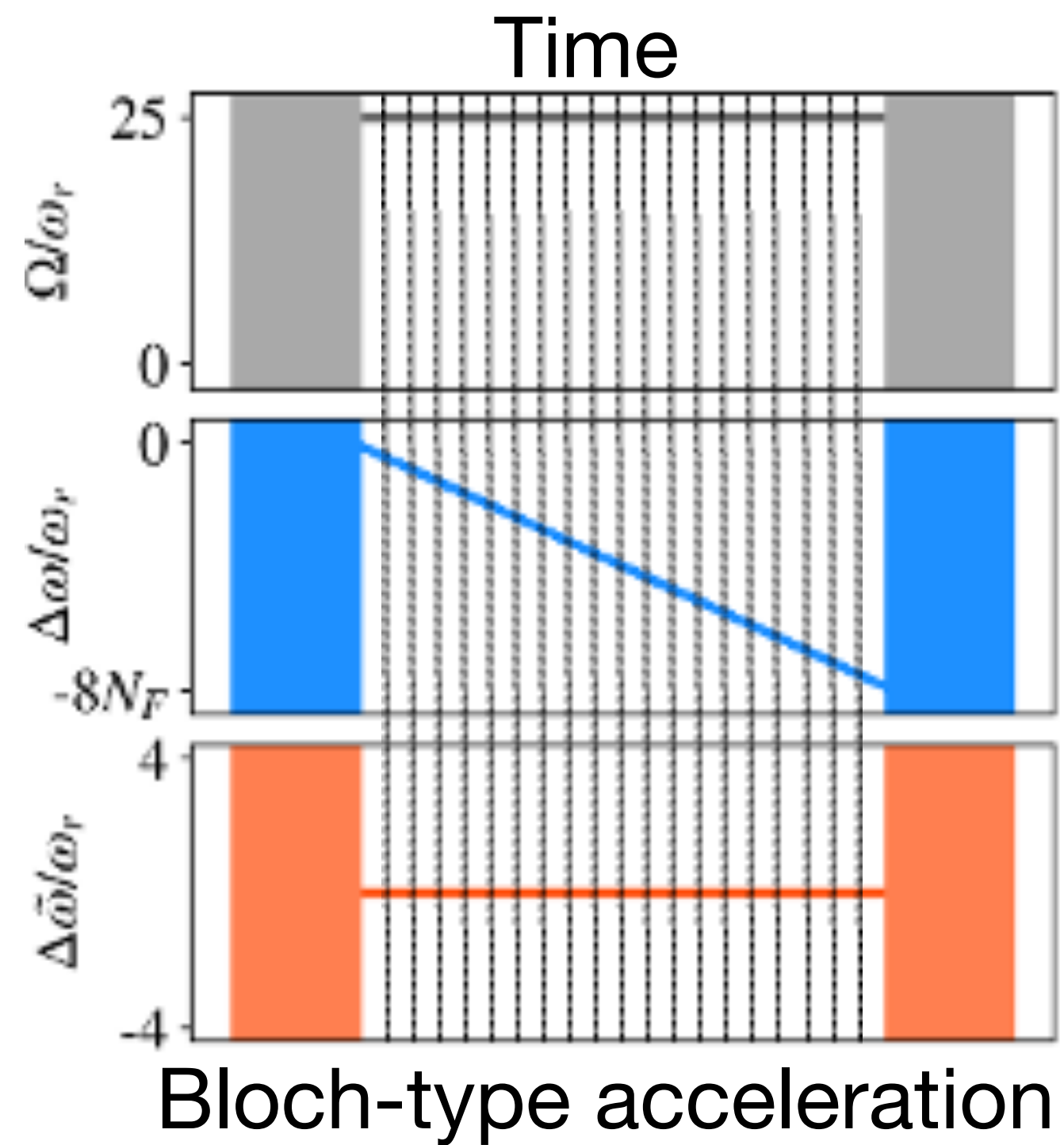
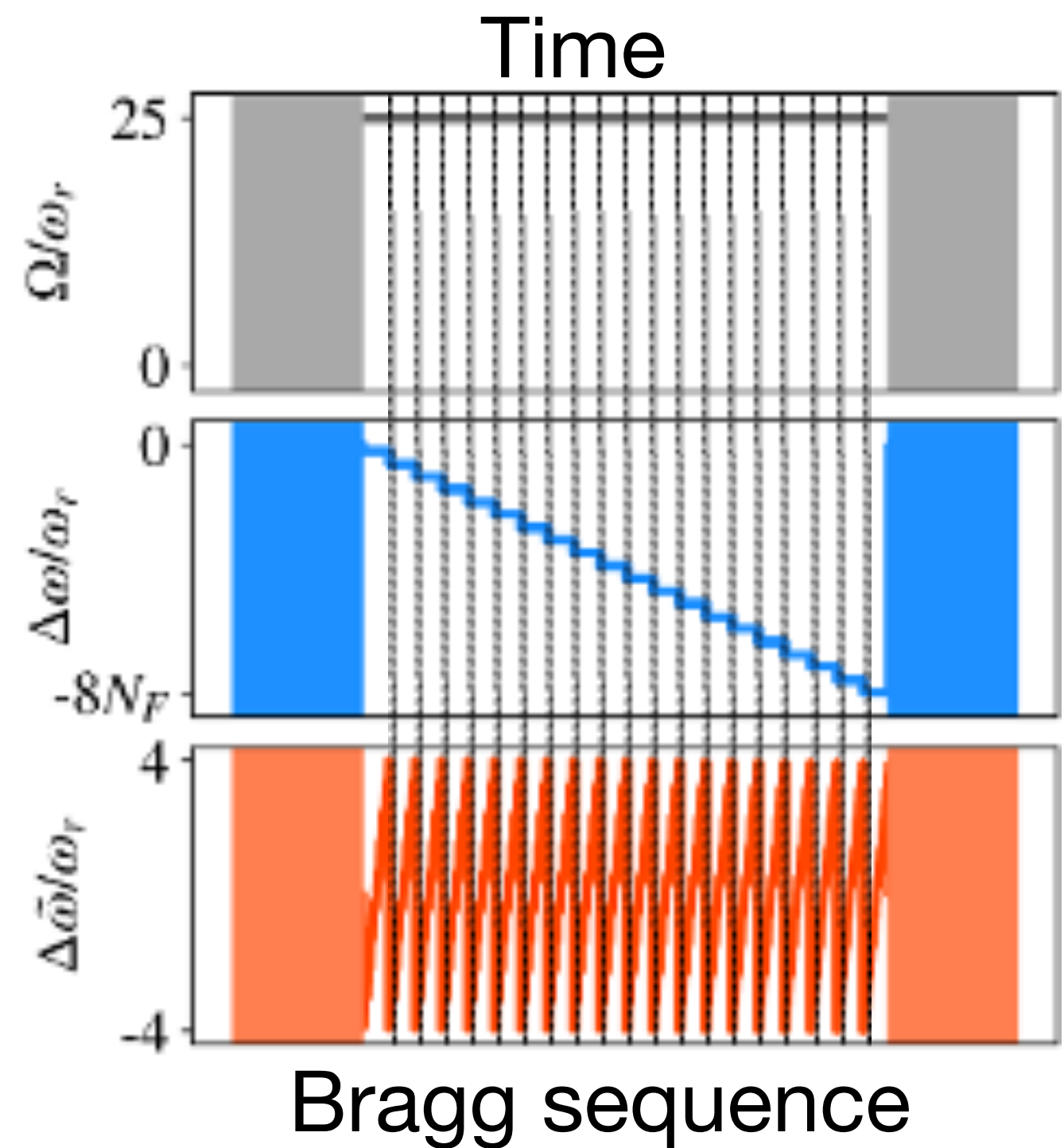
Stroboscopic stabilization at time $m\tau$ in the accelerated frame



Defines the optimal accelerated state!

Large Momentum Transfer: à la Floquet

The Floquet states $|w_n\rangle$ depend on the specific sequence



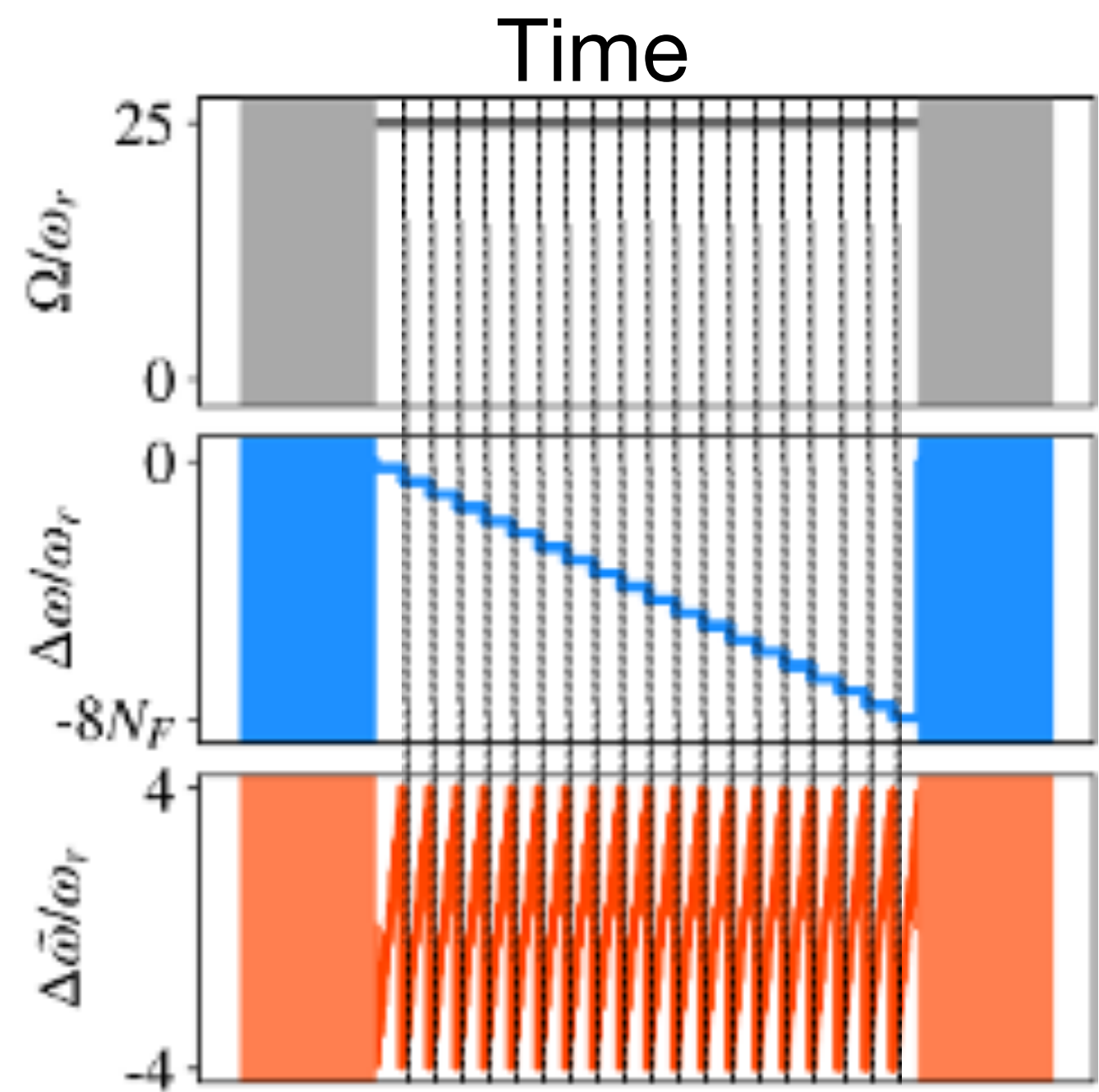
Periodic hamiltonian $H(t_0) = H(t_0 + \tau)$



Floquet states $|w_n\rangle$

Large Momentum Transfer: à la Floquet

The Floquet states $|w_n\rangle$ depend on the specific sequence

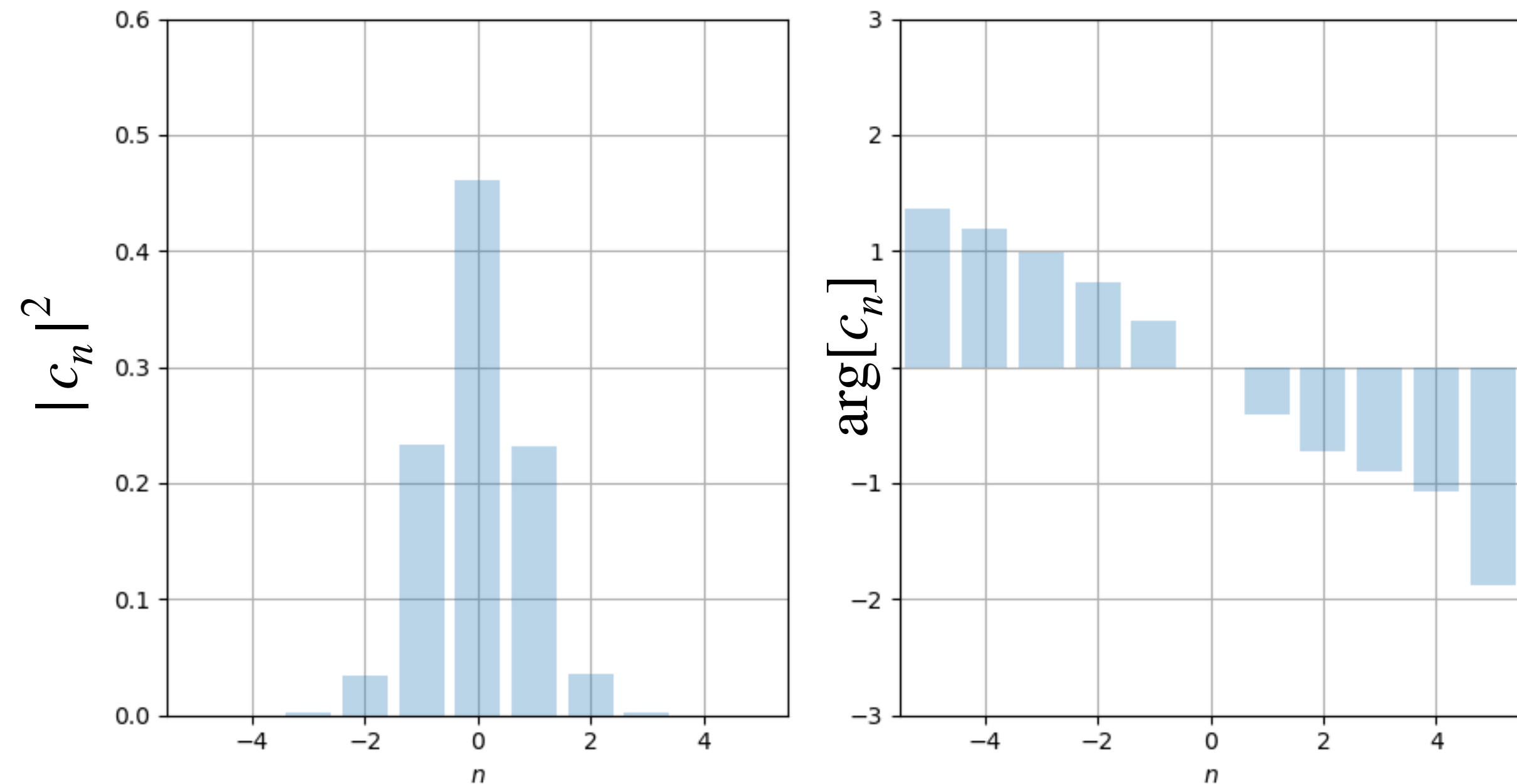


Arbitrary but periodic in
accelerated frame

Numerical
diagonalization of U



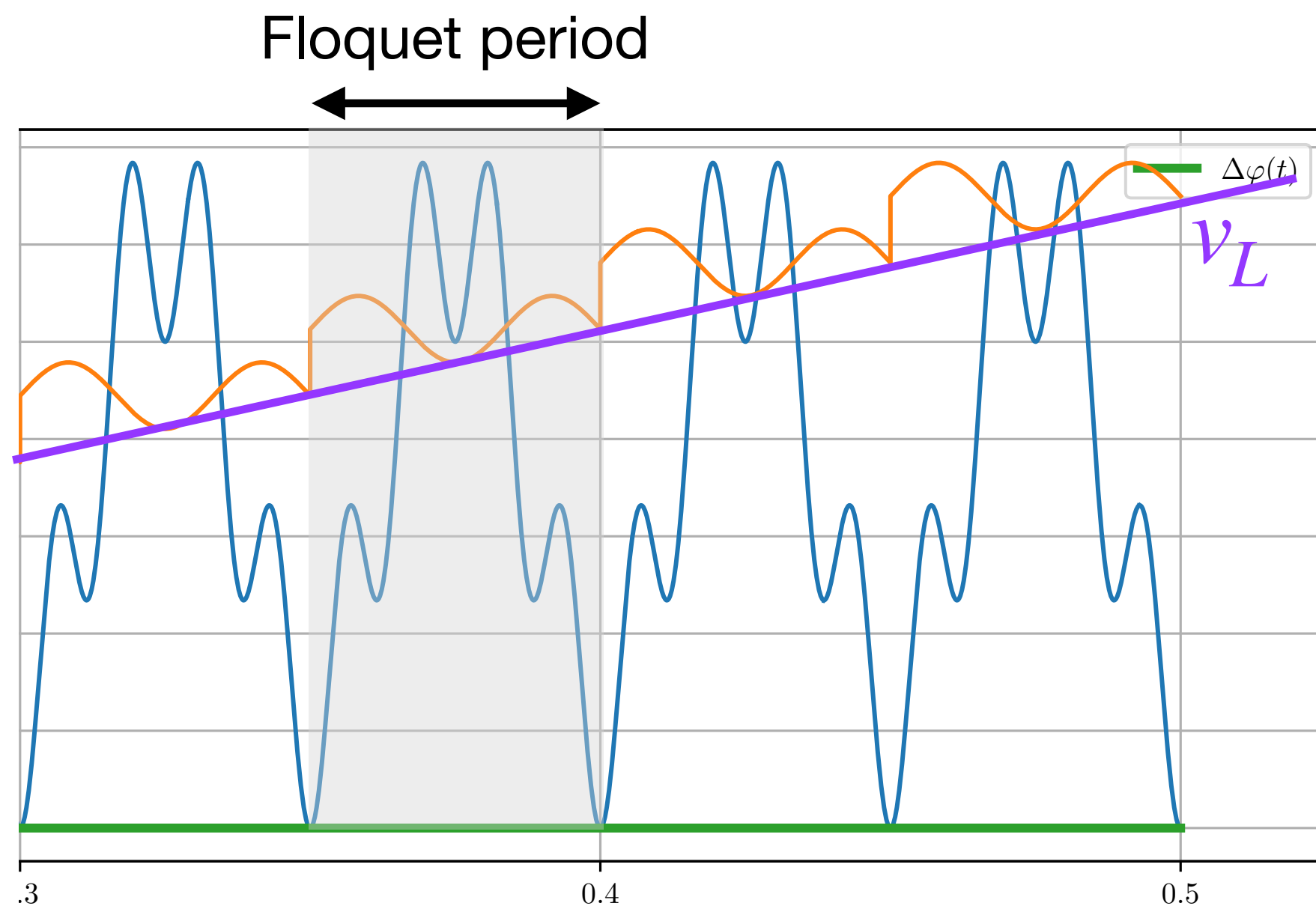
Example: square π -pulse $\tau = 5\mu s$



$$|w_0\rangle = \sum_n c_n |2n\hbar k\rangle$$

LMT Atomic Optics à la Floquet

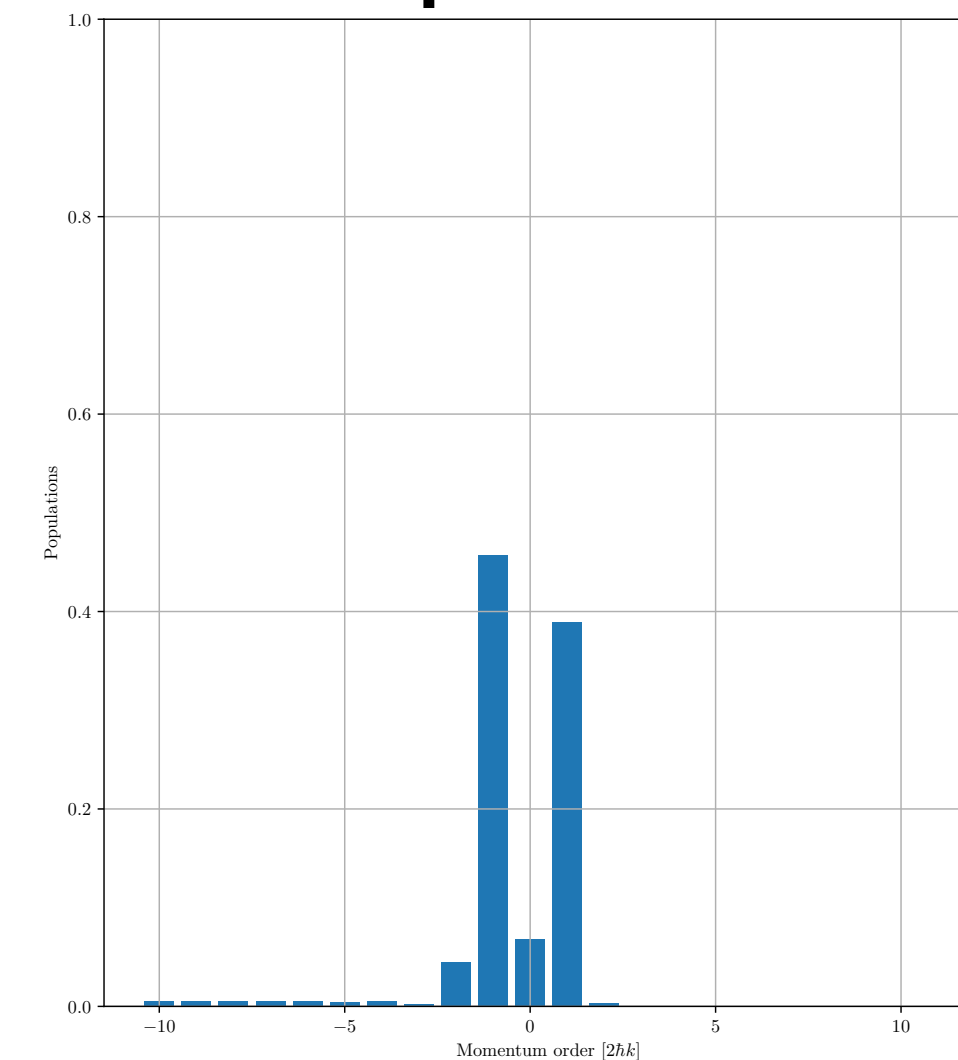
Zoology of specific Floquet states



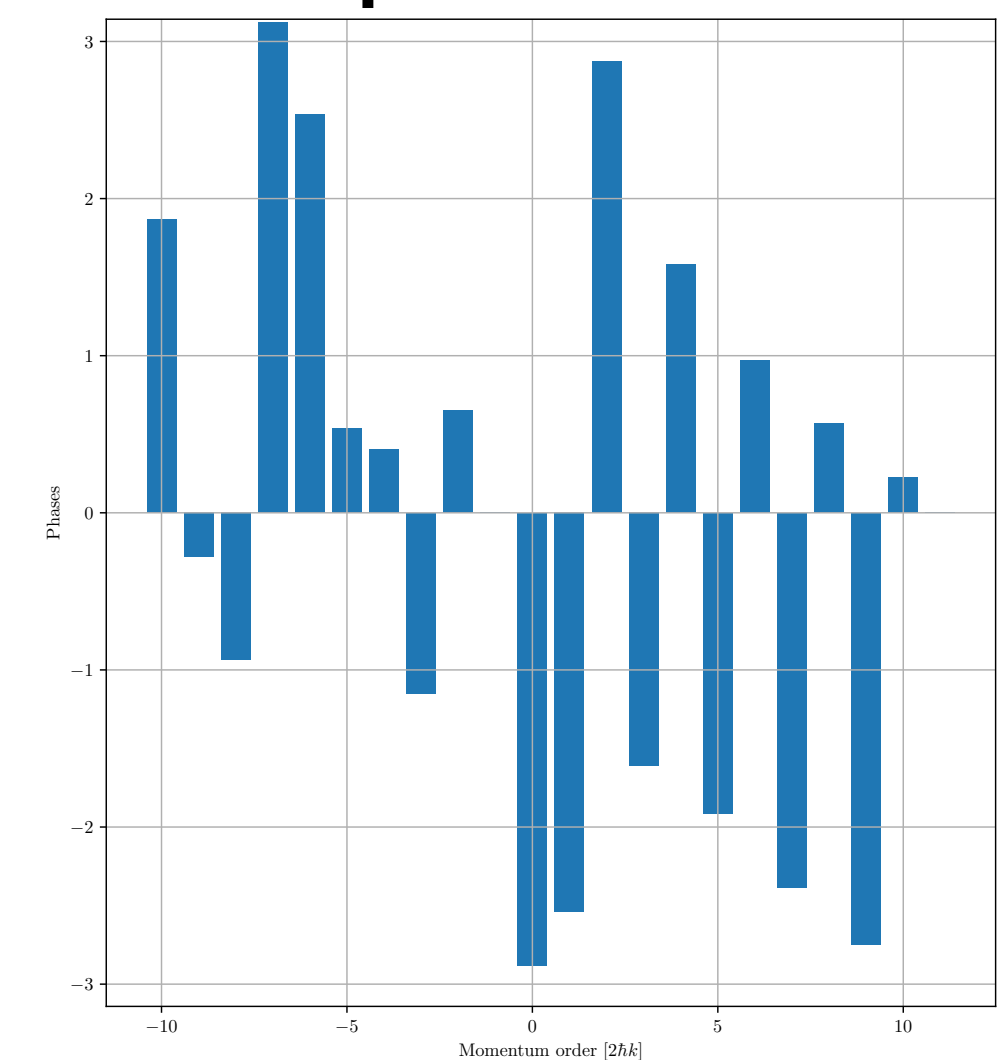
Numerical
diagonalization of U



amplitudes



phases

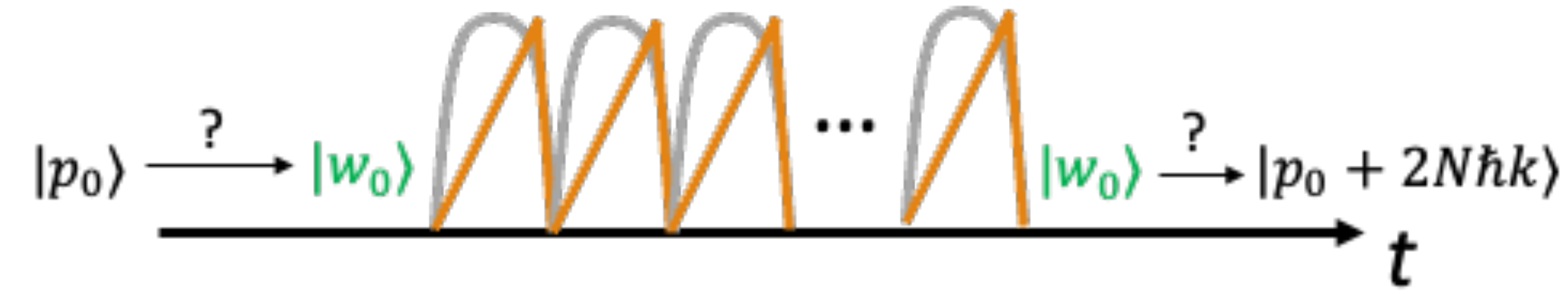
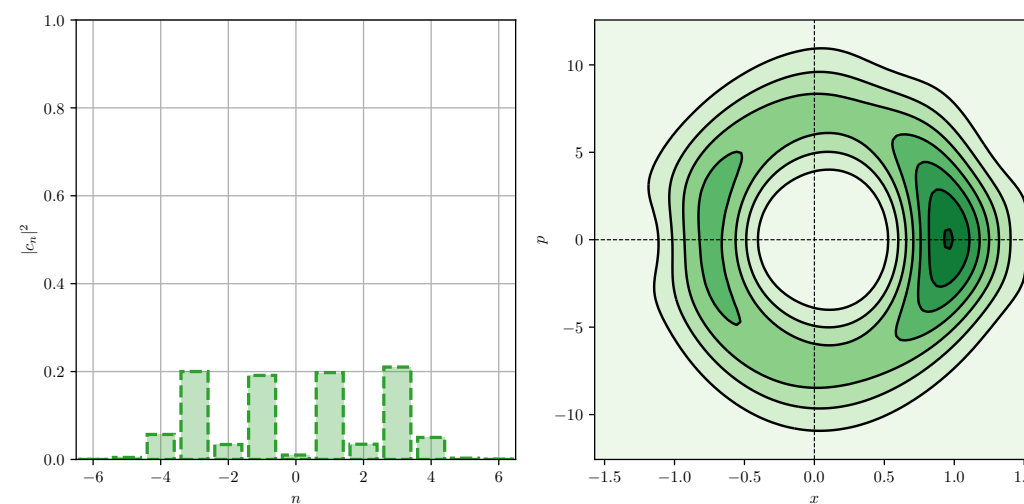
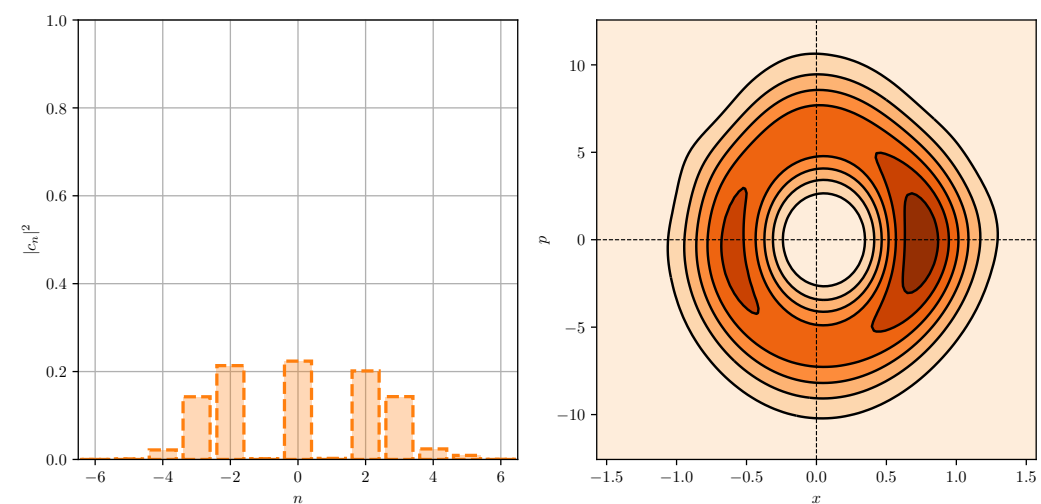
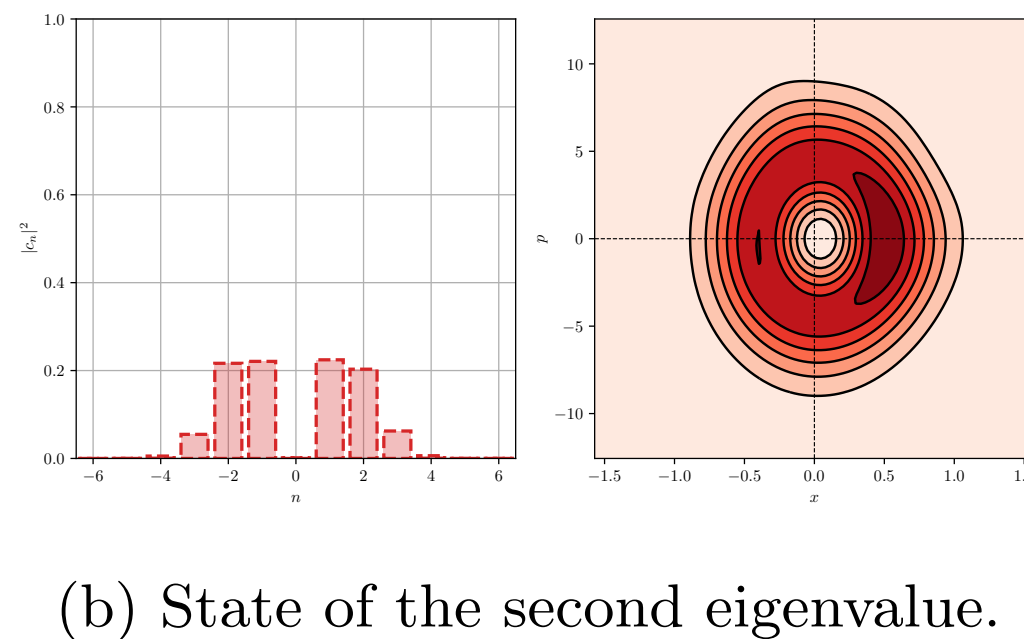
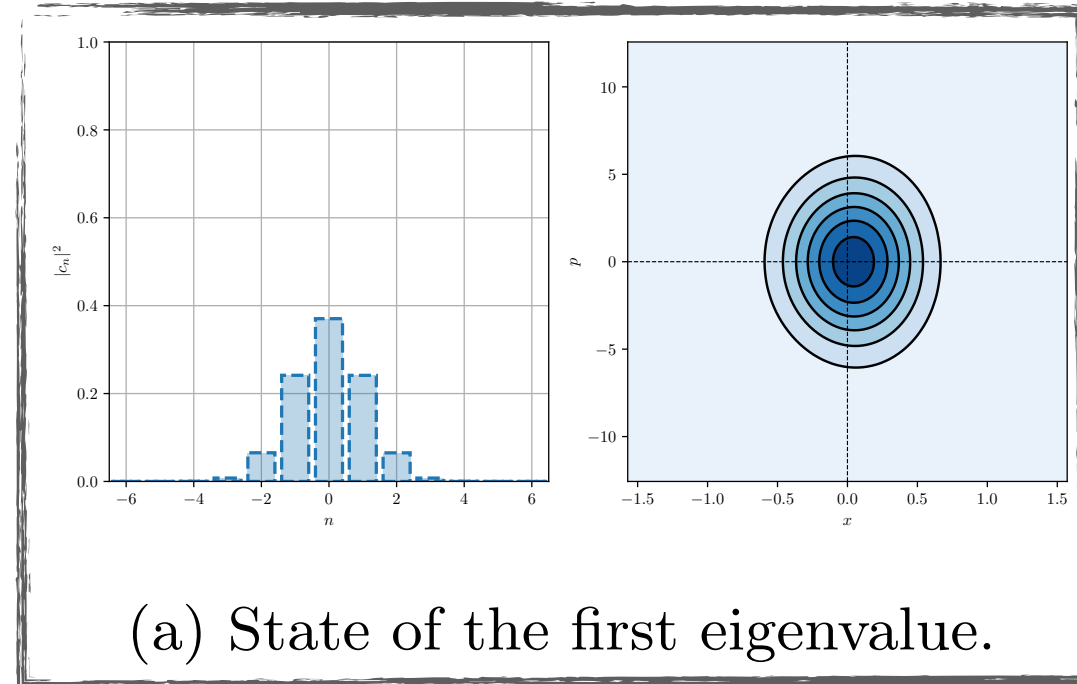


We can find Floquet states for any periodic driving in the accelerated frame

Implementation of Floquet acceleration

State engineering

For a given lattice pulse Many states $|w_n\rangle$



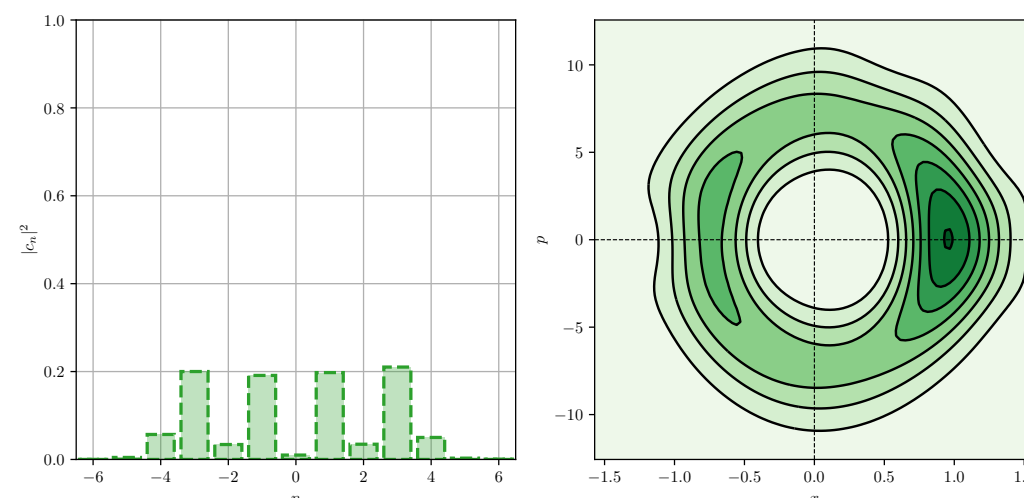
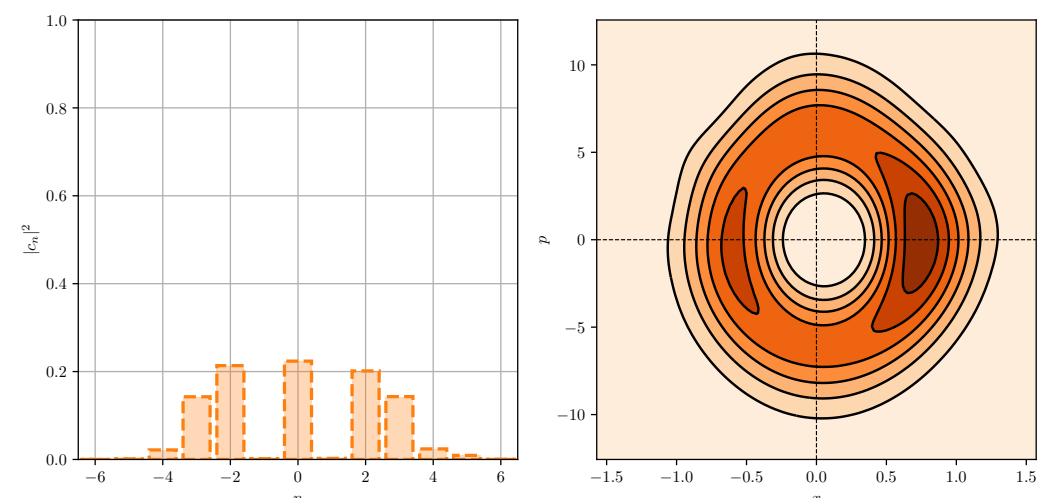
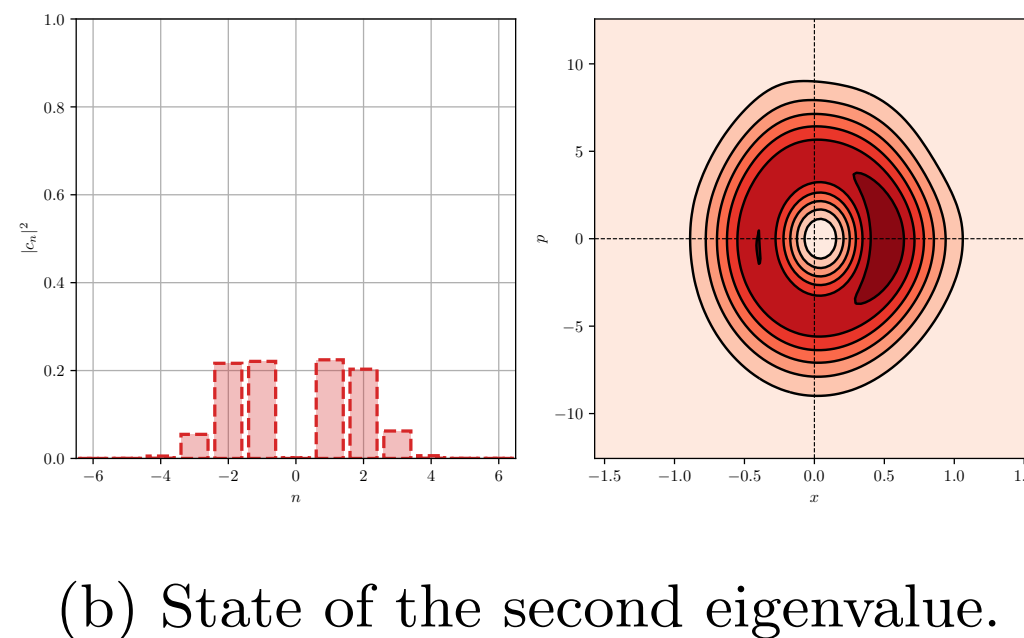
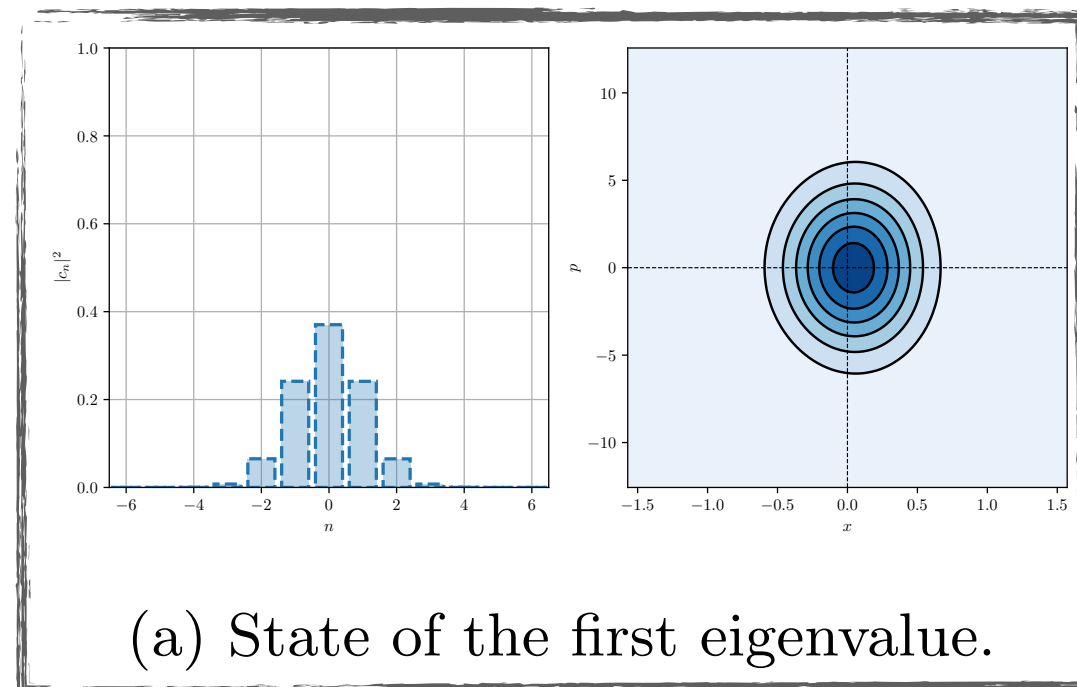
$|w_0\rangle$ = highest projection onto the initial state

Implementation of Floquet acceleration

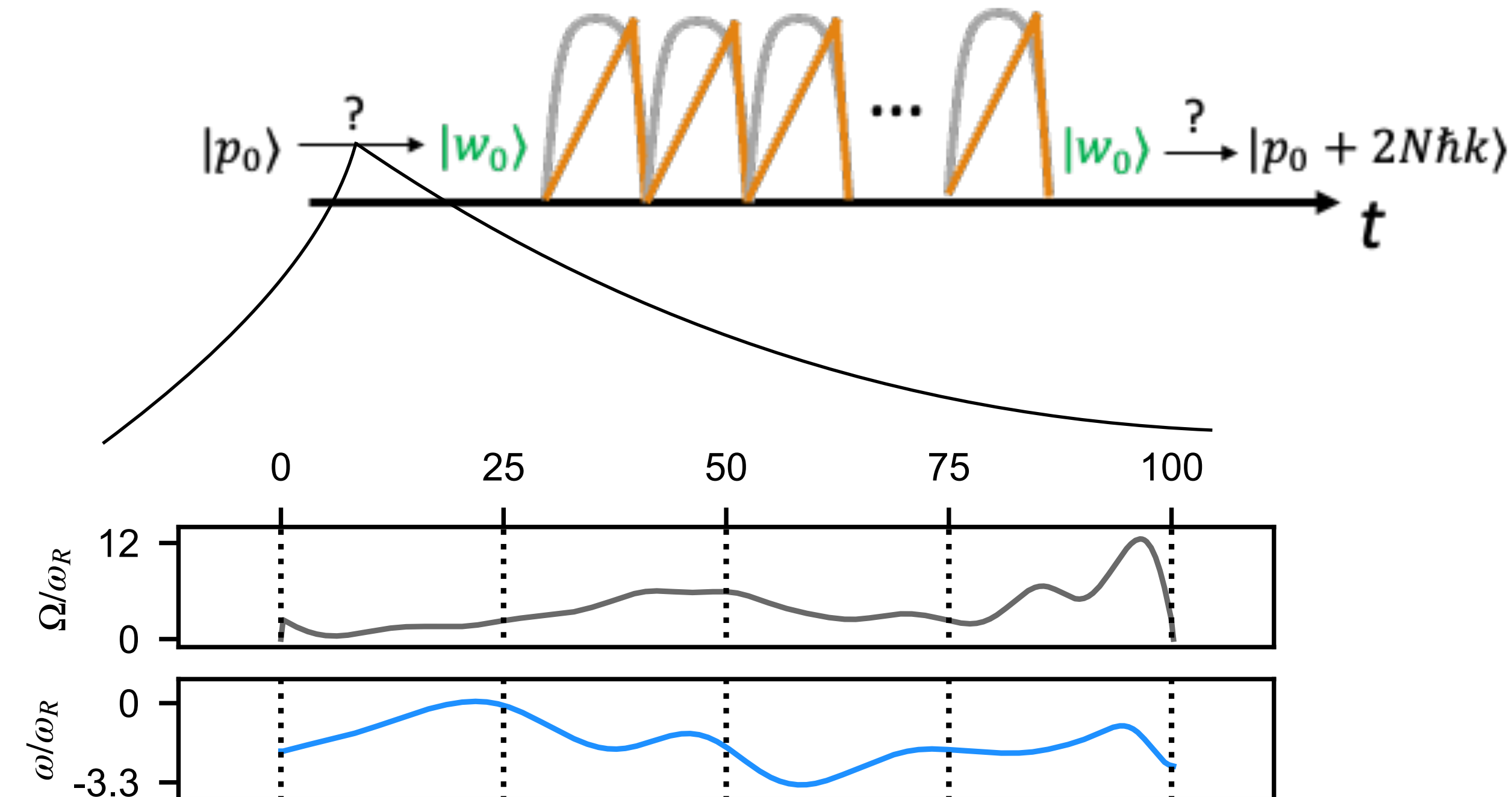
State engineering

For a given lattice pulse many states $|w_n\rangle$

State-to-State preparation $|p_0\rangle \rightarrow |w_0\rangle$



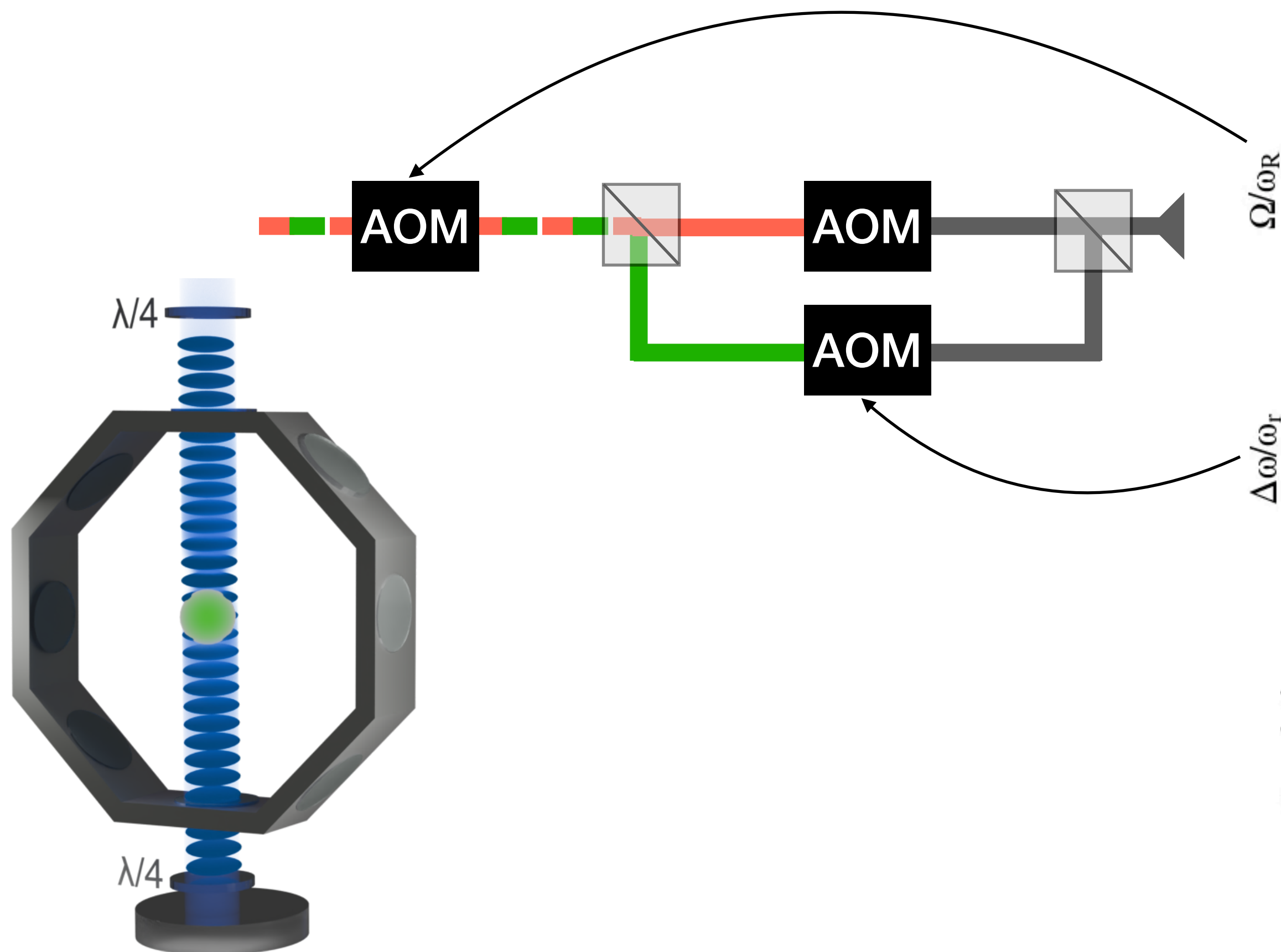
$|w_0\rangle$ = highest projection onto the initial state



Optimal Control Theory (OCT)

Optimal Control

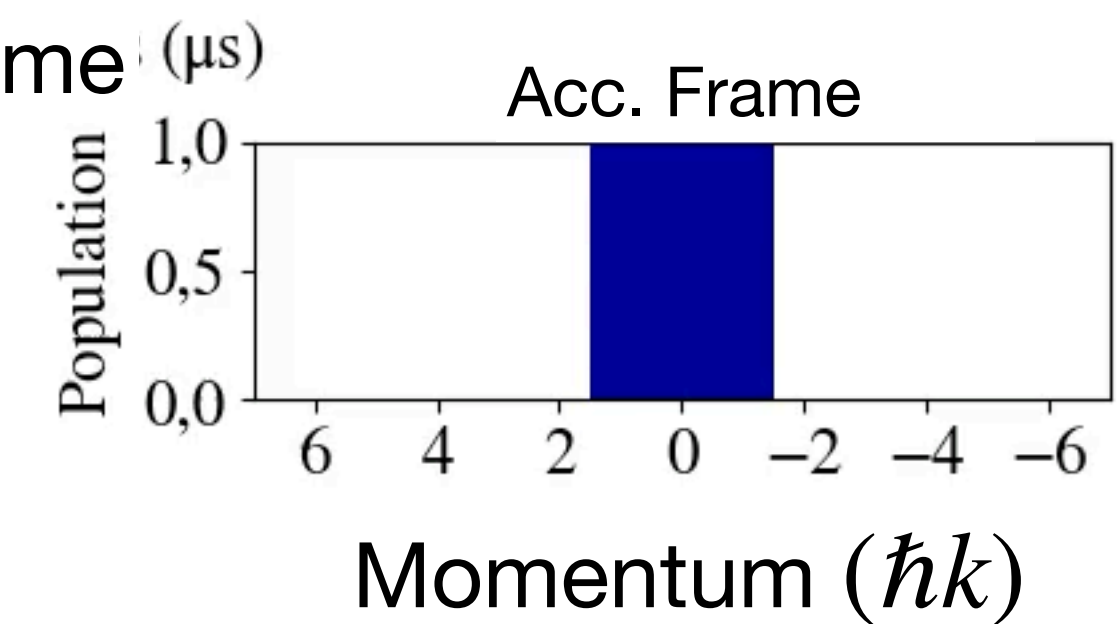
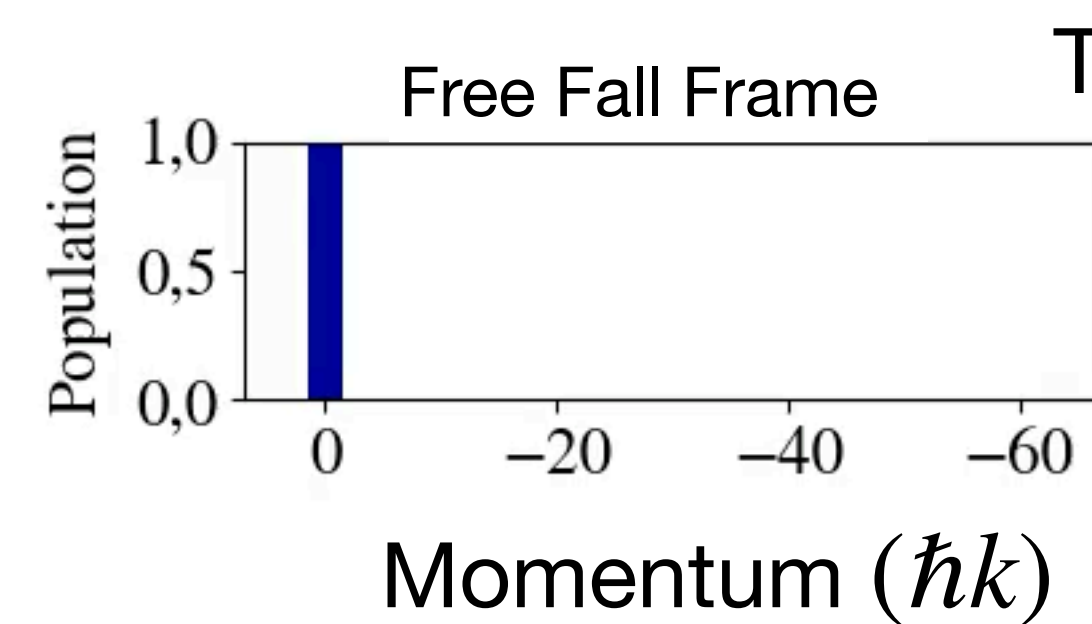
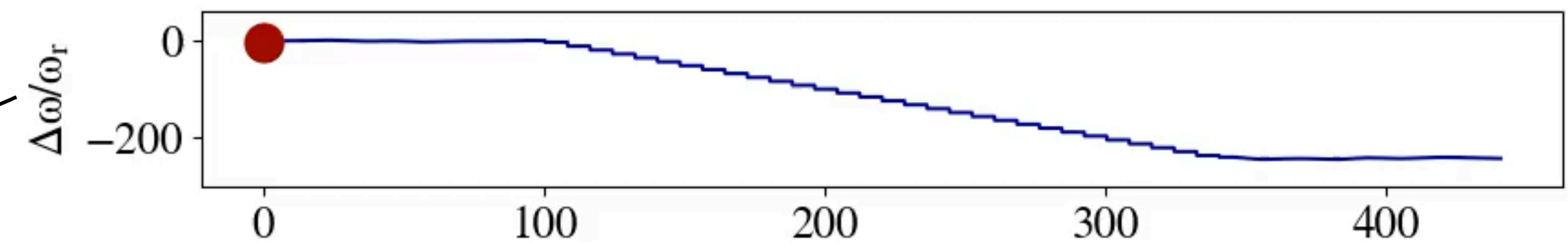
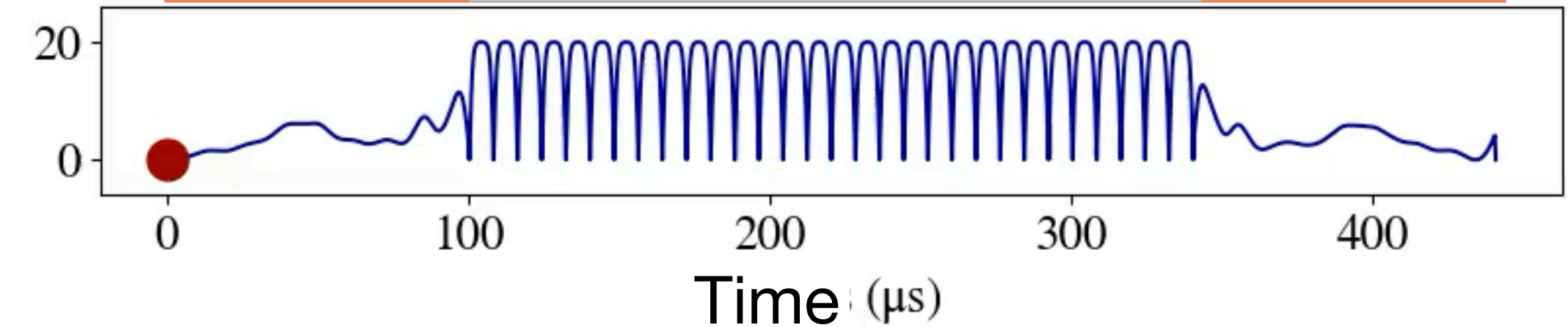
Floquet acceleration sequence



$|p_0\rangle$ $|w_0\rangle$ $|w_0\rangle$ $|p_0\rangle$

OC-1

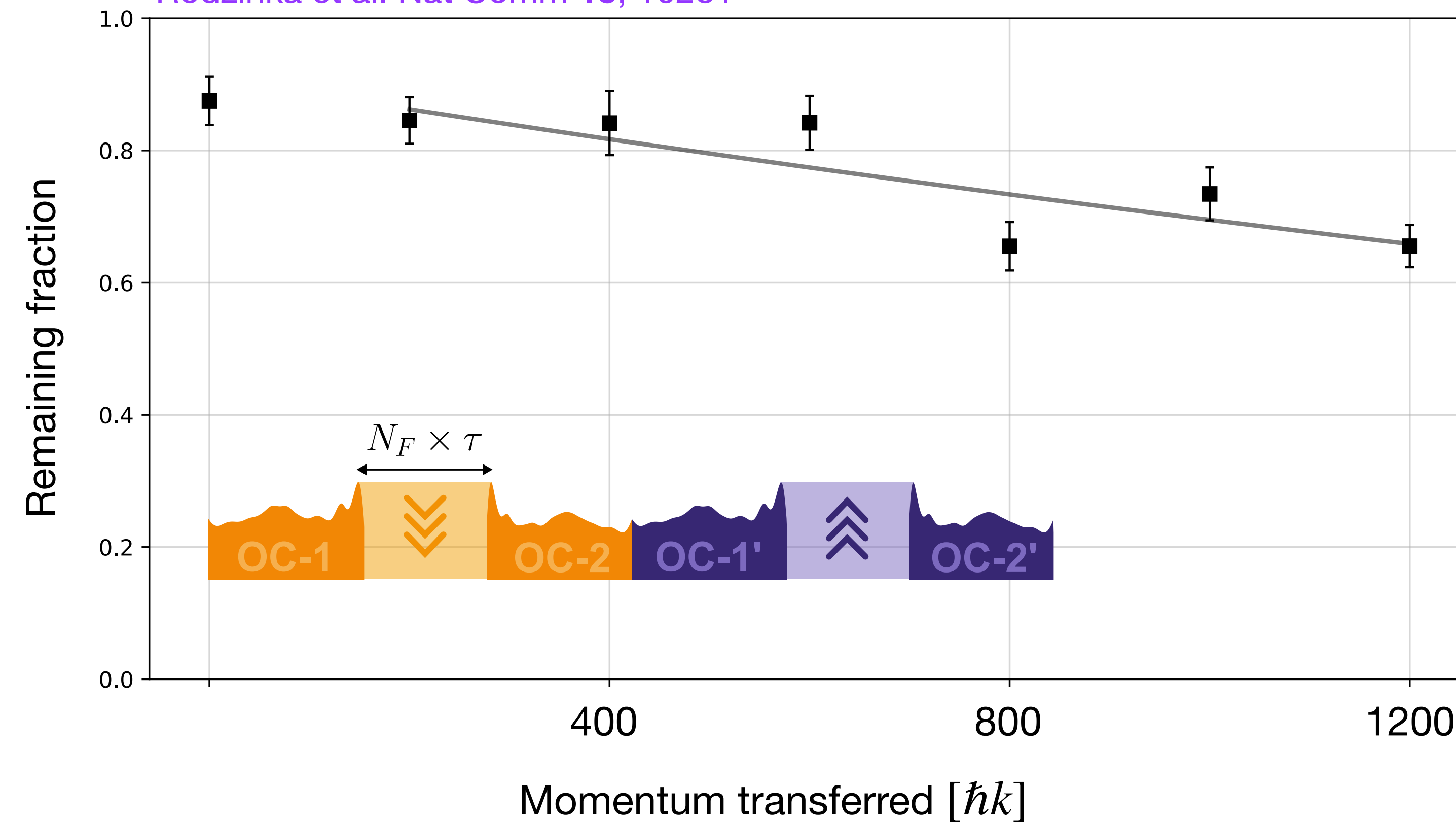
OC-2



Floquet acceleration

Normalized atom number in the fully accelerated state

Rodzinka et al. Nat Comm **15**, 10281



Fast LMT peak momentum transfer :

$$2.5 \mu s / \hbar k - a_{LMT} \sim 250g$$

Efficiency per $\hbar k$: 0.99973(5)

Limitations:

spontaneous emission: $\Delta = 30$ GHz

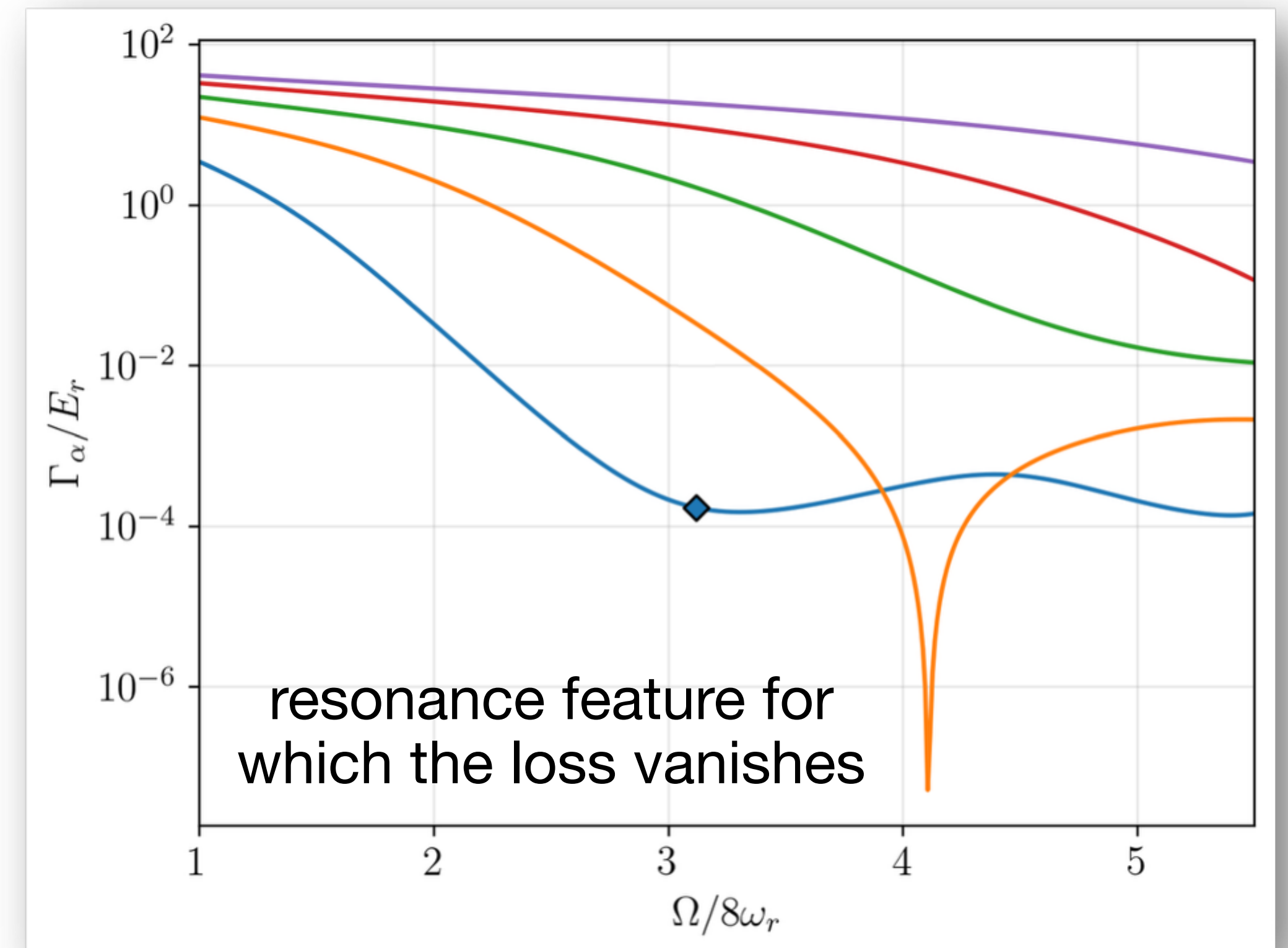
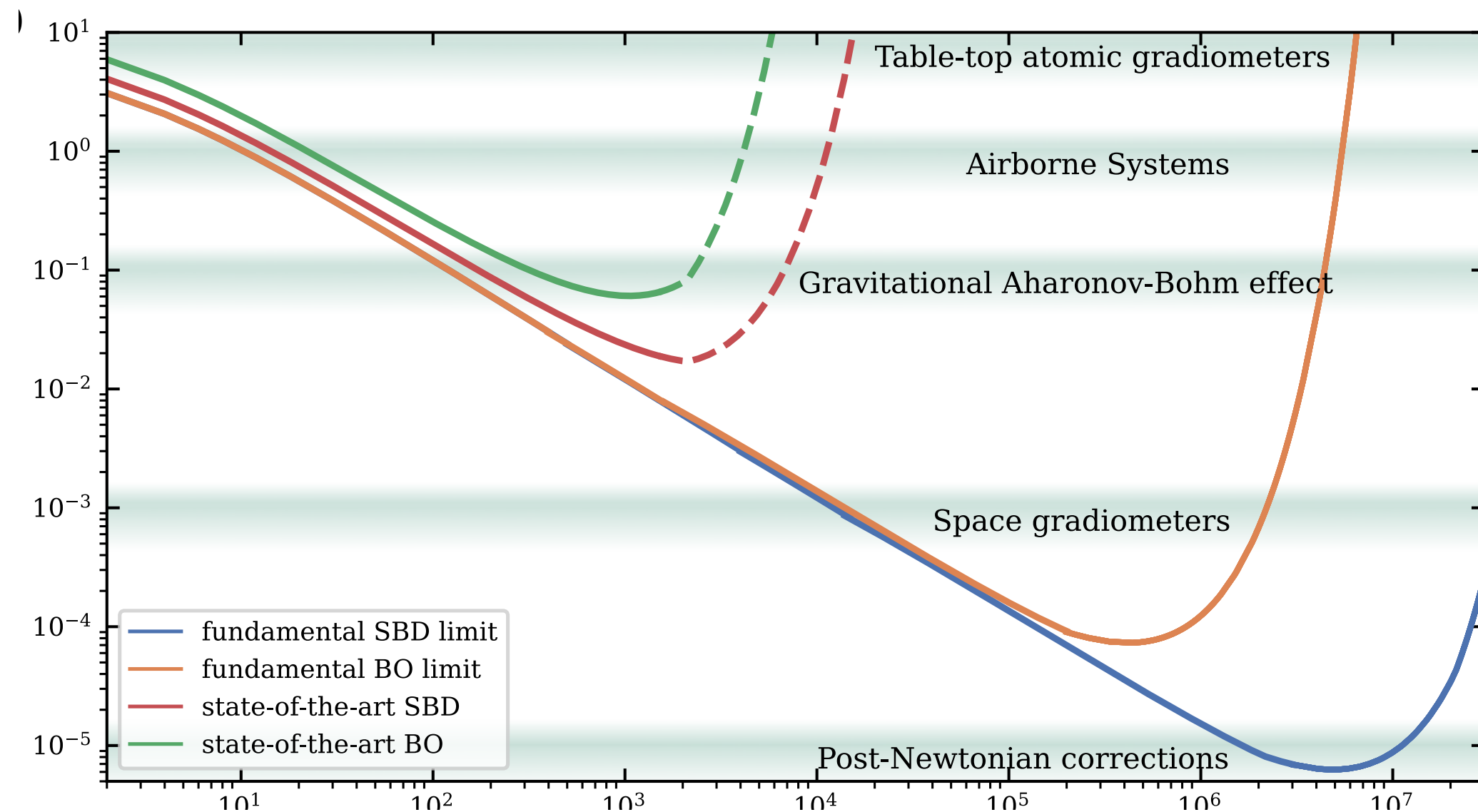
Floquet acceleration

Fundamental losses

$$\hat{U}|w_n\rangle = |w_n\rangle e^{i\epsilon_n\tau/\hbar} = |w_n\rangle e^{-i\frac{E_n\tau}{\hbar}} e^{-\frac{\Gamma_n\tau}{2\hbar}}$$

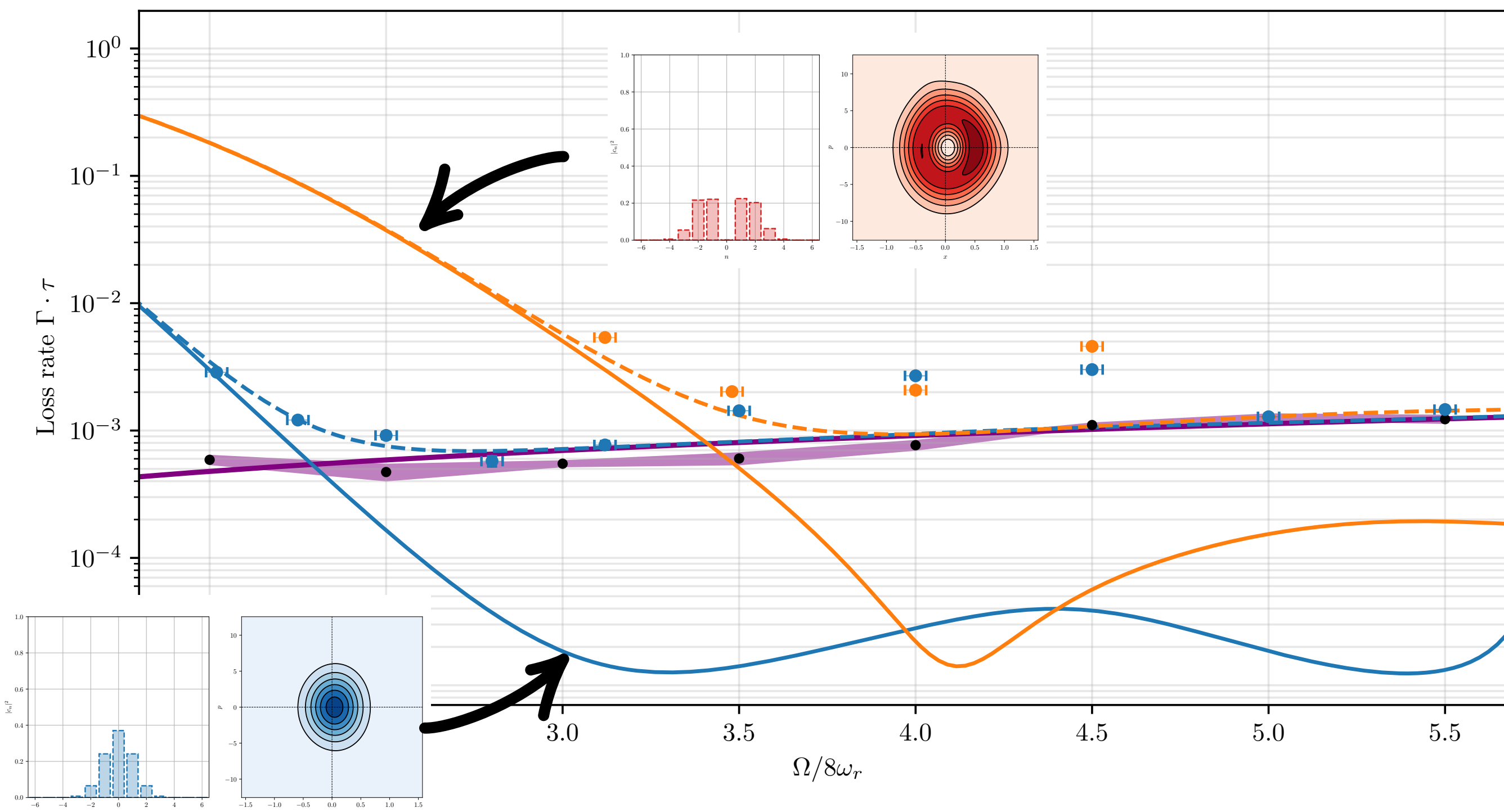
Alibabaei et al. arXiv:2602.00365v2

$$\epsilon_n = E_n - i\frac{\Gamma}{2}$$

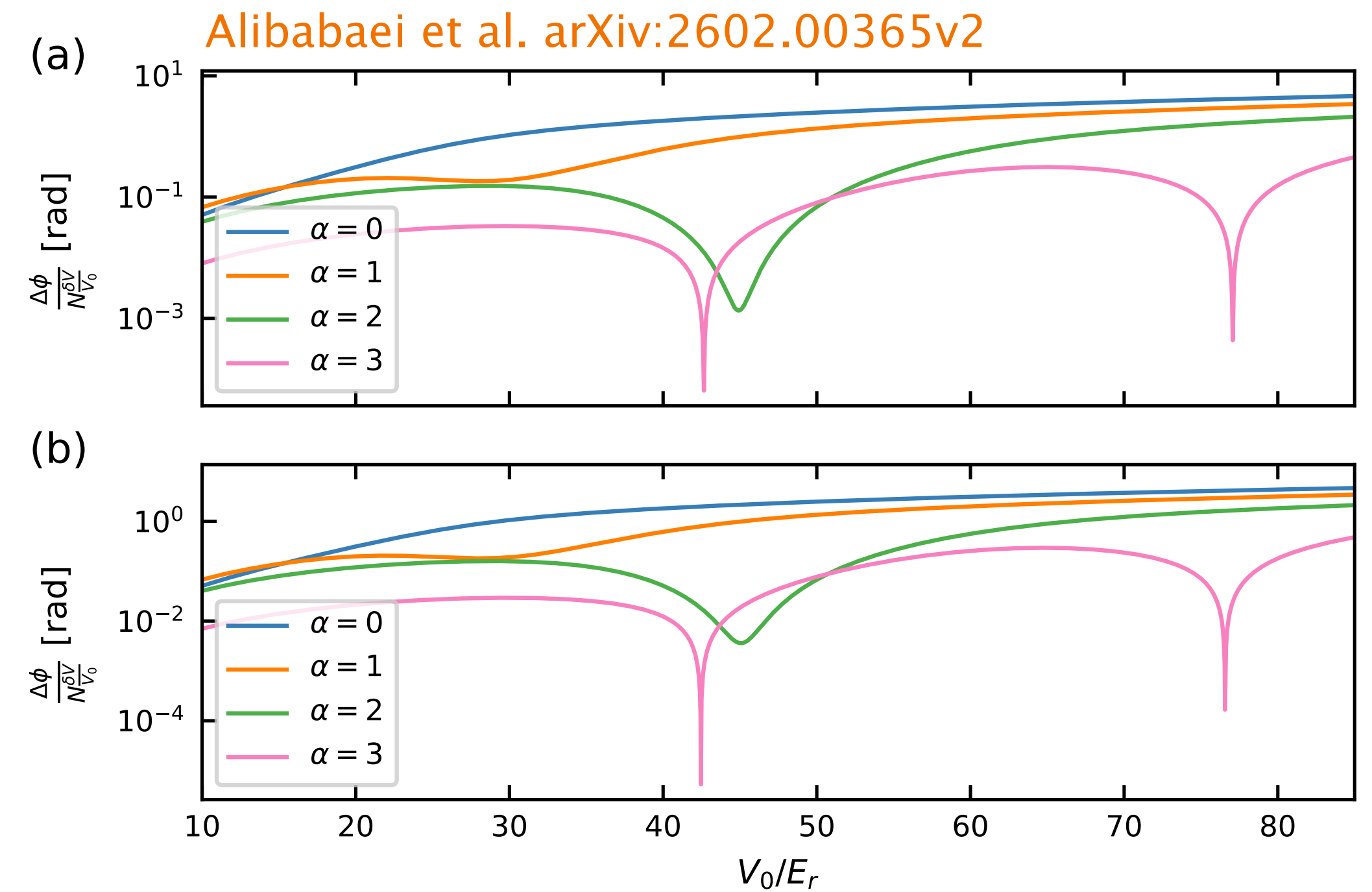


Floquet acceleration

Using Excited Floquet states ?



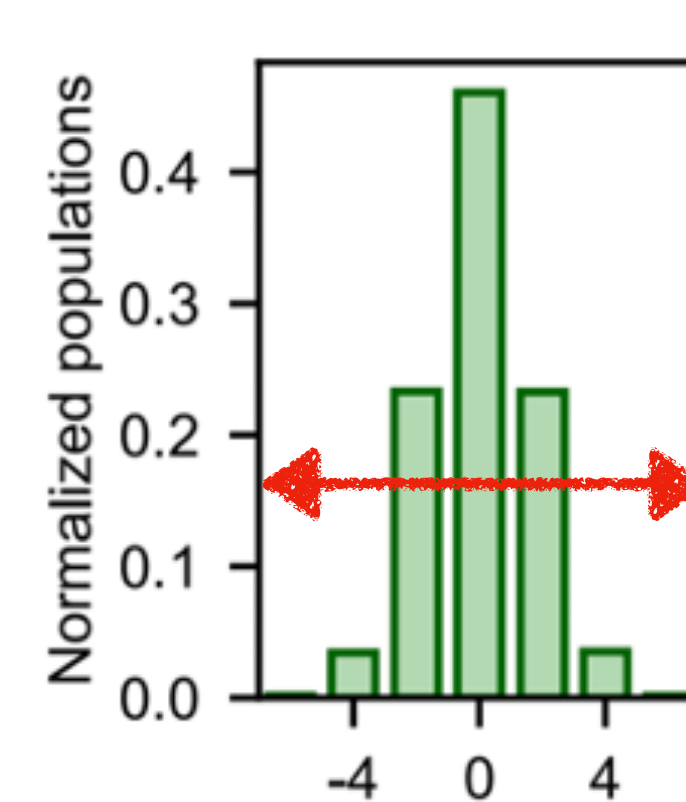
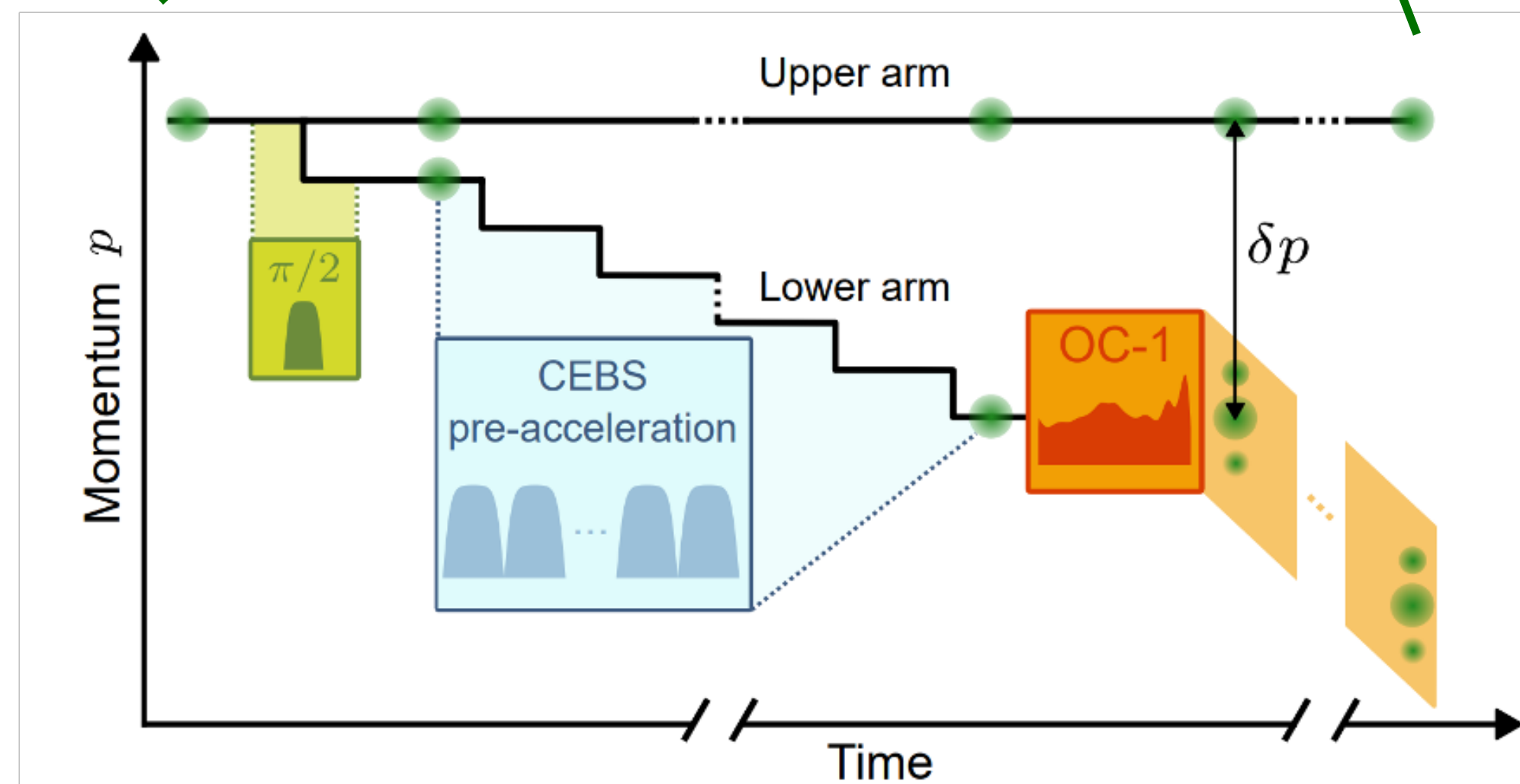
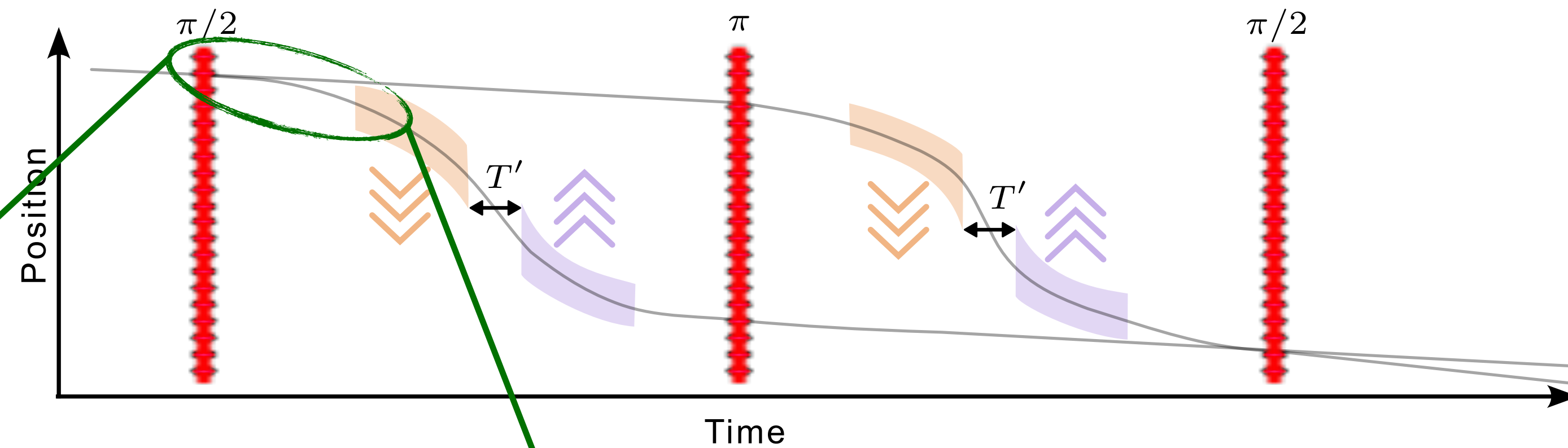
☑ Preparing excited Floquet states



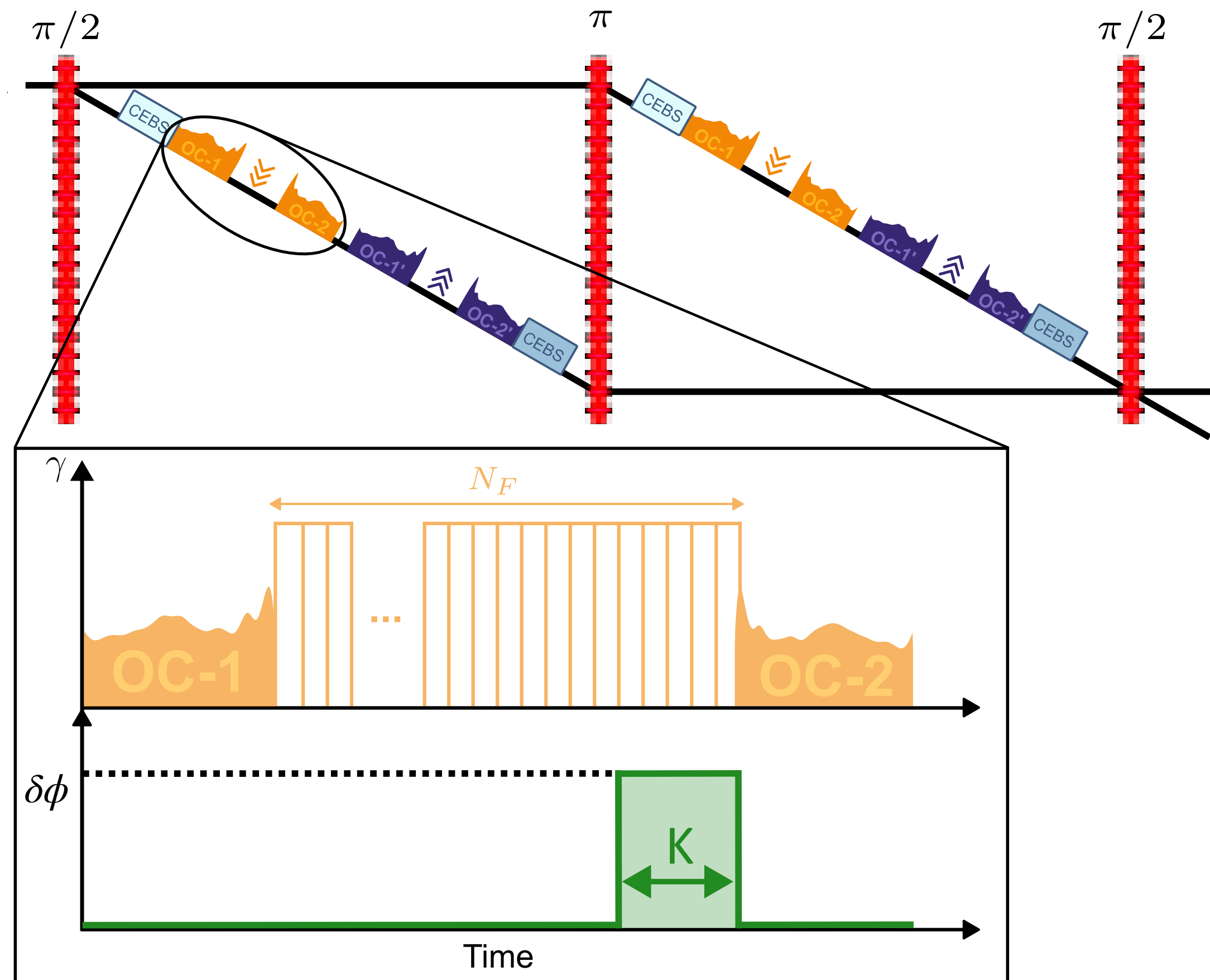
Phase-sensitivity to Lattice depth fluctuations.

Large Momentum Transfer Atom Optics

Floquet-LMT-Interferometer



Interference fringe scanning

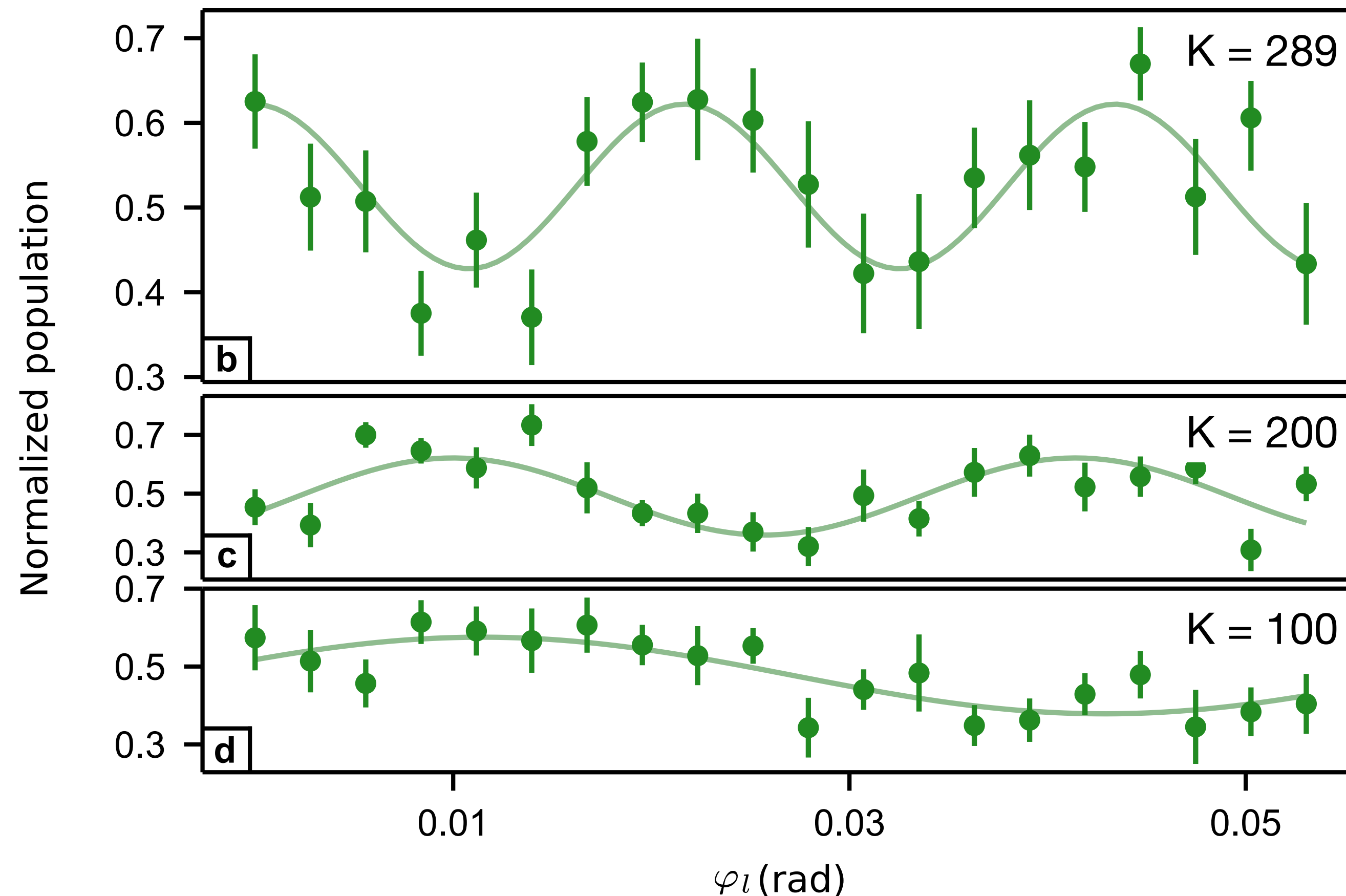


- ▶ During K acceleration pulses, we add a laser phase jump $\delta\phi$
- ▶ Signal: $\propto V \sin(K\delta\phi + \phi_0)$
- ▶ We can adjust $K = N_F$ to be sure that the detected fringes correspond to the LMT interferometer.

Large Momentum Transfer Atom Optics

Floquet-LMT-Interferometer

Rodzinka et al. Nat Comm **15**, 10281



► LMT - Interferometer $600\hbar k$

- limit = detection volume

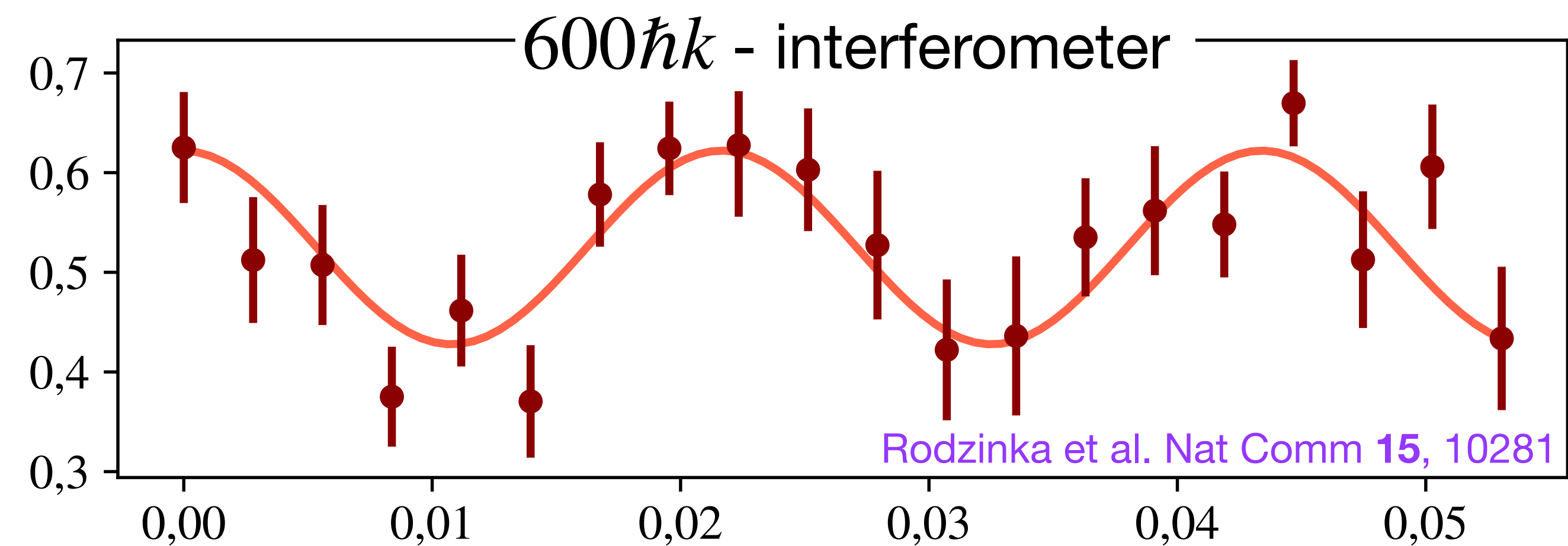
► Visibility: $18\% \pm 4\%$

- spontaneous emission

- pre-acceleration efficiency

- dephasing due to quasi-energy $\epsilon(\Omega)$

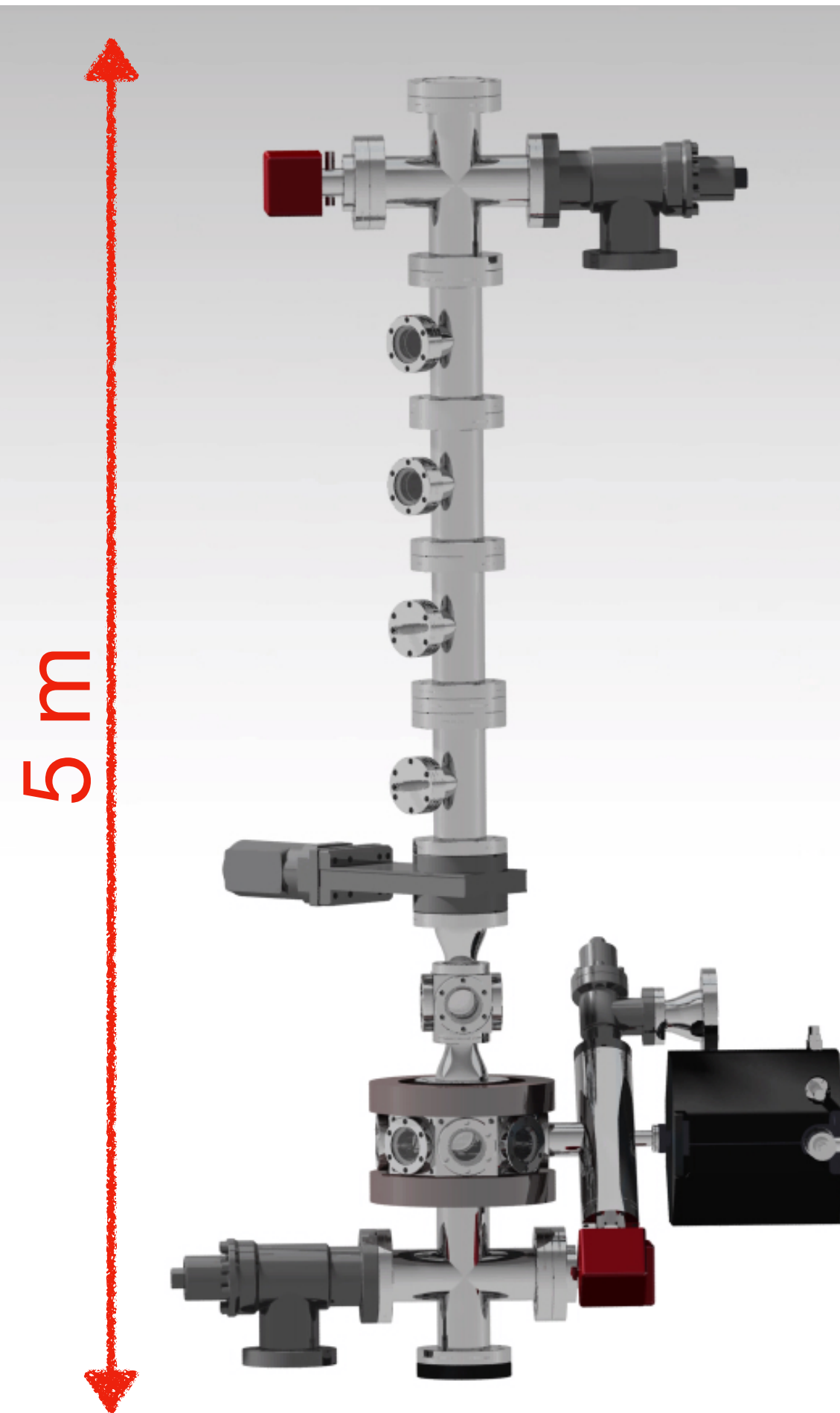
LMT-interferometers



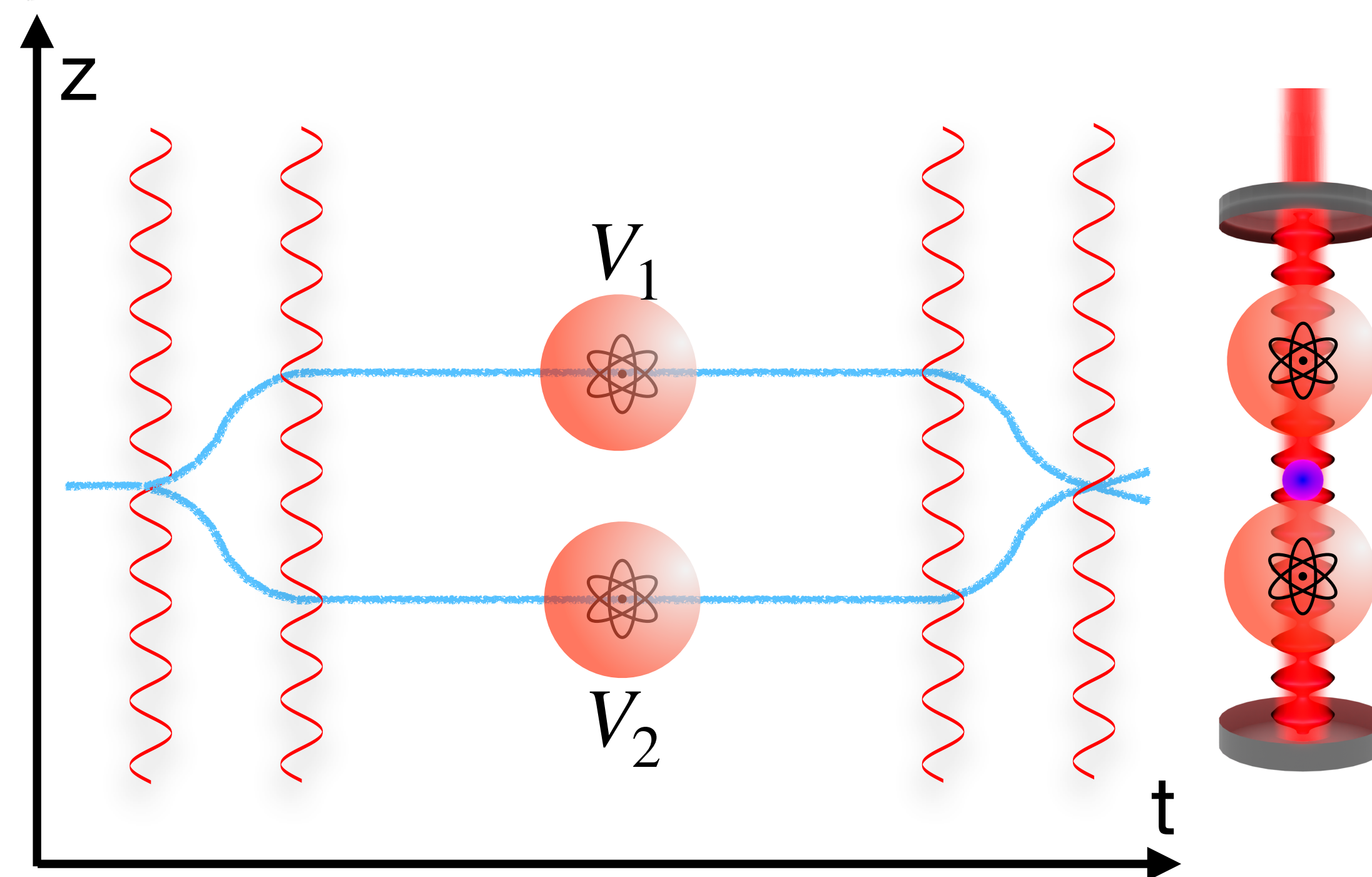
- ✓ Floquet approach for sequential and continuous acceleration.
- ✓ New QOCT implementation for navigating large Hilbert spaces.
- ✓ Fast and very efficient LMT.
- ✓ Demonstrates 600 $\hbar k$ atom interferometer.

Impulsion Transférée ($\hbar k$)	Equipe	Date
102 (Bragg)	Stanford (M. Kasevich)	2011
112 (Bragg)	Seattle (S. Gupta)	2018
240 (Bloch)	Berkeley (H. Muller)	2020
408 (Bloch)	Hanovre (E. Rasel)	2021
400 (1 photon)	Stanford (J. Hogan)	2022
600 (Floquet)	Toulouse (A. Gauguet)	2024

A new machine



- ▶ 5m BEC Fountain → time of flight ~ 2 s
- ▶ Large spatial separation (cm - m)
- ▶ Controlled Potential on the interferometer arms



Quantum phase shift

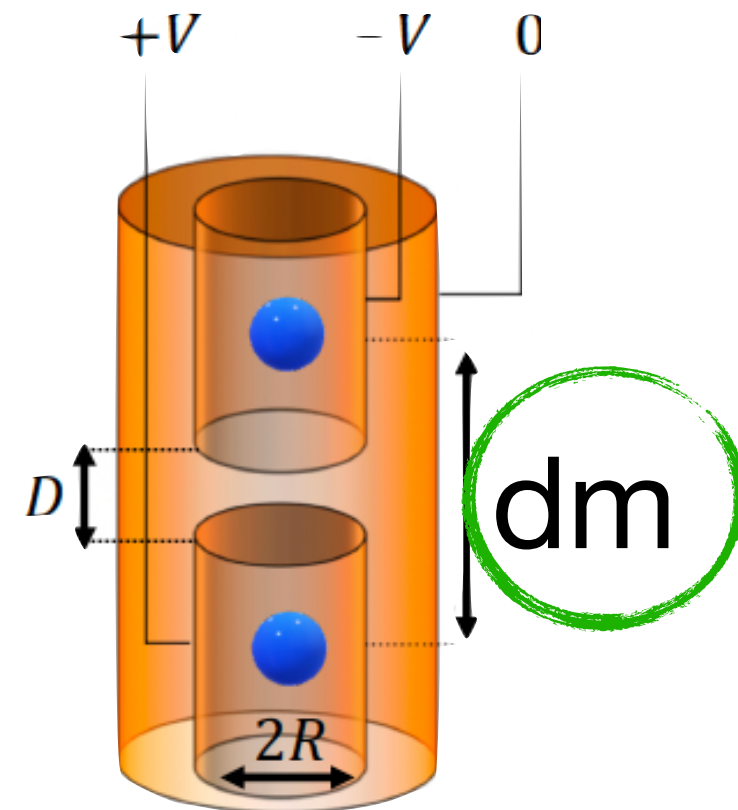
$$\Delta\phi_{AB} = - \int \frac{V_1(t) - V_2(t)}{\hbar} dt$$

Long term focus

Metrology with Aharonov-Bohm phases

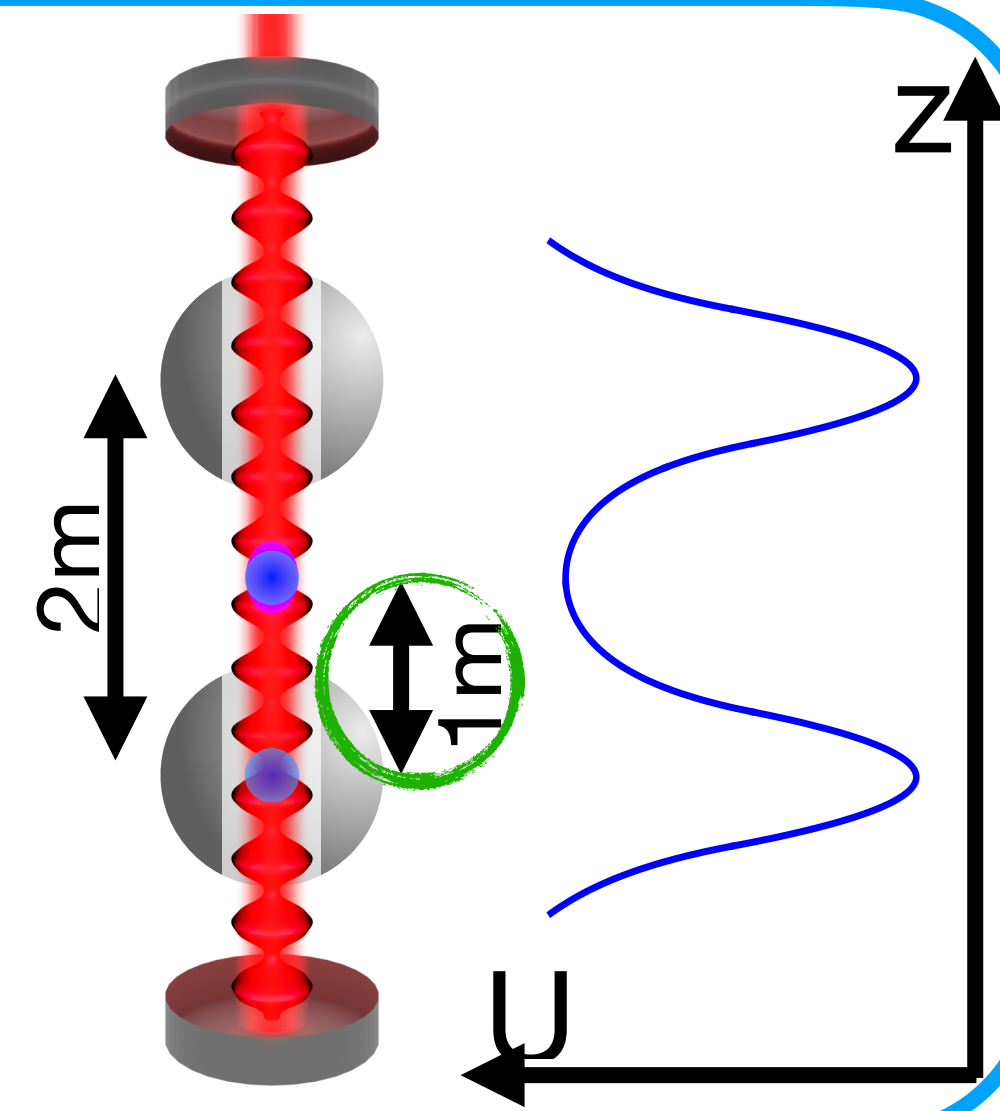
► Atom neutrality

- Improvement x1000
- Particle physics
- Astrophysics



► gravitational Aharonov-Bohm

- Gravitational redshift with no-force
- Newtonian constant G



A new approach to metrology that uses AB-phase shifts!

A century of matter waves



1923

$$\lambda = \frac{h}{p}$$

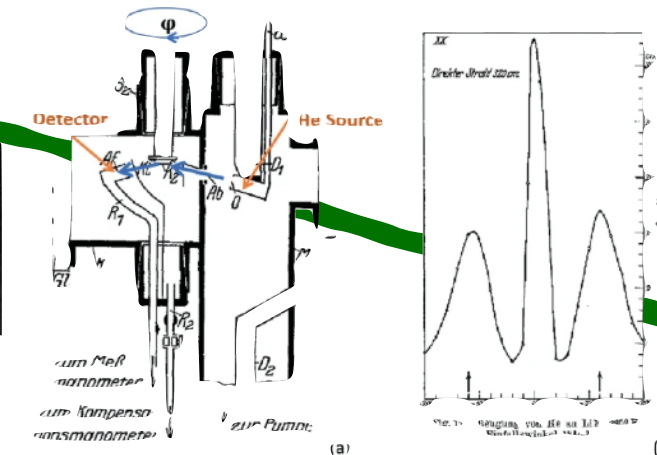


1926

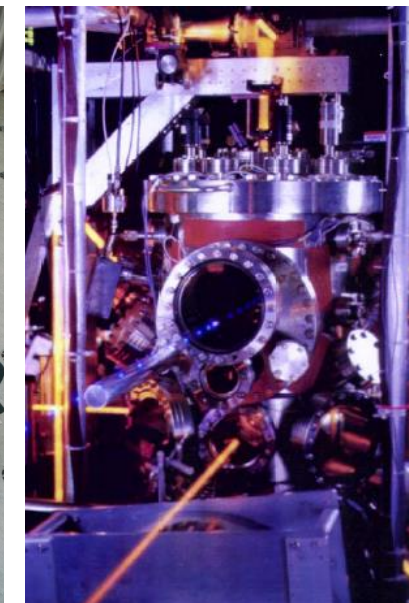
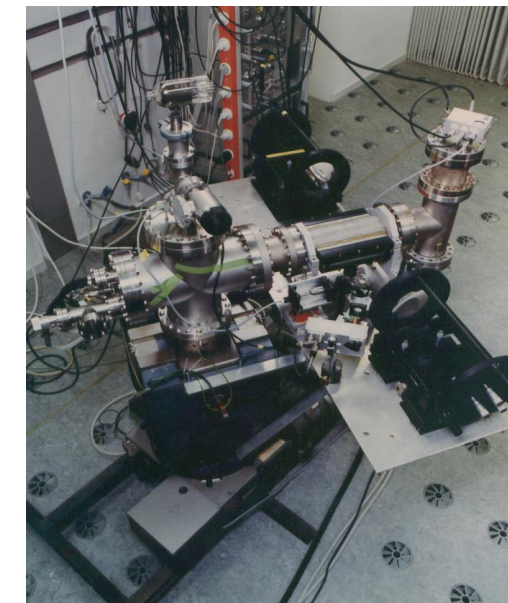
$$i\hbar \frac{\partial \Psi(t, r)}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi(t, r) + V(r) \Psi(t, r)$$



electron diffraction 1927



Helium diffraction 1929



1991

Carnal and Mlynek PRL 66 2689

Keith, Ekstrom, Turchette, Pritchard PRL 66 2693

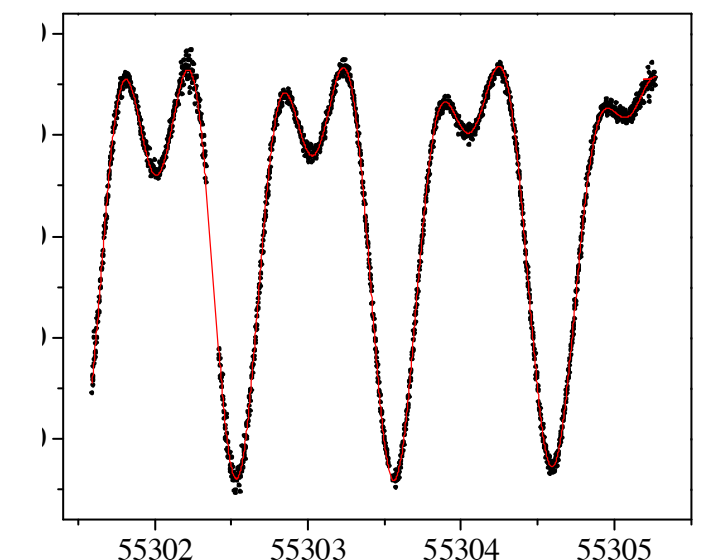
Riehle, Witte, Helmcke, Bordé PRL 67 177

Kasevich and Chu PRL 67 181

• Precision in atom interferometers

• BEC manipulation

2000s



• Inertial sensors (Earth, space) 2010s

• Large scale, quantum metrology

...

★ Progress in quantum engineering

★ Exciting projects, on Earth and in Space,
will lead the research of new Physics

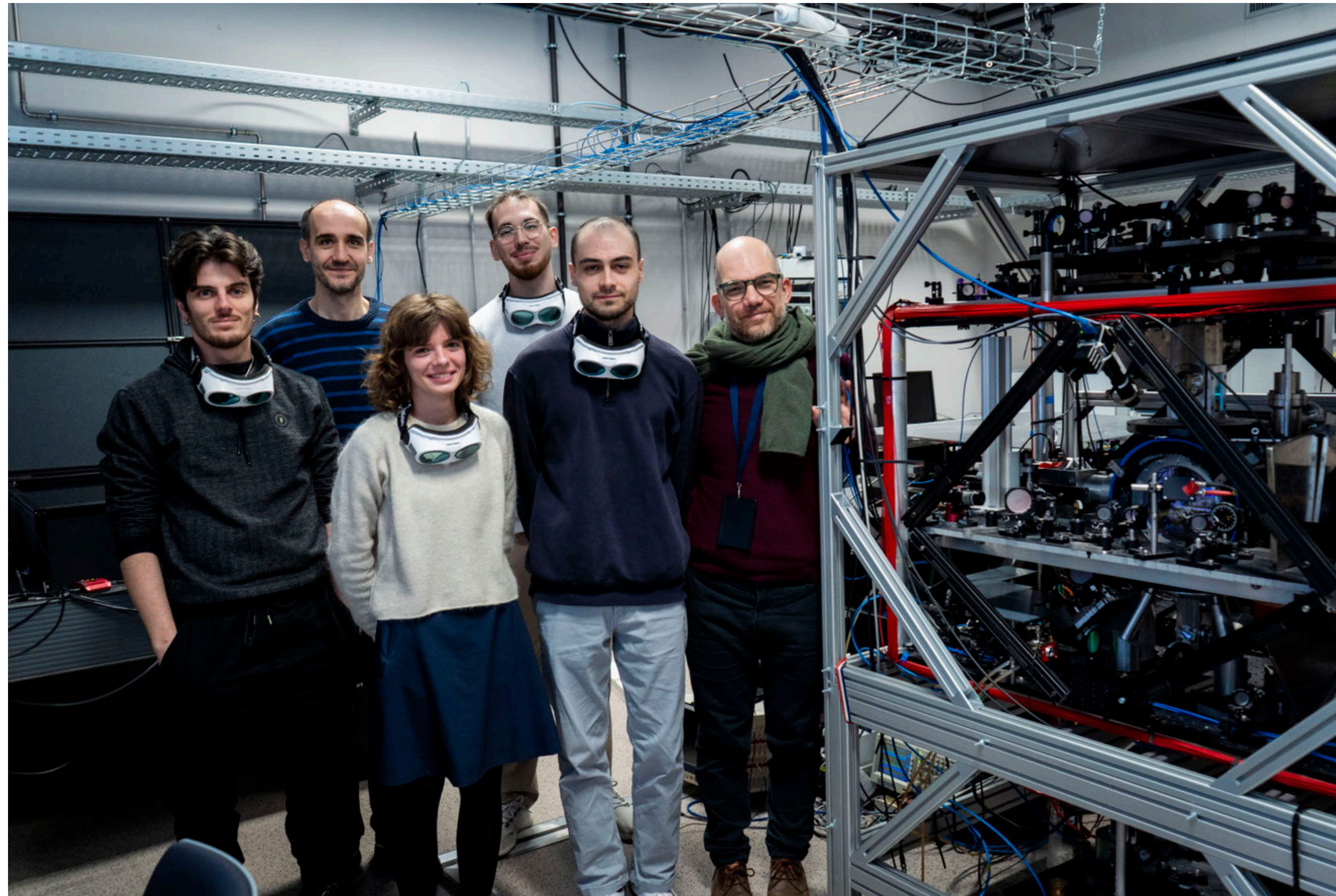
GW, New physics, Quantum sensing



A. Béguin
CR - LTE



T. Rodzinka
Q-Ctrl



B. Allard (MdC)

L. Calmels
soutenance 20 avril

S. Beldjoudi (PhD)

C. Bigard (PhD)

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Join us !

