Frequently repeated measurements of position and momentum of a quantum particle

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We present a model that focuses on the frequent measurement of a quantum particle's position and momentum, addressing the measurement problem in quantum mechanics. We propose an open system framework designed to describe a particle under interrogation by a grid of detectors uniformly distributed in phase space. We treat the particle as an open system coupled to the 'reservoir' of detectors. The evolution of the density operator ρ_S is assumed to be described by the Gorini-Kossakowski-Sudarshan-Lindblad equation:

$$\dot{\rho_S} = i[\rho_S, H_S] + \mathcal{L}_{relax}(\rho_S), \tag{1}$$

where $\mathcal{L}_{relax}$ is of the Lindblad form:

$$\mathcal{L}_{relax} = -\frac{1}{2}\sum_{\alpha}\left(C_\alpha^\dagger C_\alpha \rho_S + \rho_S C_\alpha^\dagger C_\alpha\right) + \sum_{\alpha} C_\alpha \rho_S C_\alpha^\dagger, \tag{2}$$

for which the jump operator C_α, corresponding for the measurement process, is proportional to the projector onto the coherent state $|\alpha\rangle$:

$$C_\alpha = \sqrt{\gamma}|\alpha\rangle\langle\alpha|. \tag{3}$$

Here γ is a coupling strength determining the characteristic detection frequency and α enumerates the pre-selected subset of coherent states corresponding to the positions of meters in the phase space.

This way the formalism incorporates a wavefunction collapse postulate, where after measurement, the system is assumed to be in one of the meter's states—a coherent state of a local quantum oscillator [1]. We provide trajectories of observables generated using the Wave Function Quantum Monte Carlo method as straightforward illustrative examples. Fig.1 shows a particular, single realization of the time evolution of the probability distribution of position of a free particle initially resting at $x = 0$. Discontinuities visible in figure, correspond to the moments where the meters "click" and the wavefunction is projected onto the state of the clicking detector.

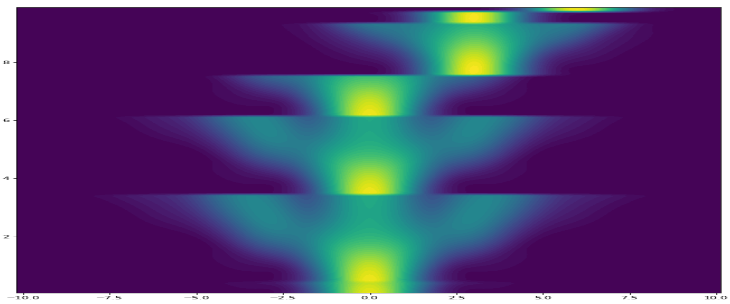

FIG. 1: Probability distribution in the position space as a function of time of a particle initially resting at $x = 0$. Jumps due to the interrogation by the meters result in discontinuities visible in figure.

The Zeno effect is predicted, particularly for a relatively sparse spatial grid of detectors.

In a limit of very frequent detection and densely distributed meters the quantum dynamics of monitored particles is equivalent to the classical stochastic process. In this case, the statistical characteristics of the monitored trajectories are analogous to those obtained in the classical dynamics of a particle undergoing Brownian motion [2].

[1] Filip Gampel and Mariusz Gajda, Continuous simultaneous measurement of position and momentum of a particle, Phys. Rev. A 107, 012420 (2023).

[2] Filip Gampel, Mariusz Gajda, On Repeated Measurements of a Quantum Particle in a Harmonic Potential, Acta Phys. Pol. A 143, S131 (2023)