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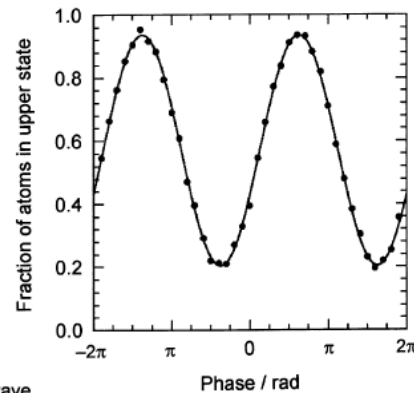
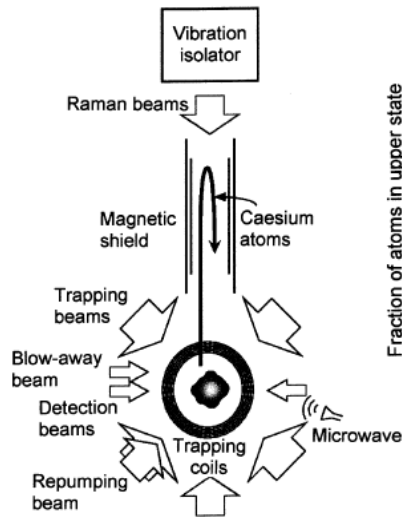
Differential Mach-Zehnder Interferometry with trapped Bose Einstein condensates

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LENS, University of Florence

9-13 September 2024, FOMO,
Crete, Greece



Atom interferometry with free falling atoms



A. Peters, K. Y. Chung, S. Chu, *Nature*,
400, 849-852 (1999)



Classical phase !

$$\phi \sim \ell/\lambda, = \frac{1}{2} g T^2/\lambda$$

High precision measurements

- Gravity, inertial forces, fundamental constants

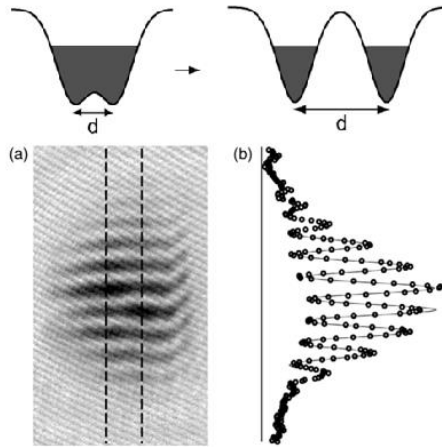
Fundamental physics tests

- Equivalence principle
- GW detection
- Dark matter detection

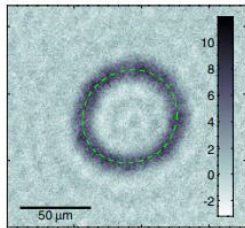
Main limitation

- Low spatial resolution
- Limited interrogation time

Trapped / Guided Atom Interferometers



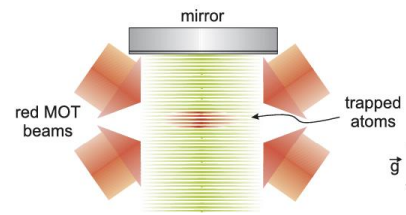
Y. Shin et al. PRL 92 050405 (2004)



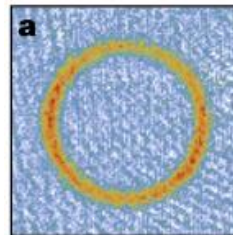
Y. Guo, et al. PRL, 124, 025301 (2020)

Main advantages

- High spatial resolution $\sim \mu\text{m}$
- Arbitrarily long interrogation times T



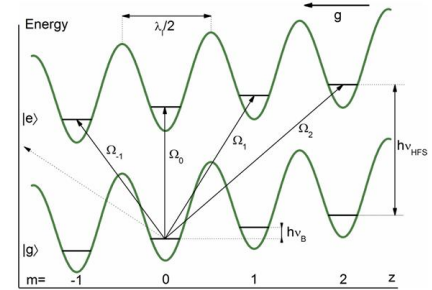
G. Ferrari et al. PRL 97 060402 (2006)



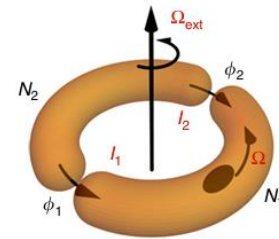
Saurabh Pandey, et al. Nature, 570, 205 (2019)

Main problems

- Control the depth and shape of the trapping potential
- Interactions in high density atomic samples

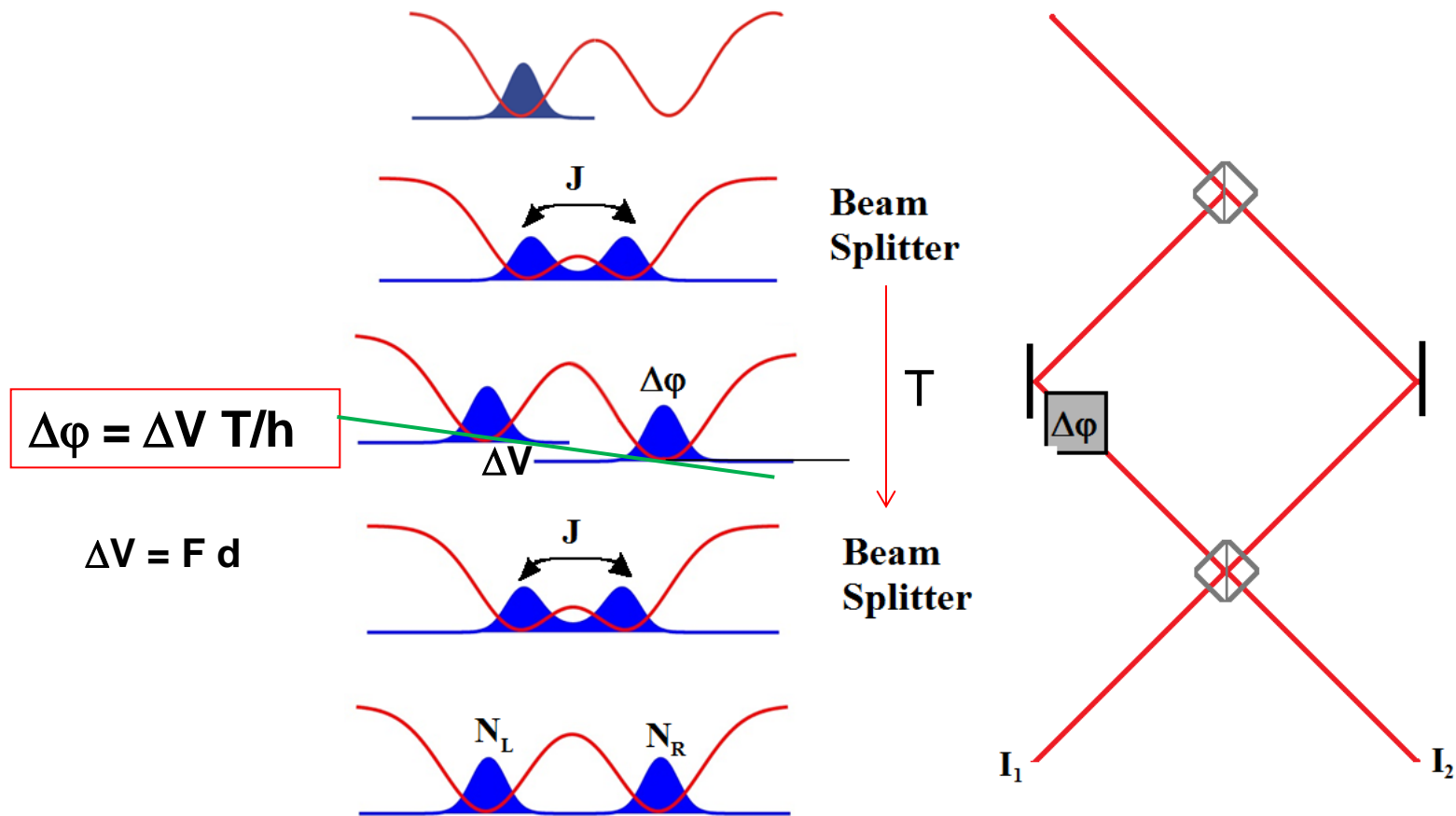


B. Pelle et al. PRA 87 023601 (2013)



C. Ryu et al. Nat. Comm. 11 3338 (2020)

Mach Zender interferometry with BEC in a double-well



- Ultimate space resolution
- Spatial coherence of a BEC allows large separation d
- Exploitation of quantum entanglement
- Creation of Maximally entangled quantum states like NOON

Second problem: interaction induced decoherence

1	H	
3	Li	4 Be
11	Na	12 Mg
19	K	20 Ca
37	Rb	38 Sr
55	Cs	56 Ba
87	Fr	88 Ra

^{41}K boson, condensed

G. Modugno et al, Science **294**, 1320 (2001)

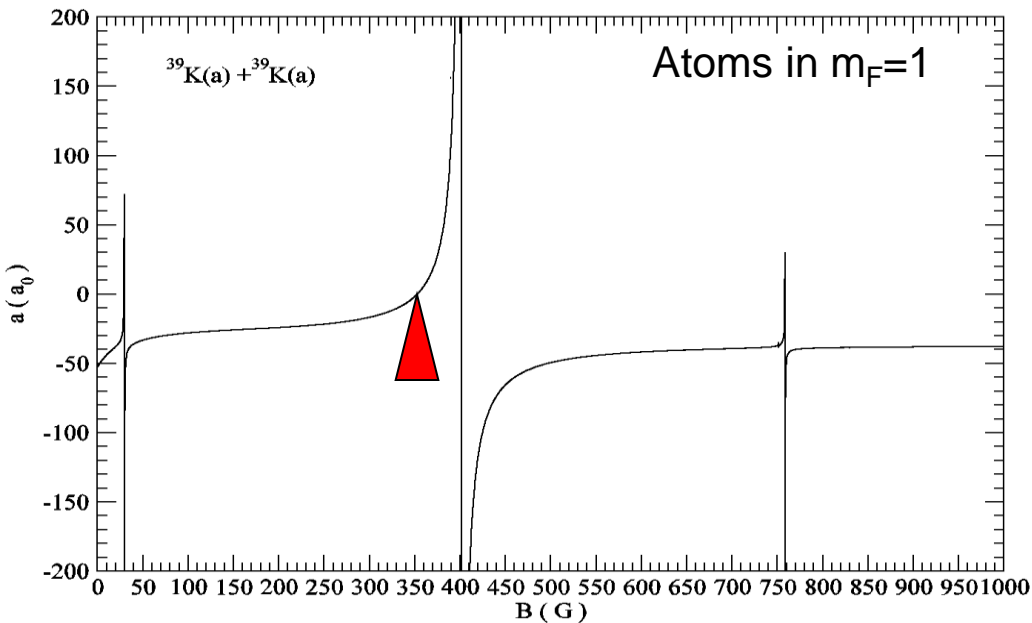
^{40}K fermion

De Sarlo et al, PRA **75**, 022715 (2007)

^{39}K boson, $a = -33a_0$

D'Errico et al. New J. Phys. **9**, 223 (2007)

Roati et al. PRL **99**, 010403 (2007)



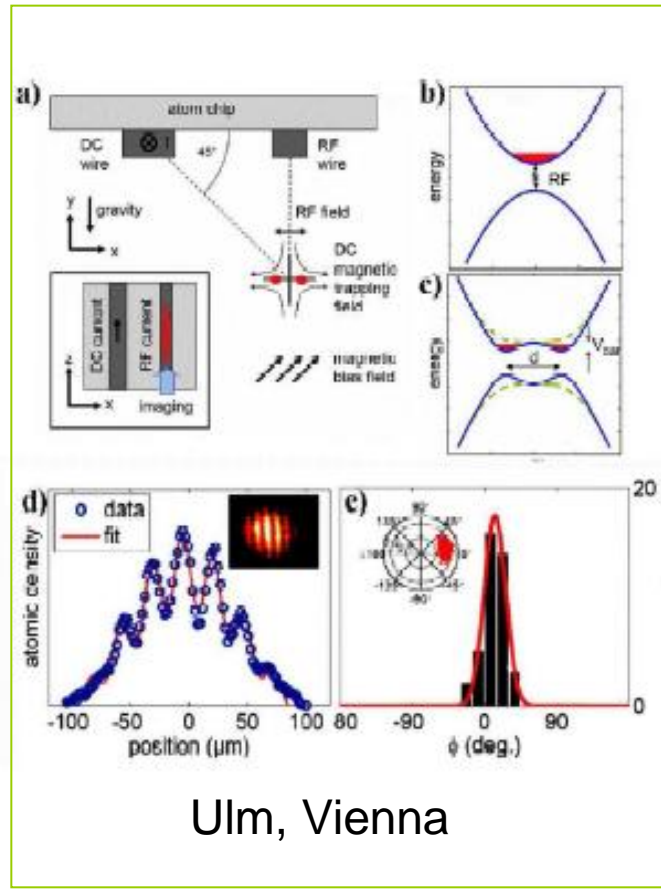
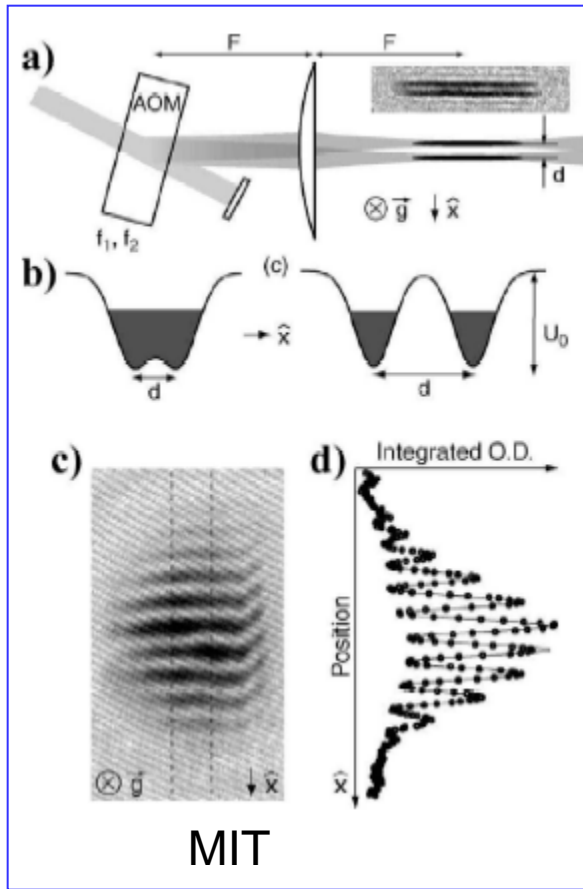
$$a(B) = a_{bg} \left(1 - \frac{\Delta}{B - B_0} \right)$$

$$a(B) \approx \frac{a_{bg}}{\Delta} (B - B_{ZC}) \rightarrow \frac{\Delta a}{\Delta B} = 0.6 a_0 / \text{G}$$

Very high degree of tunability !!

Atom interferometry on the zero crossing of the broad Feshbach resonance

First problem: how to control depth and shape of the trap?



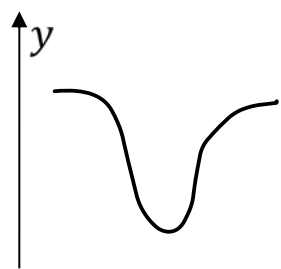
Y. Shin, et al., *Phys. Rev. Lett.* **92**, 050405 (2004)

J. Esteve, et al., *Nature* **455**, 1216 (2008).

Florian Baumgärtner, et al. *Phys. Rev. Lett.* **105**, 243003 (2010)

T. Barreda, et al., *Nat. Comm.* **4**, 2077 (2013).

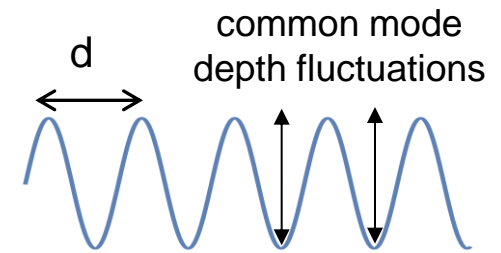
$\Delta y \sim 1 \mu\text{m}$
 $\Delta V \sim 1 \text{ kHz}$



$U \sim 500 \text{ nK}$
 $V \sim h 10 \text{ kHz}$



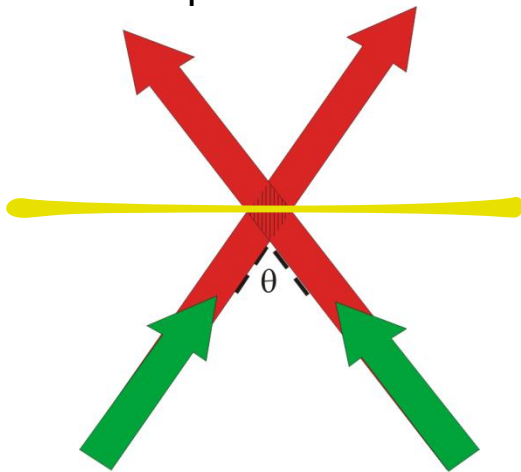
Optical lattices !!



$\Delta V = F_{\text{ext}} d$

Double-well potential: our first idea

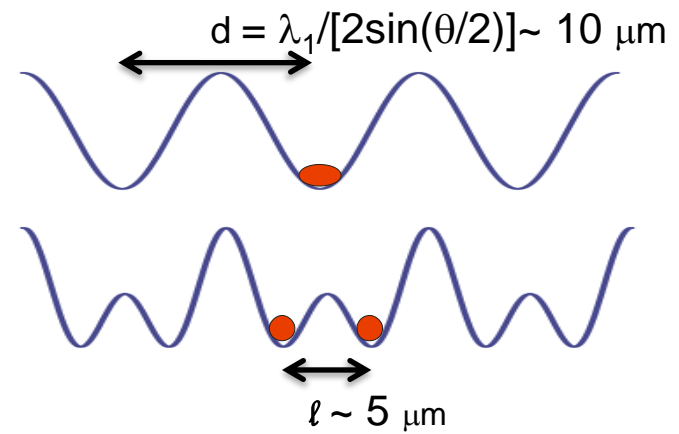
• Super-Lattice



$$\lambda_1 = 1064 \text{ nm}$$

Radial confinement

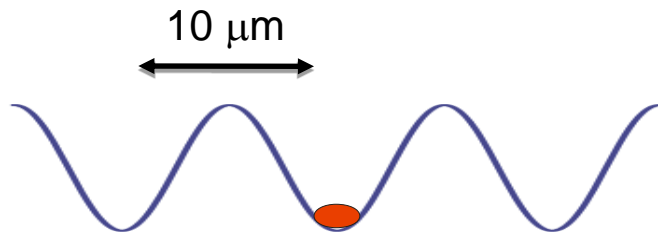
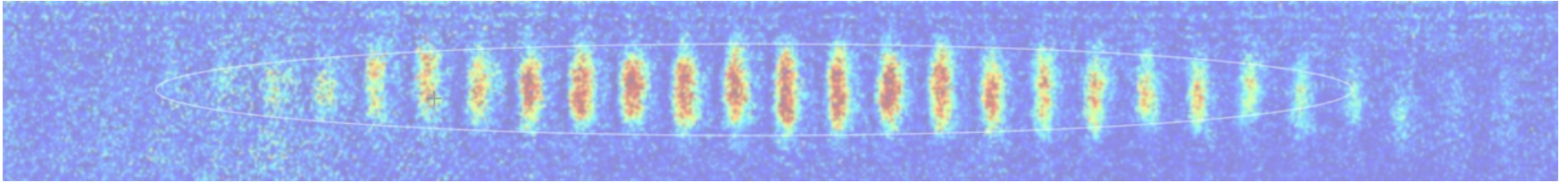
$$\lambda_2 = \lambda_1 / 2 = 532 \text{ nm}$$



J. Sebby-Strabley, et al., Phys. Rev. A **73**, 033605 2006.

Primary lattice

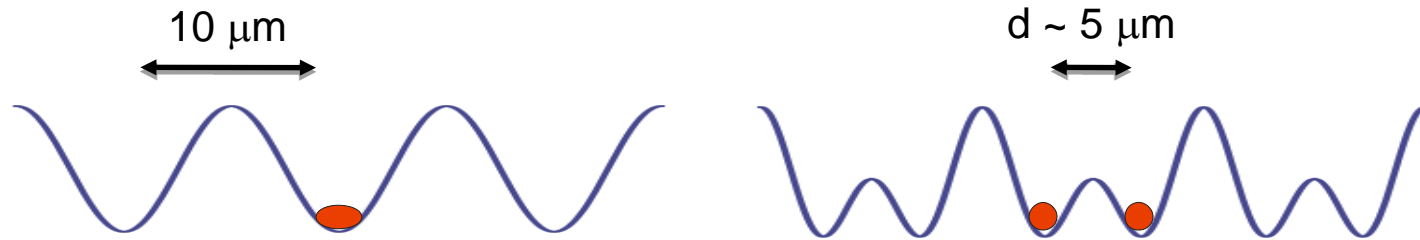
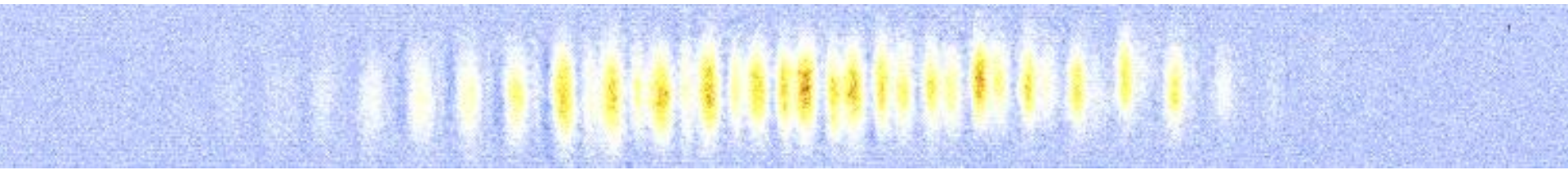
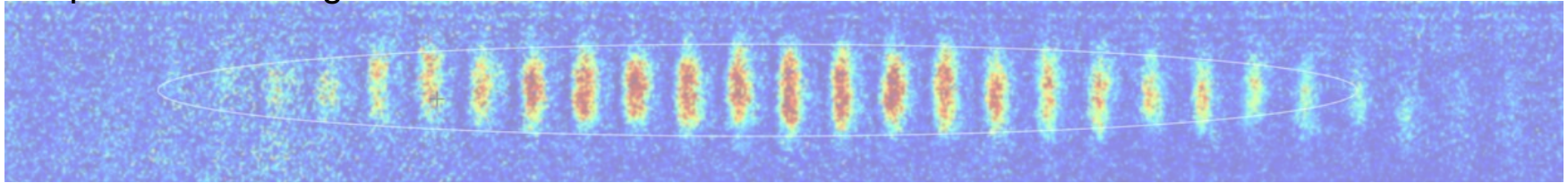
- atoms loaded into IR lattice with $N \sim 1000-4000$ per site



In situ detection: microscope objective with $1.5 \mu\text{m}$ resolution

Primary lattice + Secondary lattice

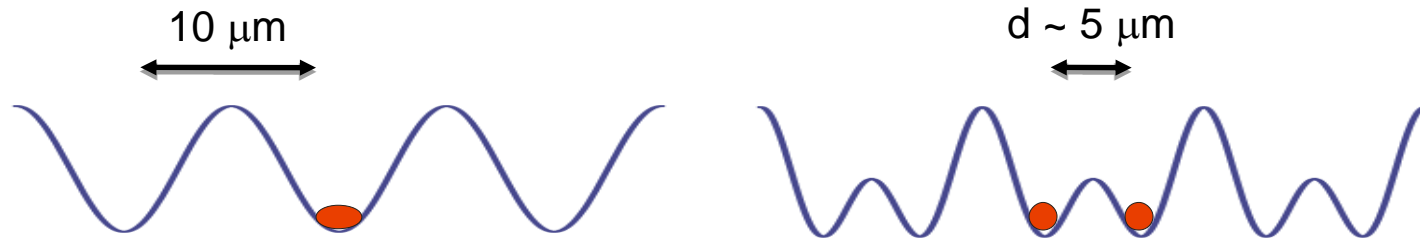
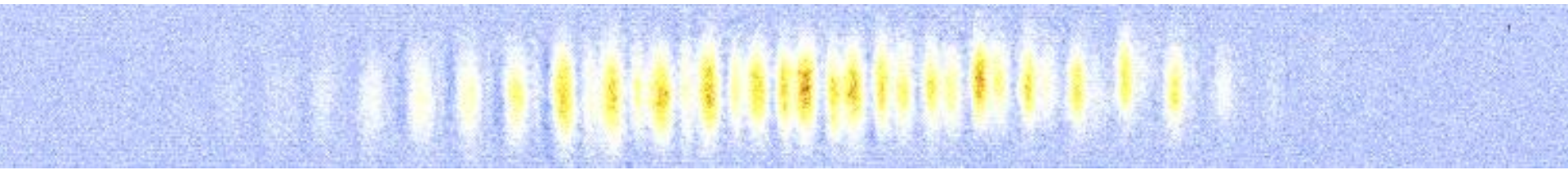
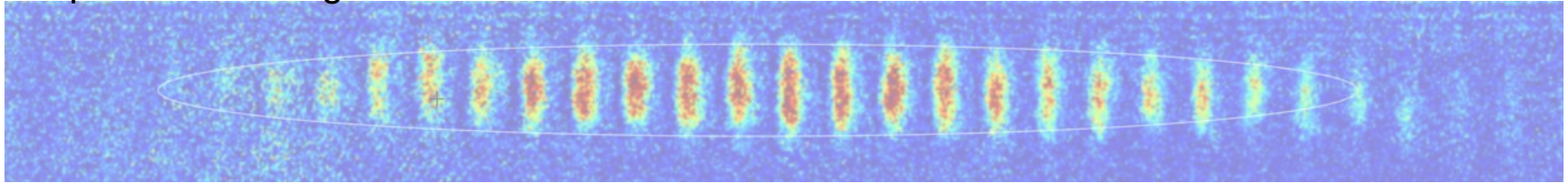
- atoms loaded into IR lattice with $N \sim 1000-2000$ per site
- split atoms with green barrier



In situ detection: microscope objective with $1.5 \mu\text{m}$ resolution

Primary lattice + secondary lattice

- atoms loaded into IR lattice with $N \sim 1000-2000$ per site
- split atoms with green barrier



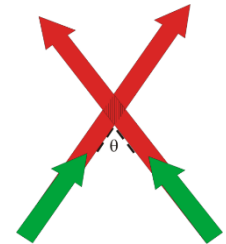
Our control of the barrier is below 1 nm, but we can center it only in one interferometer.

How to improve double well interferometry

- A large spacing superlattice is needed
- Lattices must be created retro-reflecting the laser light on a mirror to minimize sensitivity to misalignments

$$d = \frac{\lambda}{2\sin(\theta/2)}$$

$$\Delta d = \lambda \frac{\cos(\theta/2)}{4\sin(\theta/2)^2} \Delta\theta$$

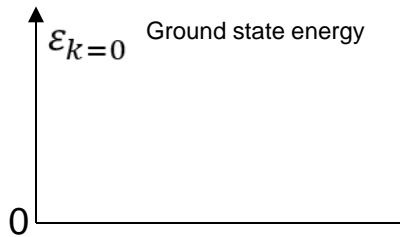


Beat-Note Lattice : the idea

Free particle

$$\Psi_k(x) = \sqrt{\frac{1}{L}} e^{ikx}$$

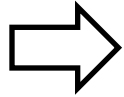
$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}$$



Beat-Note Lattice : the idea

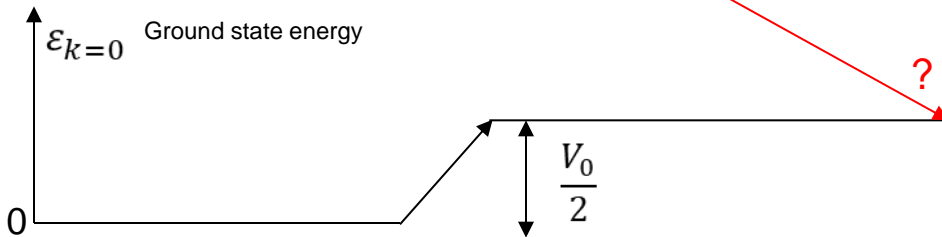
Free particle

$$\psi_k(x) = \sqrt{\frac{1}{L}} e^{ikx}$$
$$\epsilon_k = \frac{\hbar^2 k^2}{2m}$$



If an optical lattice is switched on

$$V(x) = V_0 \cos^2(k_L x) = \frac{V_0}{4} (e^{i2k_L x} + e^{-i2k_L x} + 2)$$

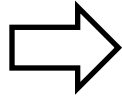


Beat-Note Lattice : the idea

Free particle

$$\psi_k(x) = \sqrt{\frac{1}{L}} e^{ikx}$$

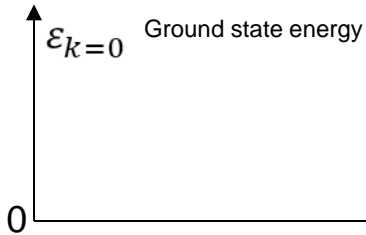
$$\epsilon_k = \frac{\hbar^2 k^2}{2m}$$



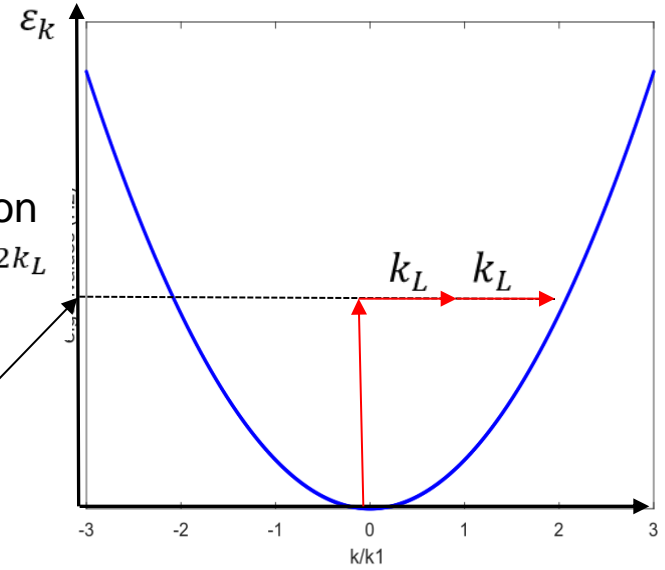
If an optical lattice is switched on

$$V(x) = V_0 \cos^2(k_L x) = \frac{V_0}{4} (e^{i2k_L x} + e^{-i2k_L x} + 2)$$

$\epsilon_{\pm 2k_L}$



$$\frac{V_0}{2}$$

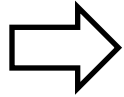


Beat-Note Lattice : the idea

Free particle

$$\psi_k(x) = \sqrt{\frac{1}{L}} e^{ikx}$$

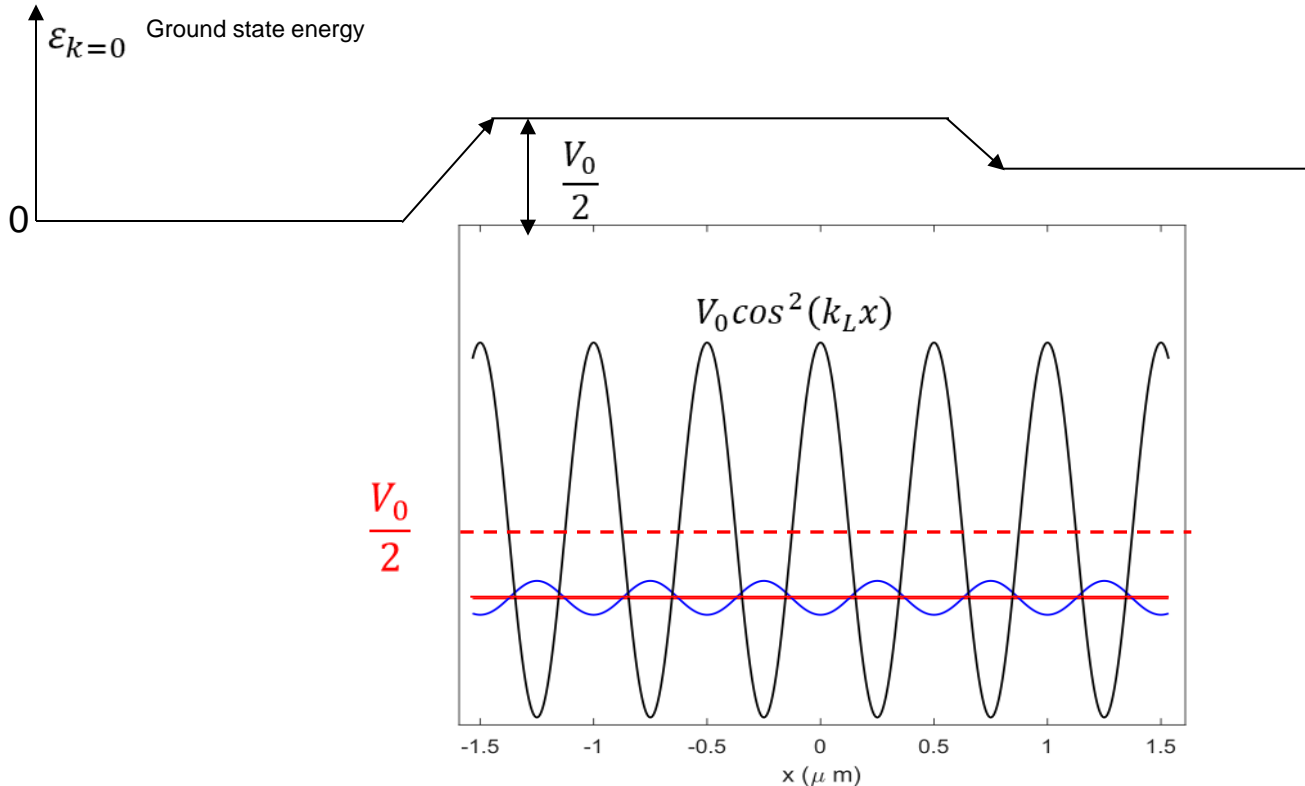
$$\epsilon_k = \frac{\hbar^2 k^2}{2m}$$



If an optical lattice is switched on

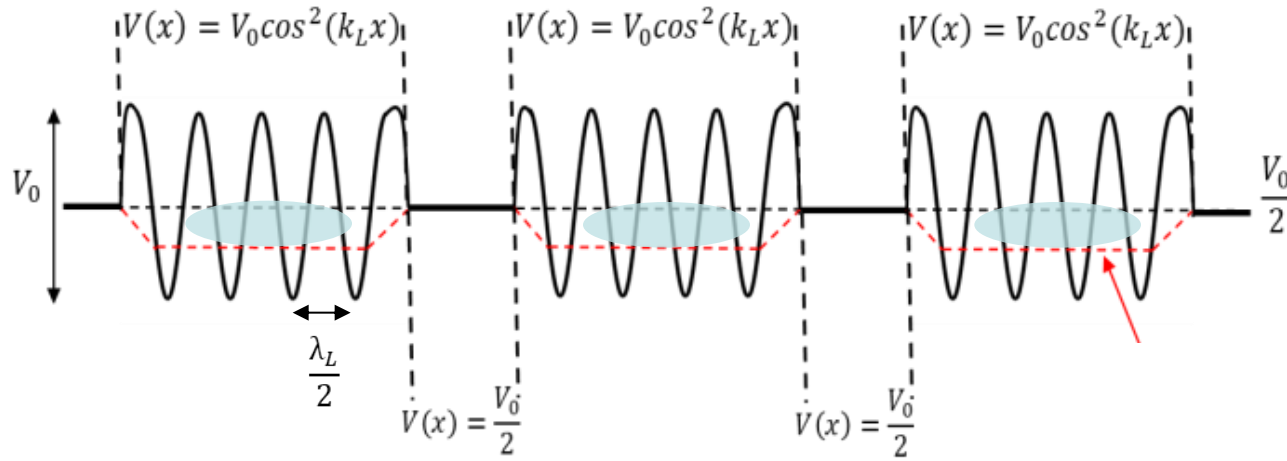
$$V(x) = V_0 \cos^2(k_L x) = \frac{V_0}{4} (e^{i2k_L x} + e^{-i2k_L x} + 2)$$

$$\epsilon_{k=0} = \frac{V_0}{2} - \frac{V_0^2}{32E_R} < \frac{V_0}{2}$$

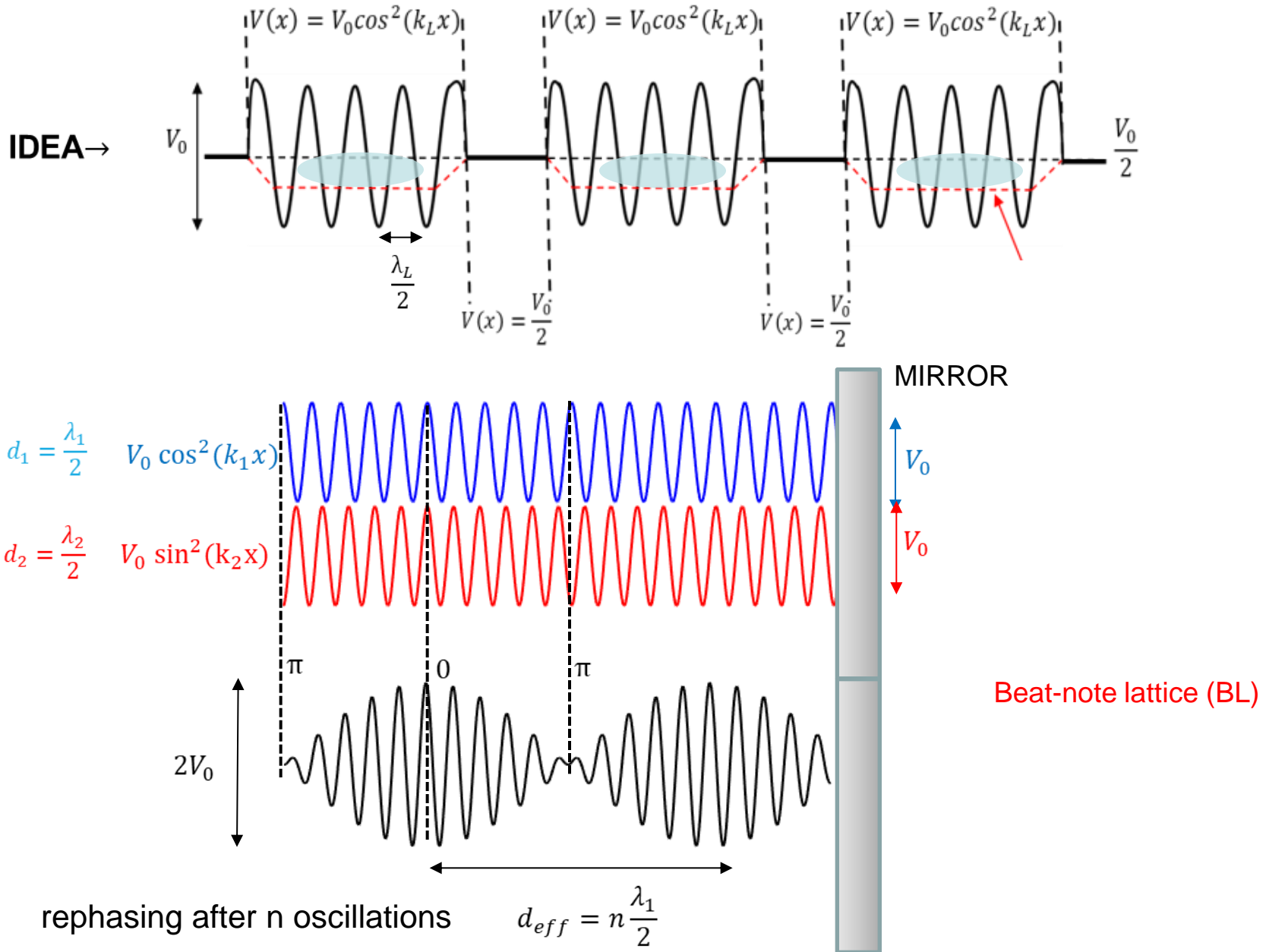


BL : the idea

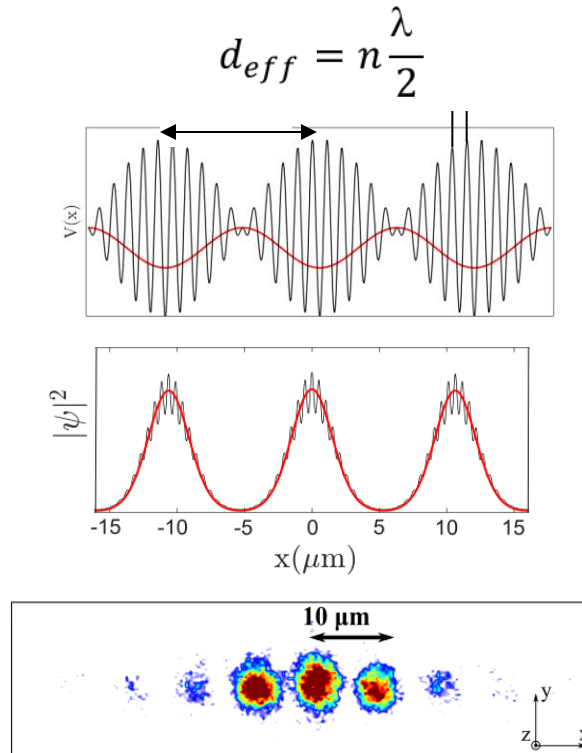
IDEA →



BL : the idea



Beat-Note Superlattice @ 10 μm



Effective potential for $V_0 < E_{R+}$

$$V_{eff}(x) = -\frac{V_0^2}{8E_{R+}} \cos^2(k_-x)$$

working conditions

$$(n+1)\lambda_1 = n\lambda_2$$

$$\lambda_2 = 1013.7 \text{ nm}$$

$$\lambda_1 = 1064.5 \text{ nm}$$

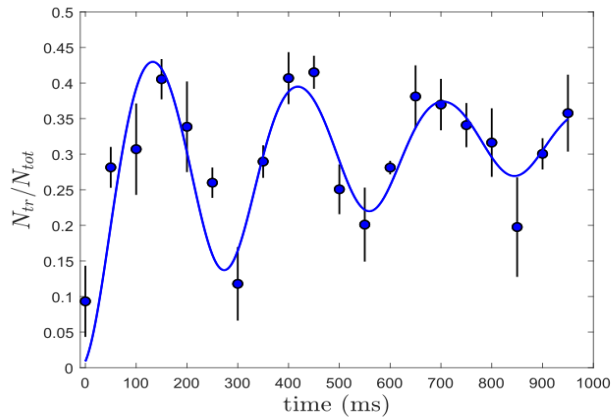
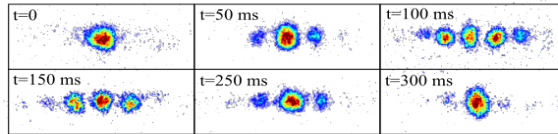
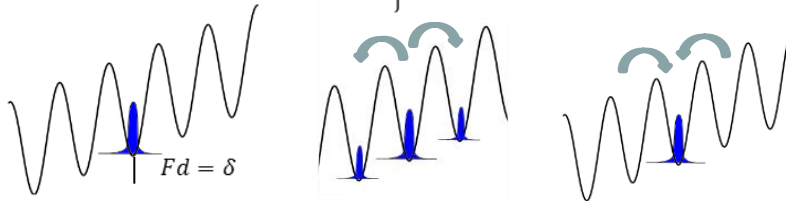
$$n=20$$

$$d_+ \cong 0.5 \mu\text{m}$$

$$d_- \cong 10.6 \mu\text{m}$$

Bloch Oscillations in a BNSL

$d \cong 10.6 \mu\text{m}$



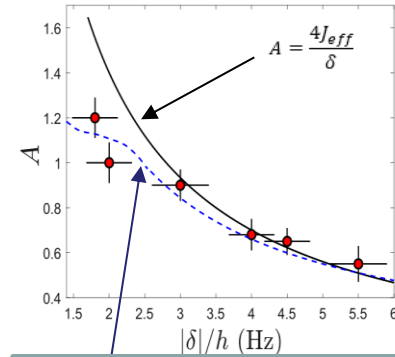
No interactions

$N \cong 5 * 10^3$ atoms
 $V_0 \cong \sim 0.3 E_R+$
 Force $\sim 3 * 10^{-4} g$
 $\nu = 3$ Hz

Bloch period

$$\tau_B = h/\delta = h/ Fd$$

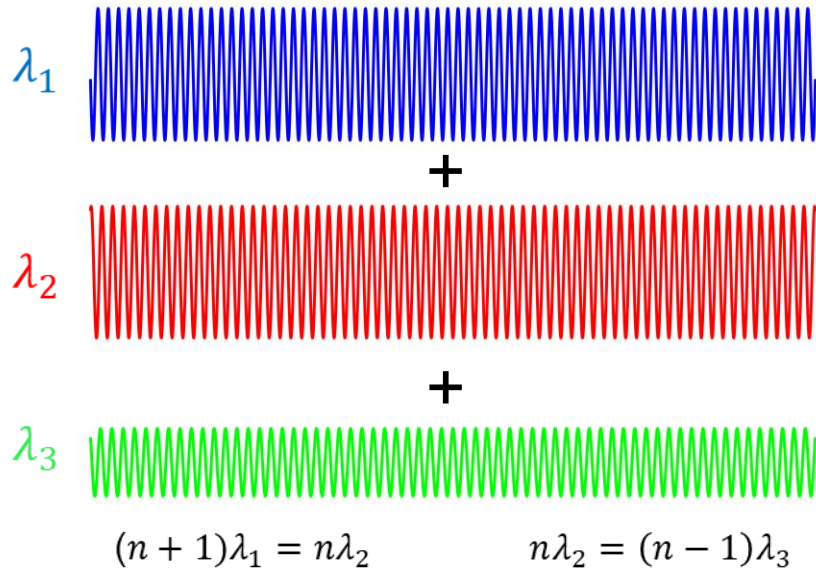
1 second coherence



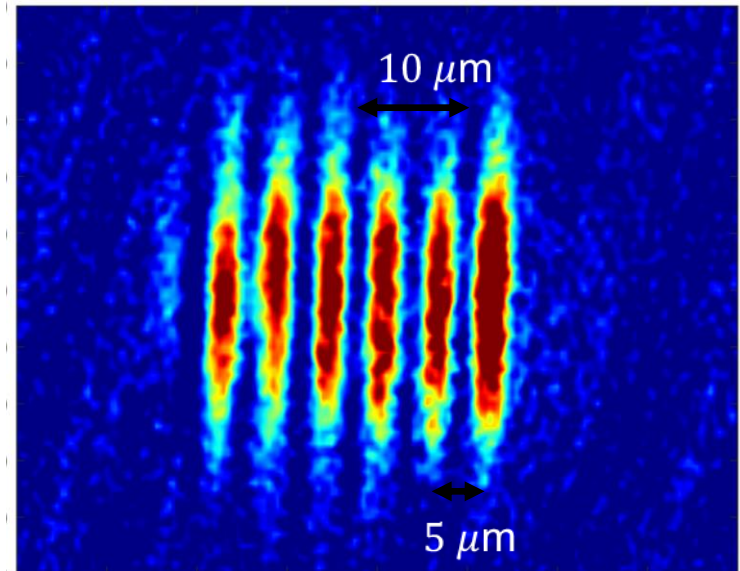
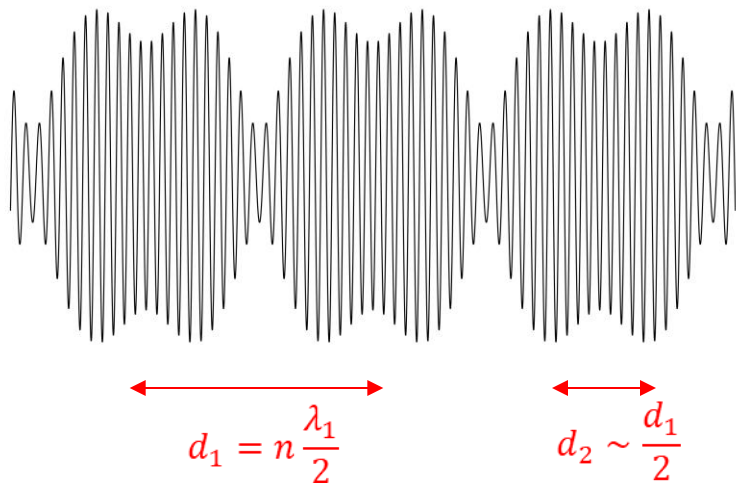
numerical simulations where a longitudinal harmonic potential of 1.5 Hz is included

L. Masi, et al. Phys. Rev. Lett. **127**, 020601 –2021

Array of DWs using Beat-note Superlattices

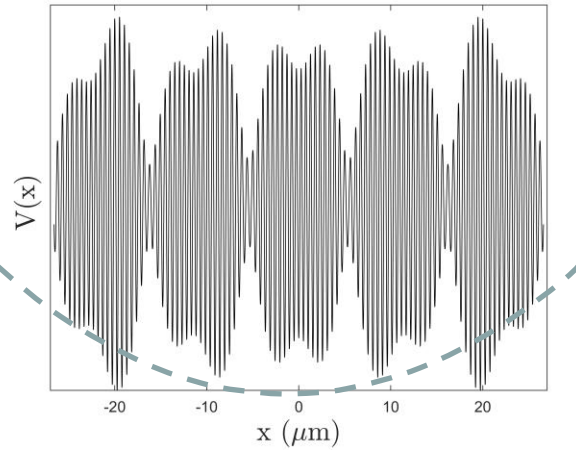


$$\begin{cases} \lambda_1 = 1013 \text{ nm} \\ \lambda_2 = 1064 \text{ nm} \\ \lambda_3 = 1120 \text{ nm} \end{cases} \longrightarrow \begin{cases} d_1 \sim 10 \mu\text{m} \\ d_2 \sim 5 \mu\text{m} \end{cases}$$

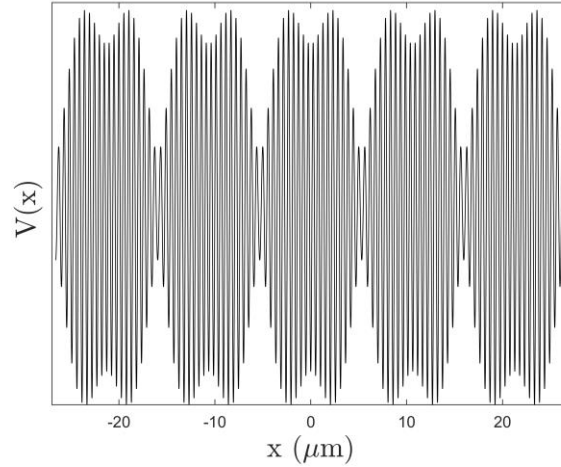


Array of DWs using Beat-note Superlattices

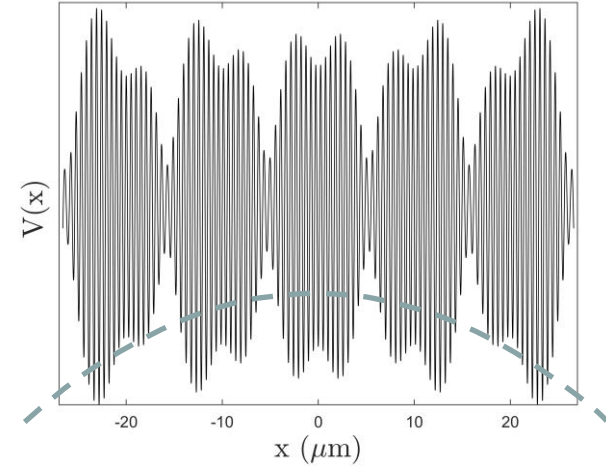
$\lambda_3 - \varepsilon$



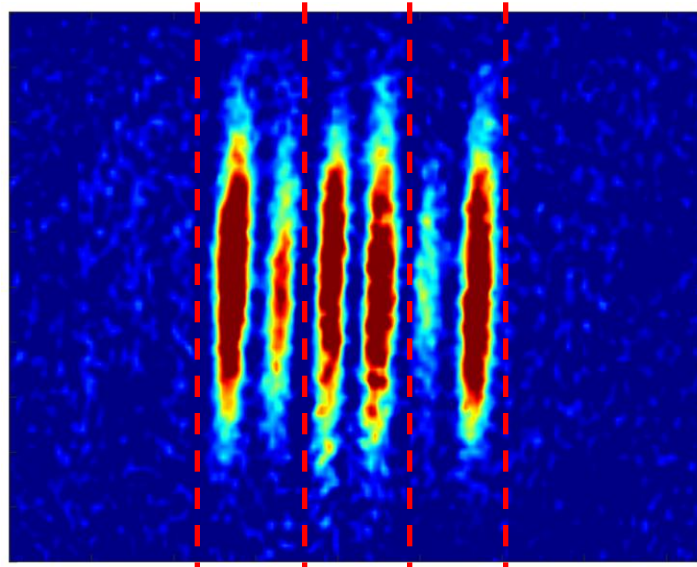
λ_3



$\lambda_3 + \varepsilon$



The two BNSL with $d=10 \mu\text{m}$ are not in counterphase anymore and they have different periodicities

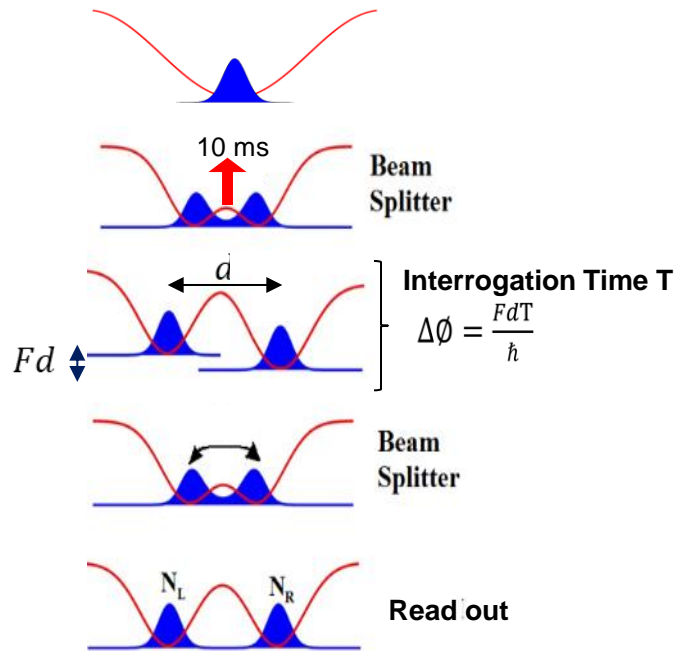


Some numbers:

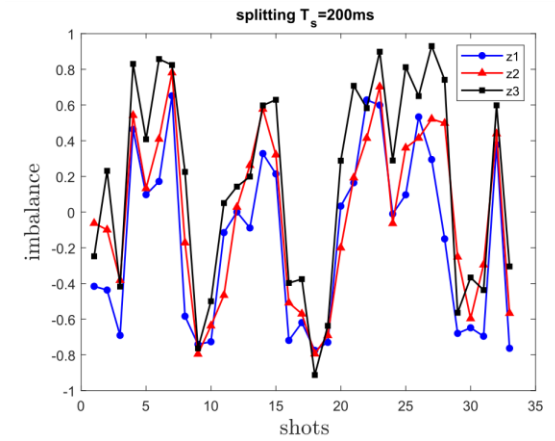
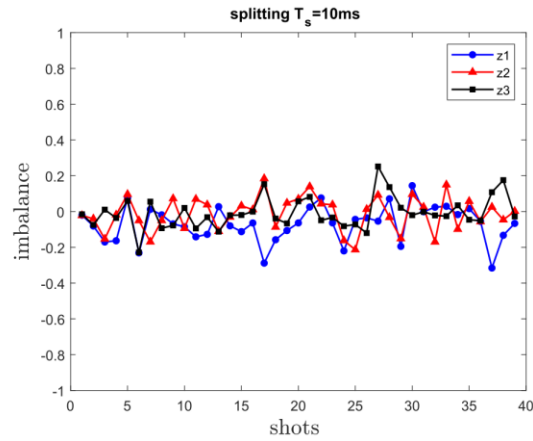
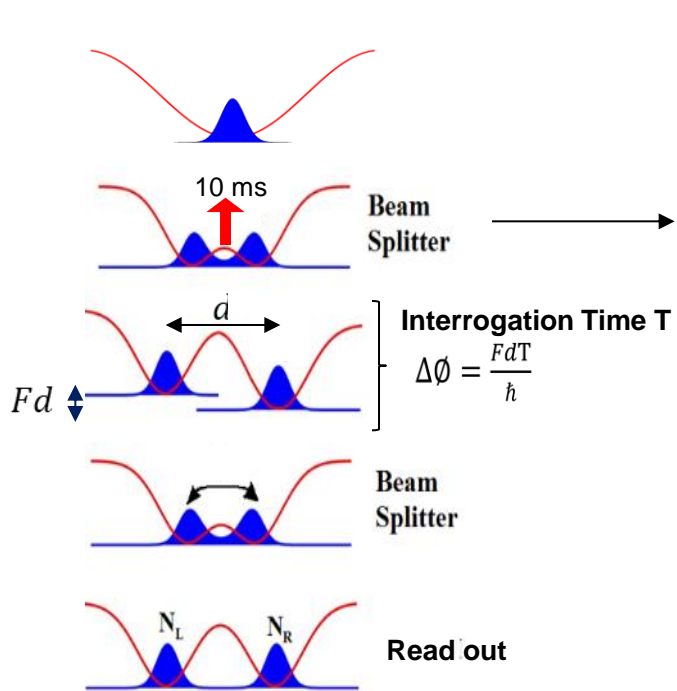
$$\Delta\lambda \sim 0,01 \text{ nm}$$

$$\Delta E_{i,i+1} \sim 2 \text{ Hz}$$

Mach-Zehender

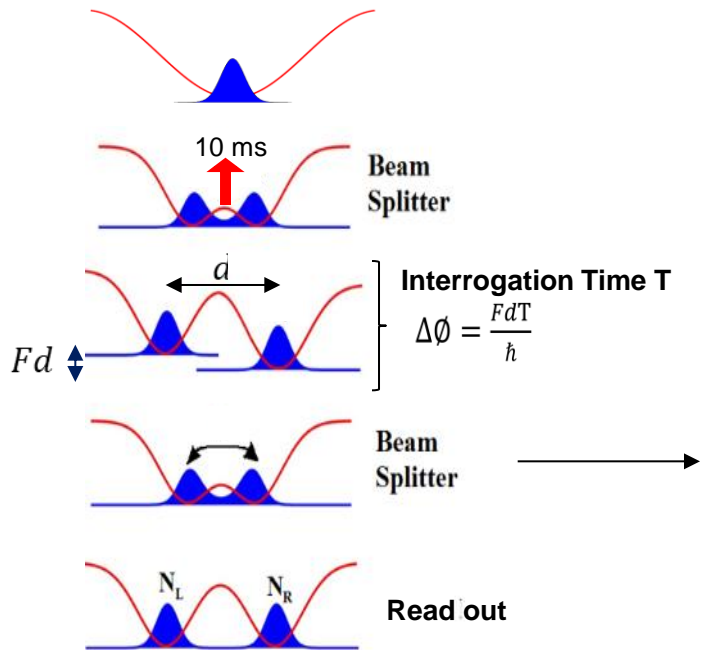


How to measure the interferometric phase ?

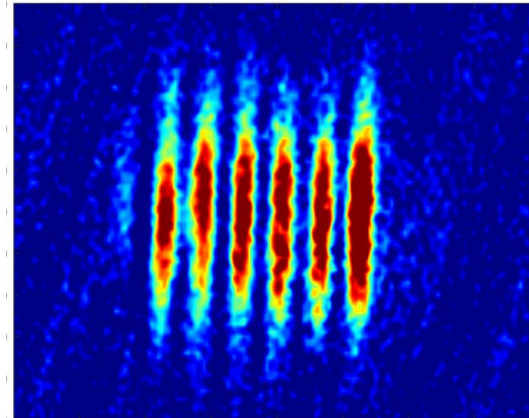


$$Z = \frac{N_L - N_R}{N_L + N_R}$$

How to measure the interferometric phase ?



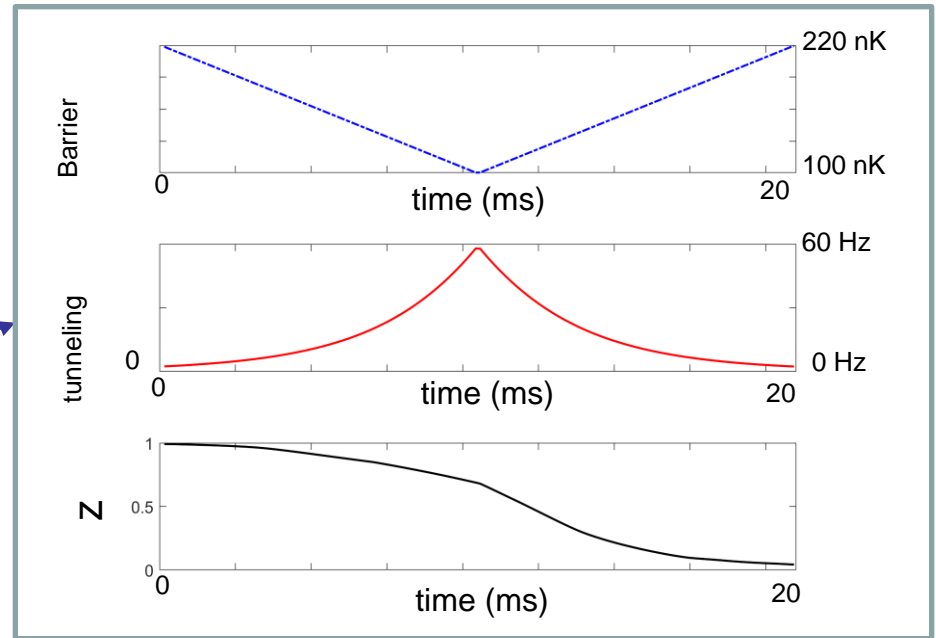
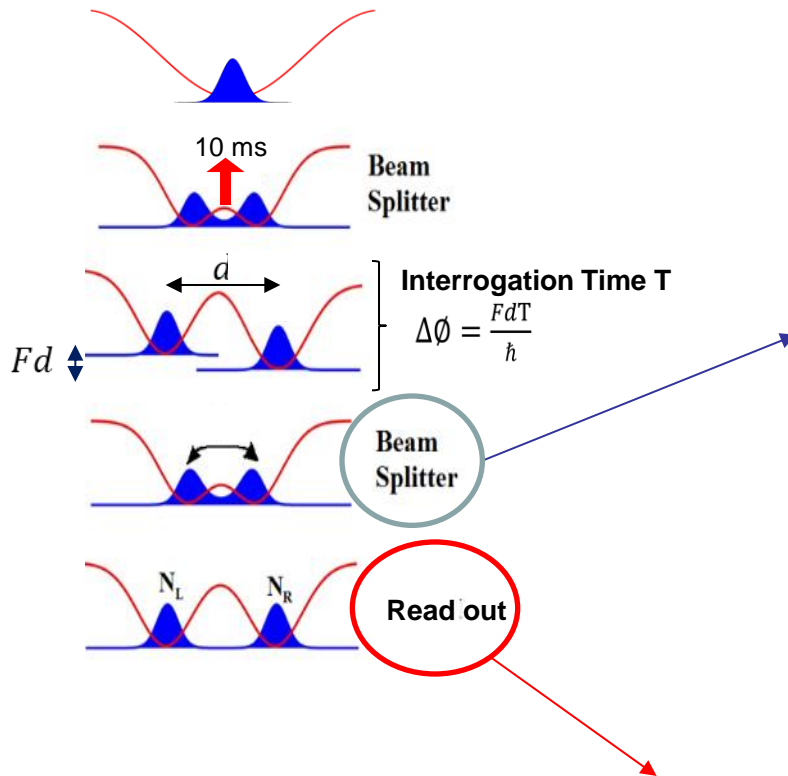
?



Interference after free expansion not possible !!

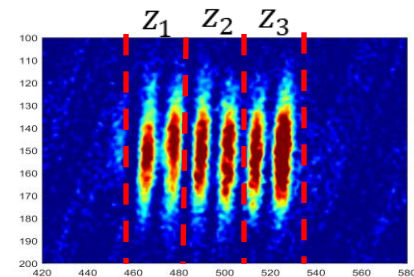
T. Berrada et al., Integrated Mach Zehnder Interferometer for Bose Einstein Condensates, Nature Comm. 4, 2077, (2013)

Second Beam Splitter



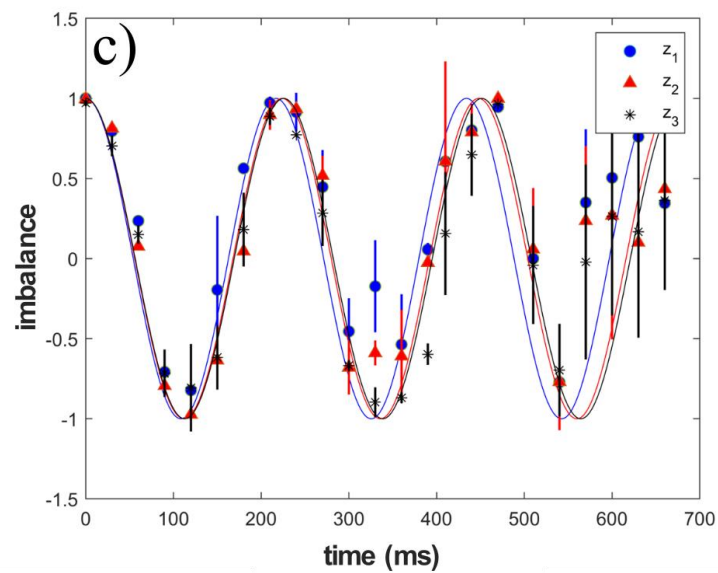
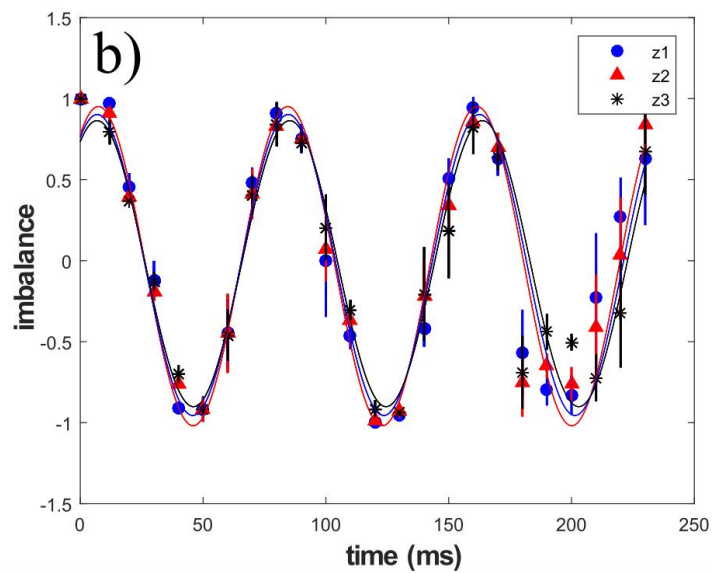
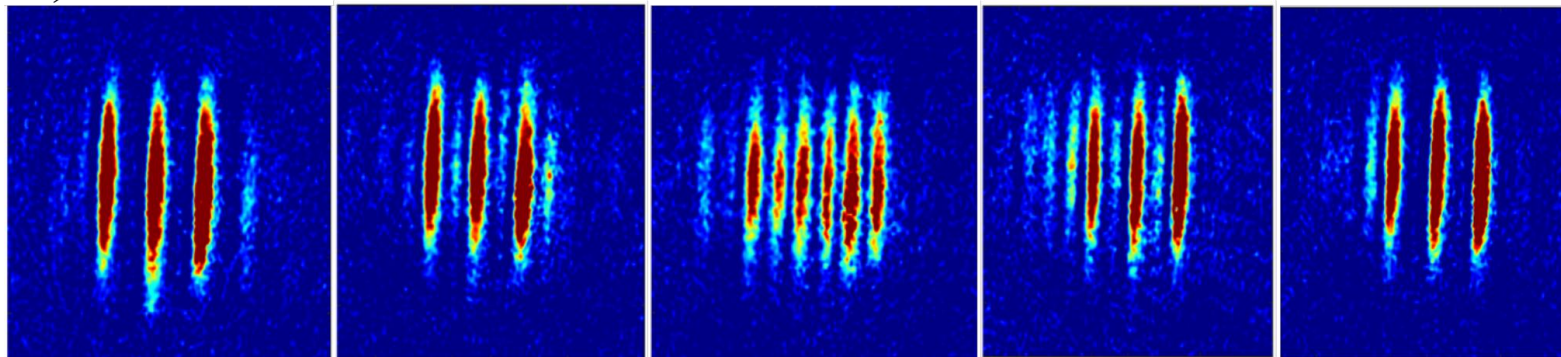
Atomic Imbalance

$$Z = \frac{N_L - N_R}{N_L + N_R}$$



Rabi oscillations

a)

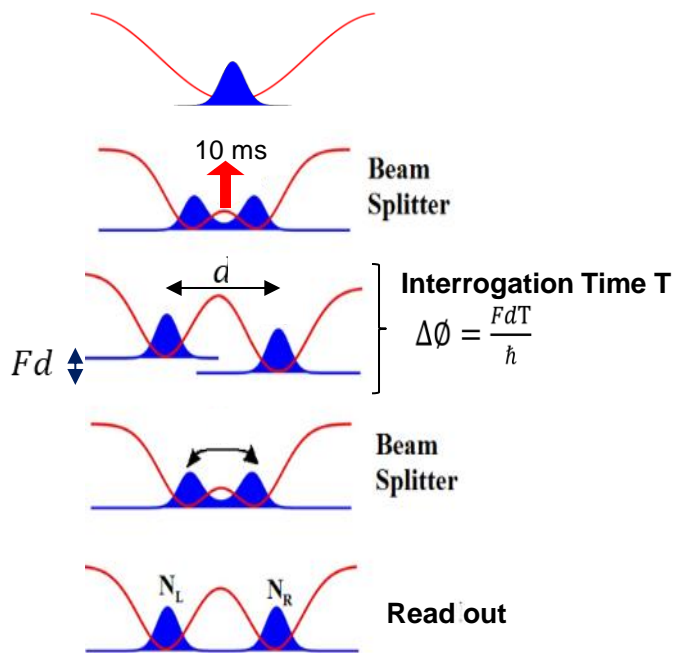


$$z = \frac{N_L - N_R}{N_L + N_R}$$

$J/h \sim 13 \text{ Hz}$

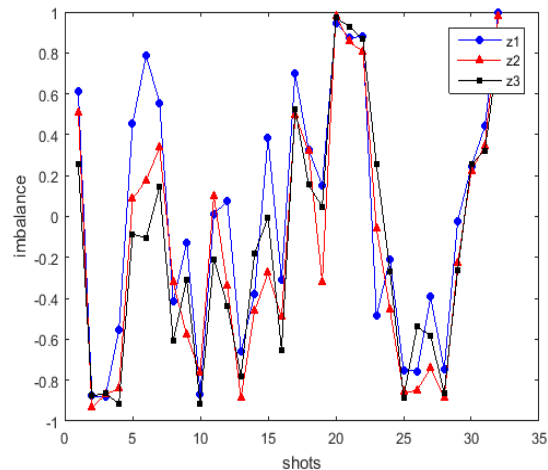
$J/h \sim 4 \text{ Hz}$

Interferometric sequence with 3 DWs



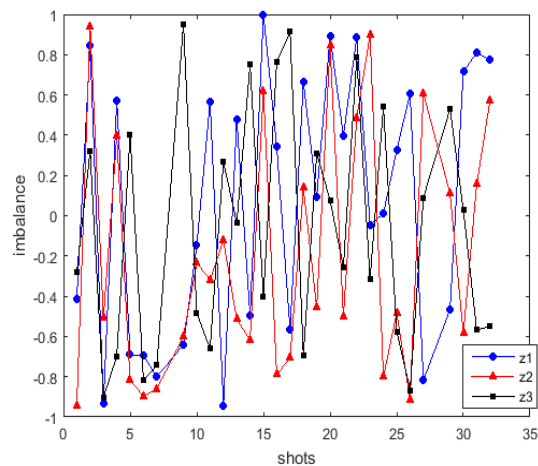
T=20 ms

Correlated output

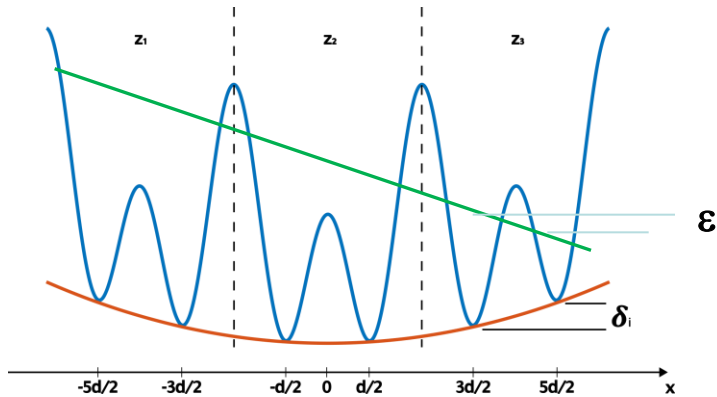


T=100 ms

Lost the correlations ?



Interferometric sequence with 3 DWs



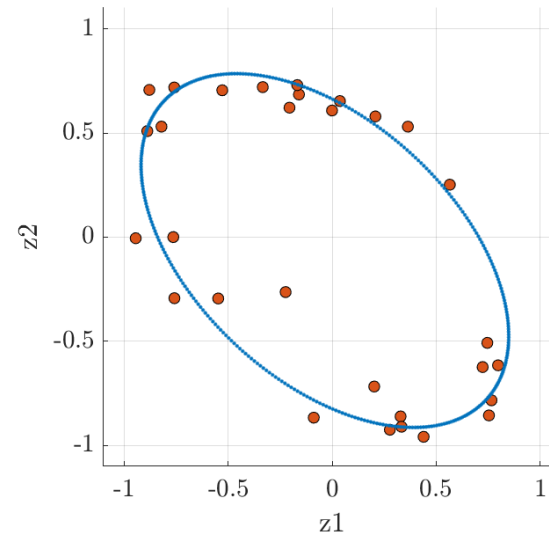
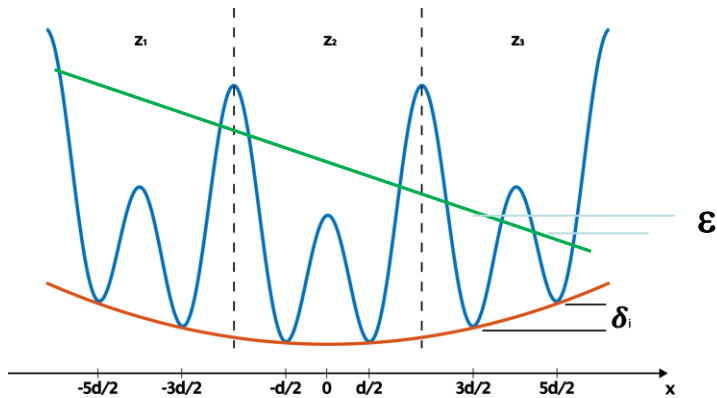
$$z_1 = \sin(\varphi_1) = \sin(-\delta T + \varepsilon T)$$

$$z_2 = \sin(\varphi_2) = \sin(\varepsilon T)$$

$$z_3 = \sin(\varphi_3) = \sin(\delta T + \varepsilon T)$$

Analysis on 2 DWs

The noise ε distributes the data on an ellipse with an eccentricity that depends on δ



We can measure
 $\varphi_2 - \varphi_1 = \delta T$

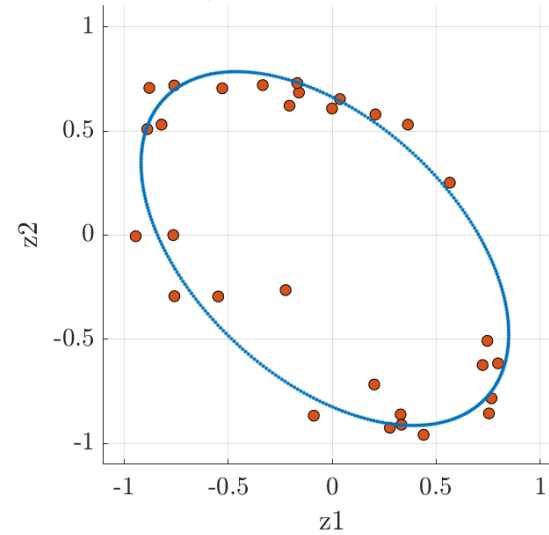
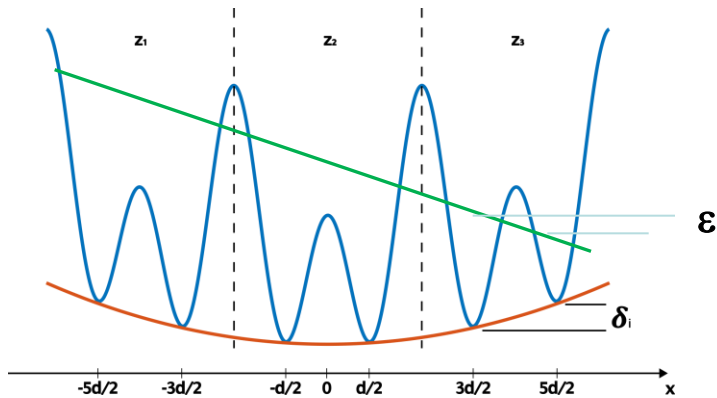
$$z_1 = \sin(\varphi_1) = \sin(-\delta T + \varepsilon T)$$

$$z_2 = \sin(\varphi_2) = \sin(\varepsilon T)$$

$$z_3 = \sin(\varphi_3) = \sin(\delta T + \varepsilon T)$$

Gradiometer

The noise ε distributes the data on an ellipse with an eccentricity that depends on δ

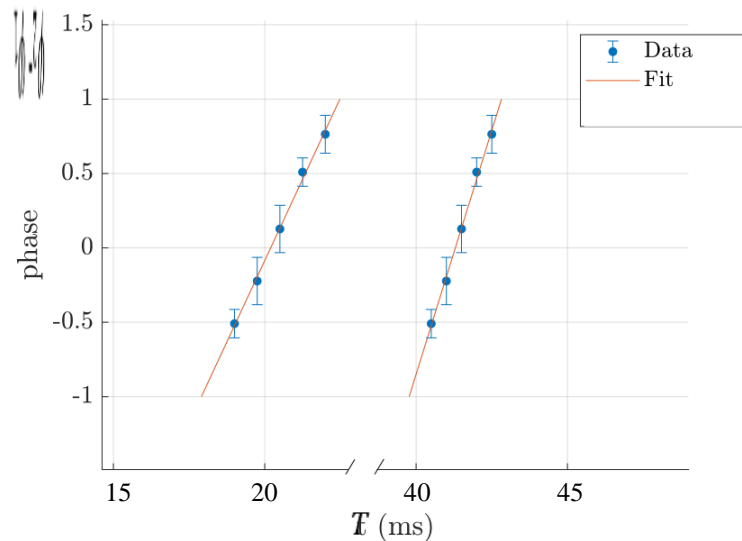


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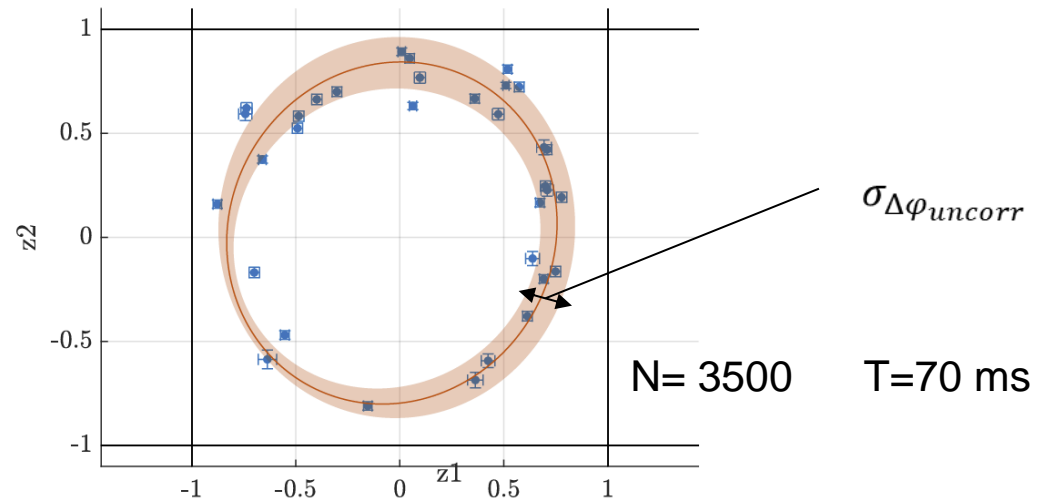


We have realized a trapped atom GRADIOTEMER

Decoherence of the differential phase

$$z_1 = \sin(\varphi_1) = \sin(-\delta T + \varepsilon T)$$

$$z_2 = \sin(\varphi_2) = \sin(\varepsilon T + \Delta\varphi_{uncorr})$$

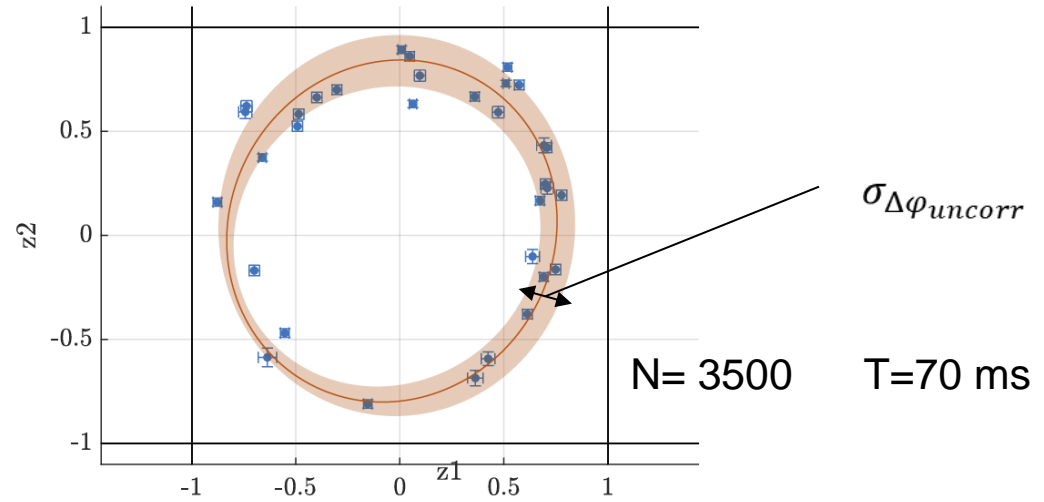


Decoherence of the differential phase

$$z_1 = \sin(\varphi_1) = \sin(-\delta T + \varepsilon T)$$

$$z_2 = \sin(\varphi_2) = \sin(\varepsilon T + \Delta\varphi_{uncorr})$$

$$\Delta\varphi_{uncorr} \propto \Delta(E_L - E_R)T \propto \Delta Z a_S T$$

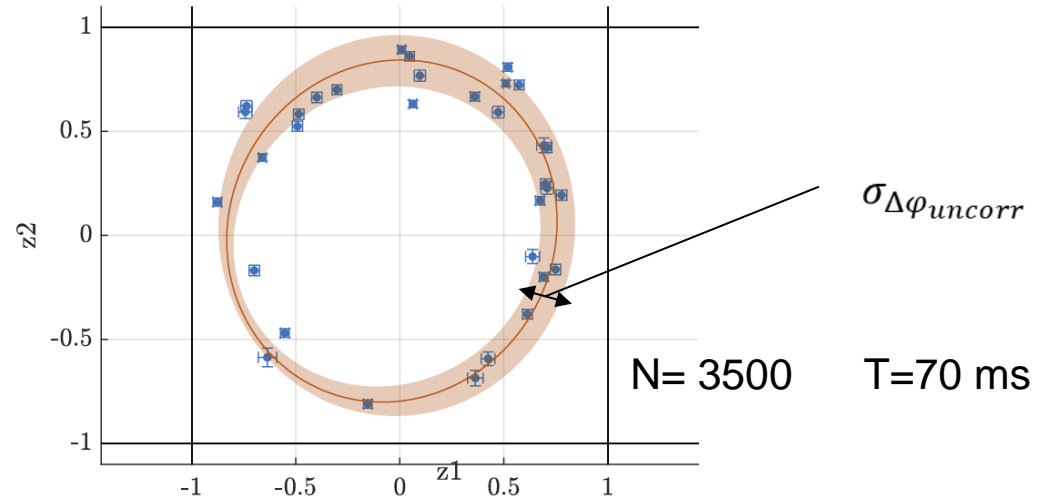


Decoherence of the differential phase

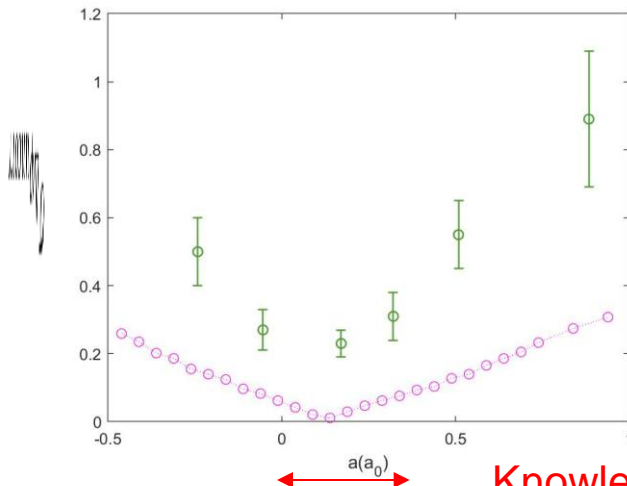
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$$\Delta\varphi_{uncorr} \propto \Delta(E_L - E_R)T \propto \Delta Z a_s T$$



$$\sigma_{\Delta\varphi_{uncorr}} \propto \sigma_{\Delta Z} a_s T \propto \frac{a_s T}{\sqrt{N}}$$



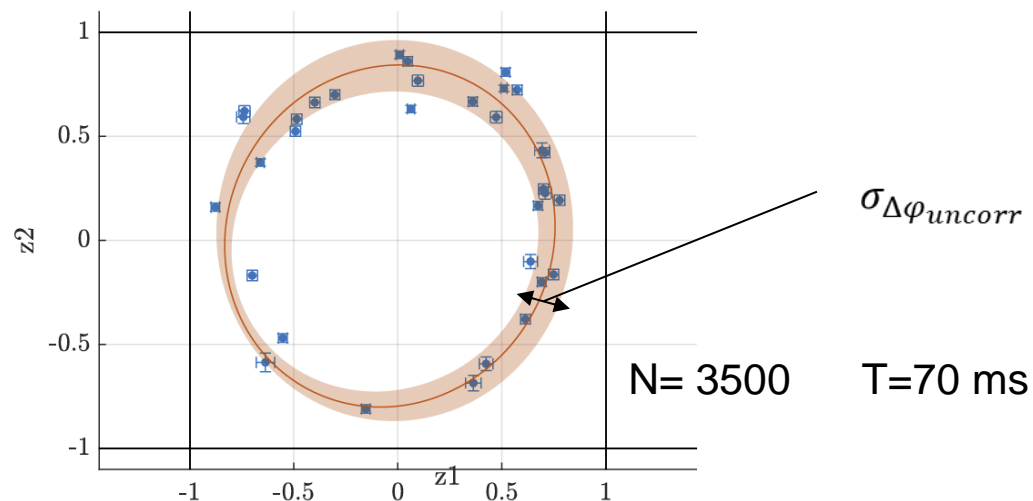
Scattering length control: $0.02 a_0$

Knowledge of the zero crossing

Decoherence of the differential phase

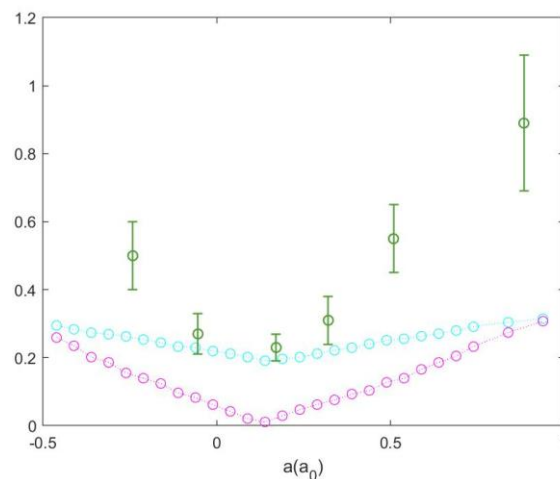
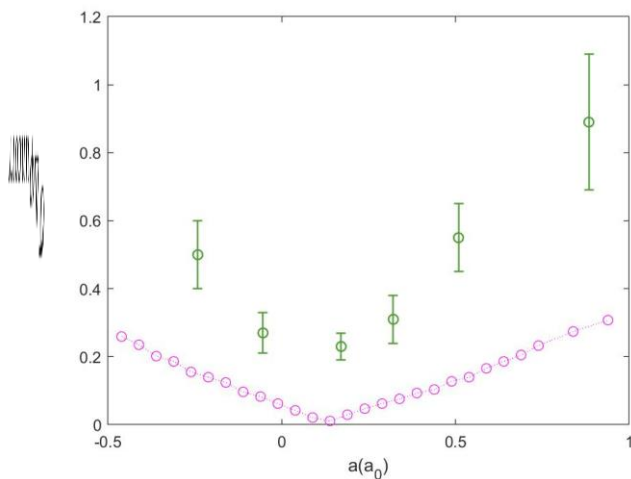
$$z_1 = \sin(\varphi_1) = \sin(-\delta T + \varepsilon T)$$

$$z_2 = \sin(\varphi_2) = \sin(\varepsilon T + \Delta\varphi_{uncorr})$$

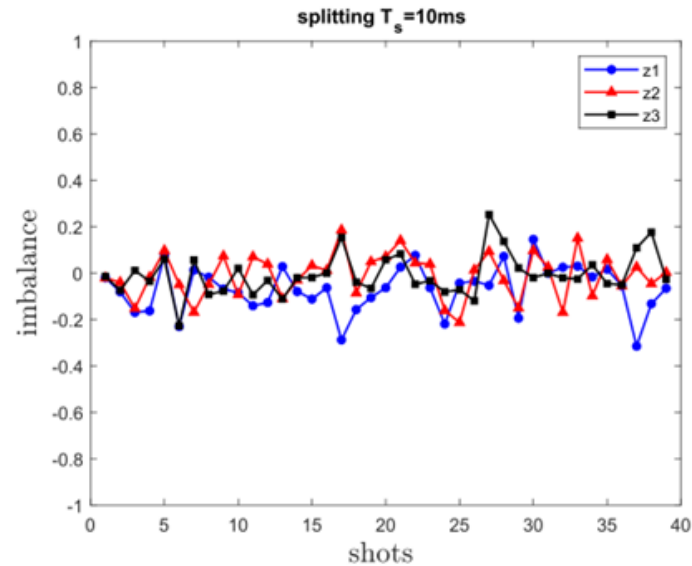


$$\sigma_{\Delta\varphi_{uncorr}} \propto \frac{a_s T}{\sqrt{N}}$$

$$\sigma_{\Delta\varphi_{uncorr}} = \sqrt{\Delta\varphi_{tech}^2 + \frac{\alpha a_s^2 T^2}{N}}$$

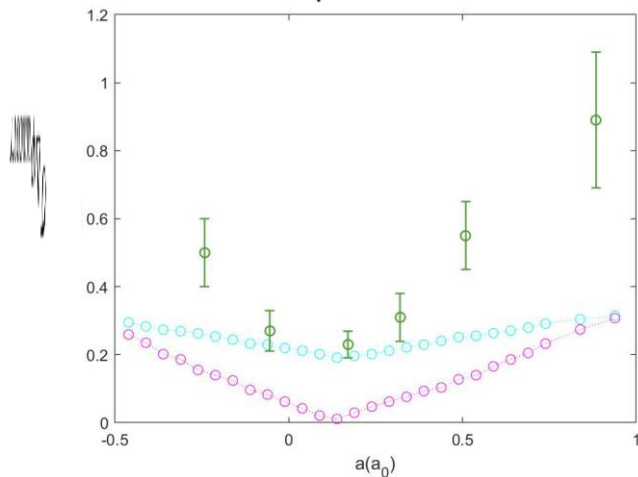


Decoherence of the differential phase

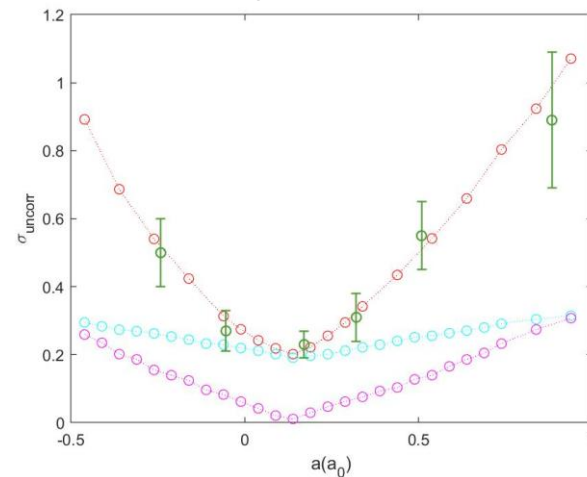


$\gamma > 1$

$$\sigma_{\Delta\varphi_{uncorr}} = \sqrt{\Delta\varphi_{tech}^2 + \frac{\alpha a_s^2 T^2}{N}}$$



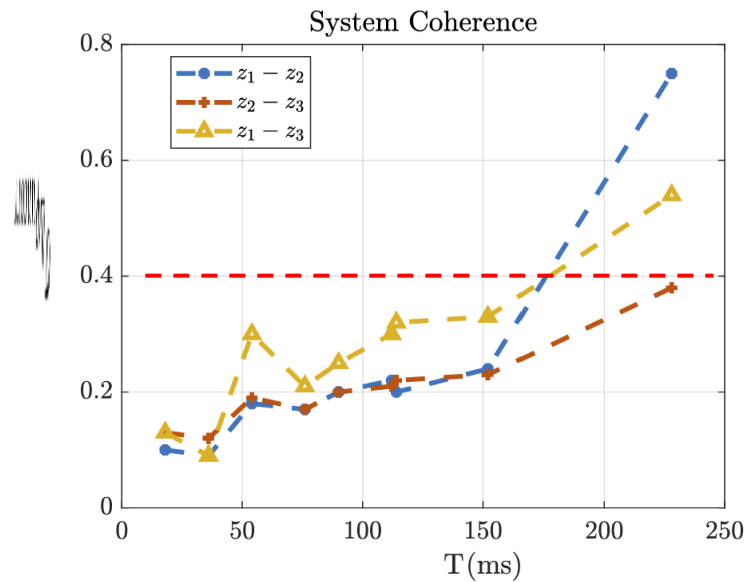
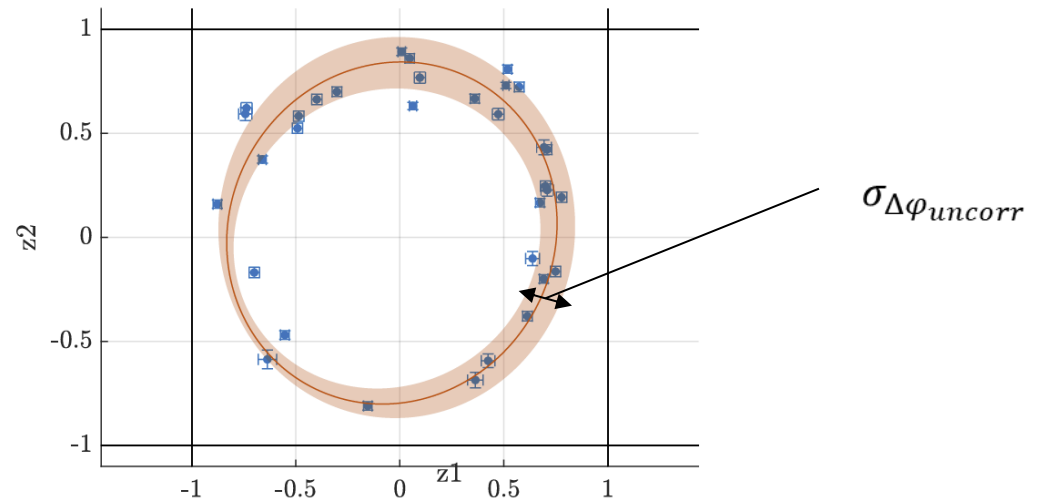
$$\sigma_{\Delta\varphi_{uncorr}} = \sqrt{\Delta\varphi_{tech}^2 + \frac{\alpha\gamma a_s^2 T^2}{N}}$$



Decoherence of the differential phase

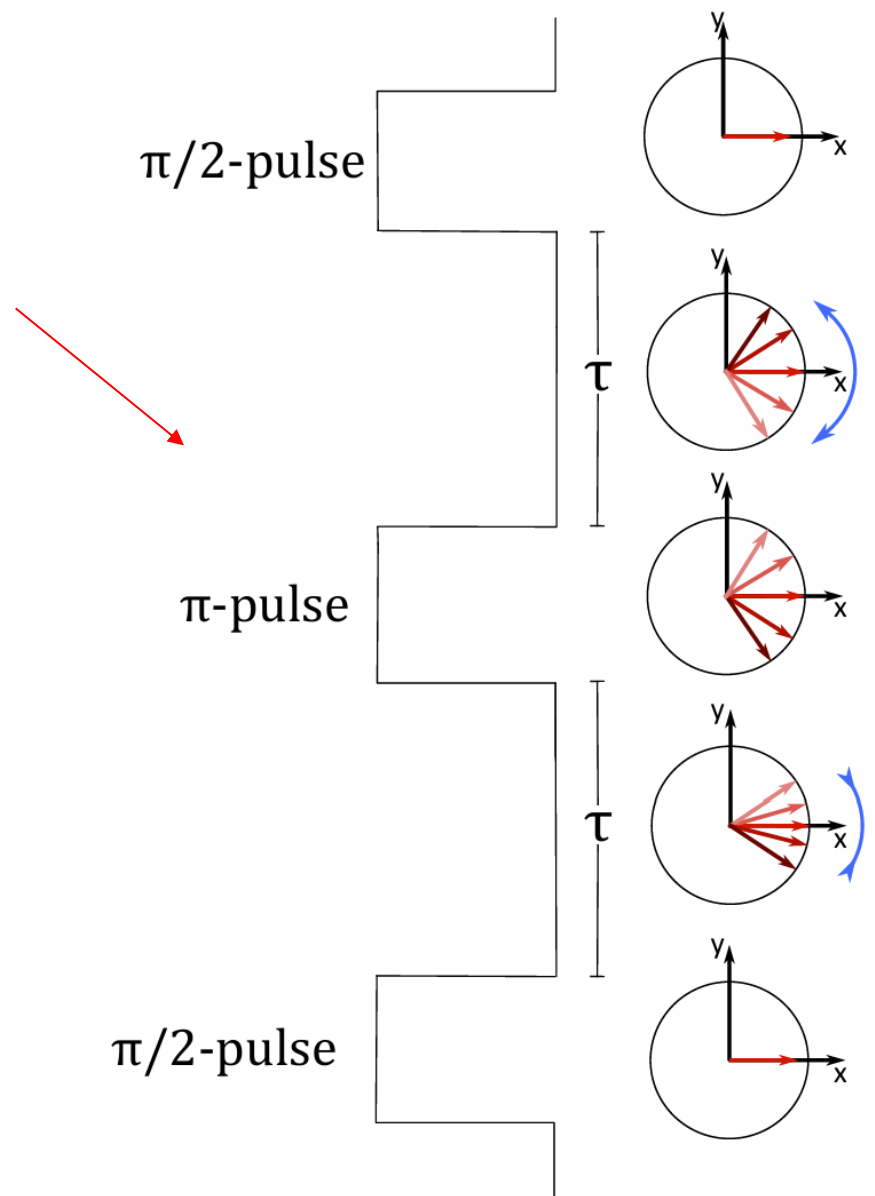
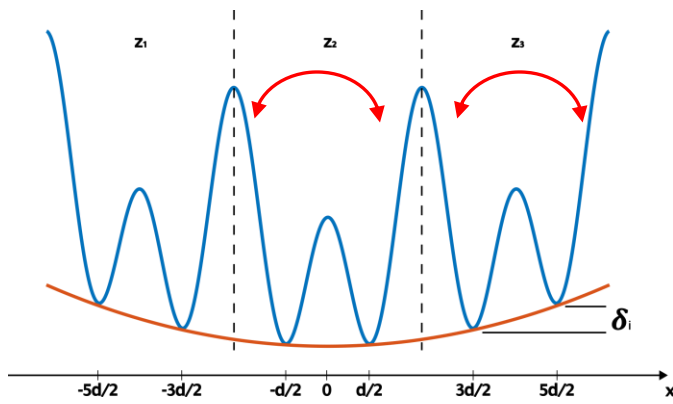
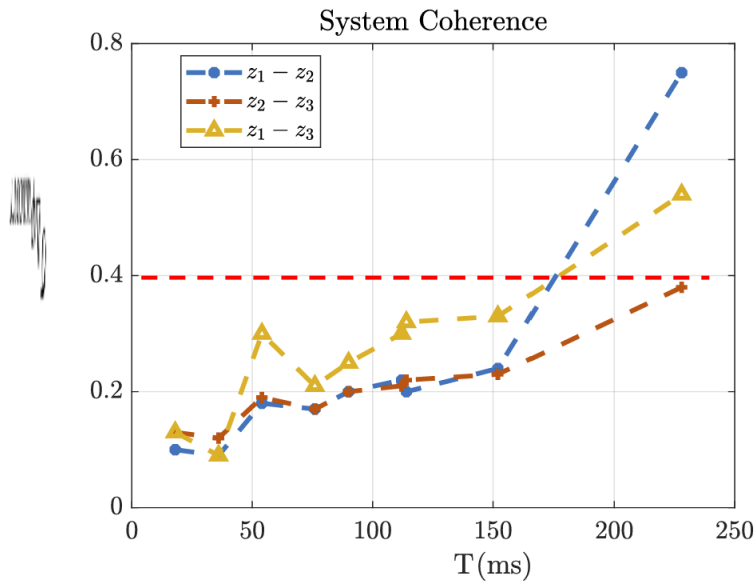
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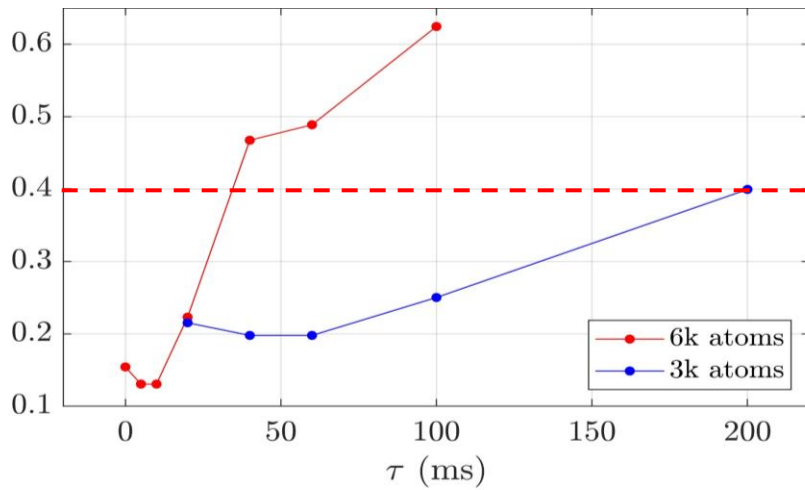
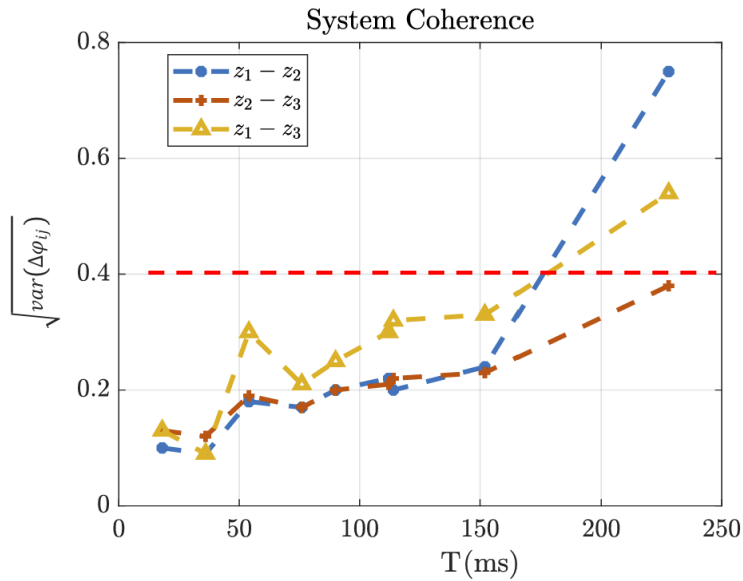


Decoherence
on the minimum
 $\sim 3 \text{ rad/s}$

Gradiometer + Spin ECHO



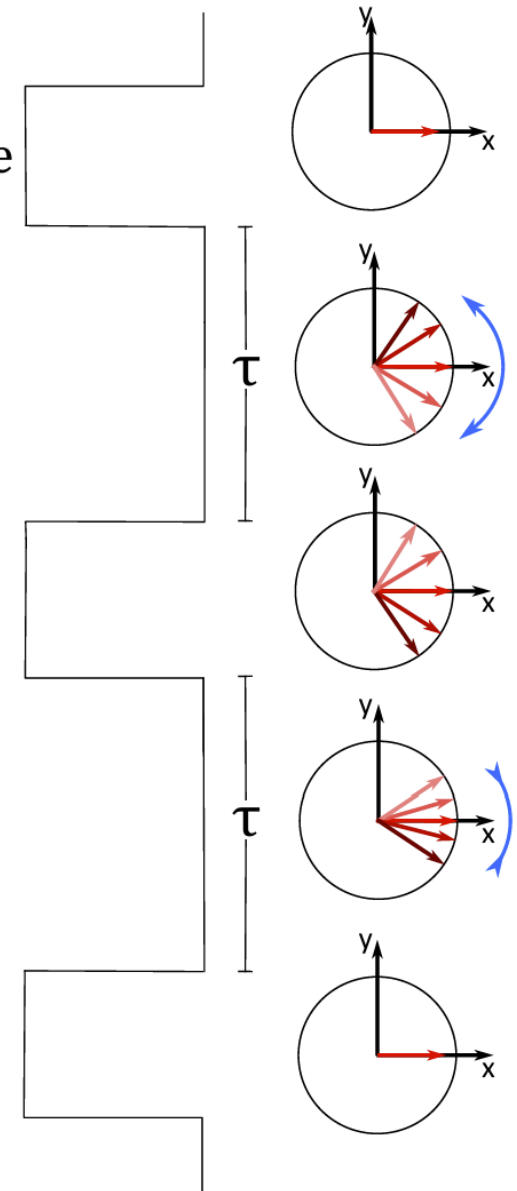
Spin ECHO



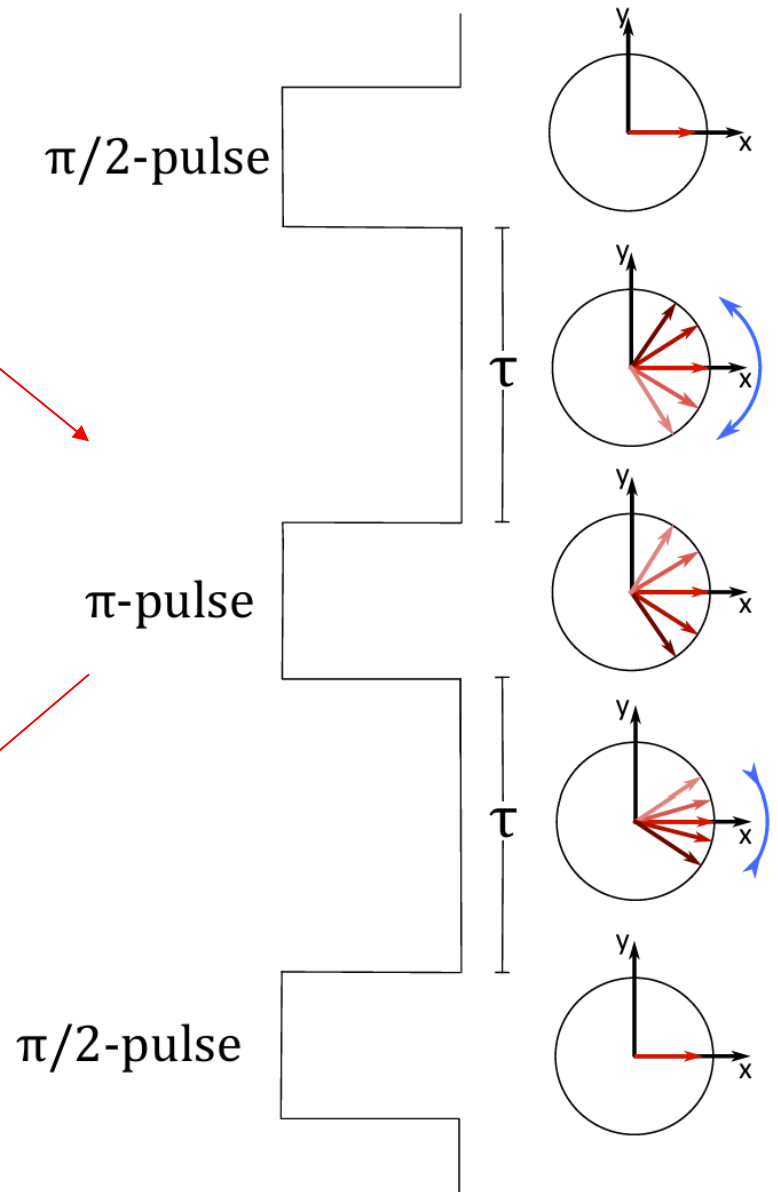
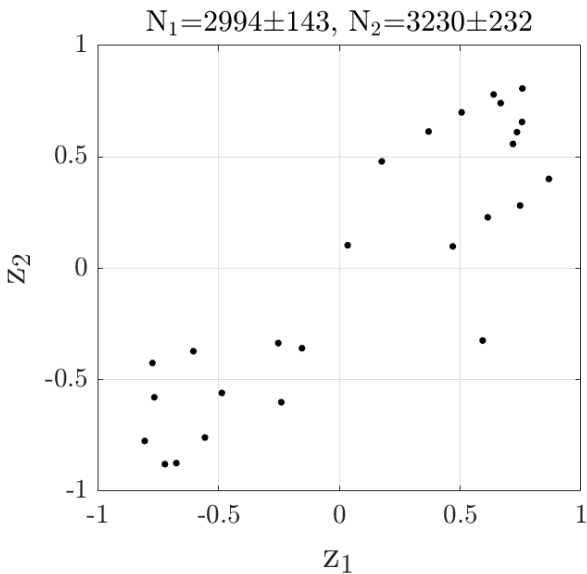
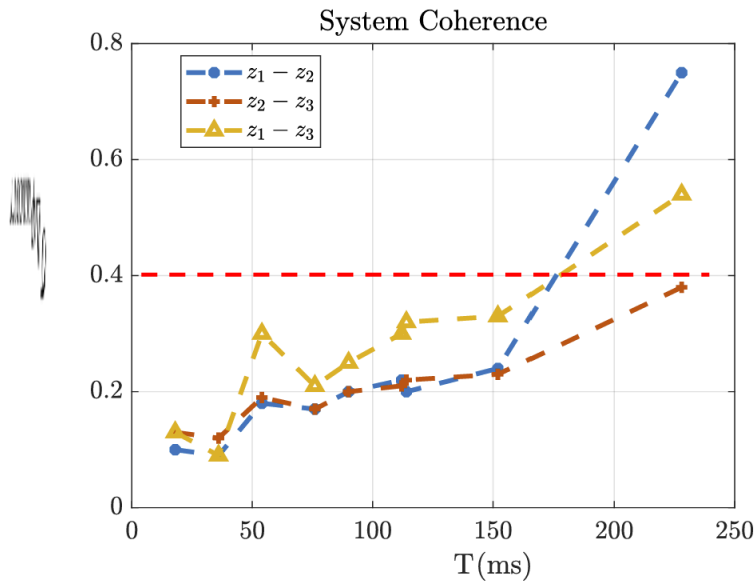
$\pi/2$ -pulse

π -pulse

$\pi/2$ -pulse

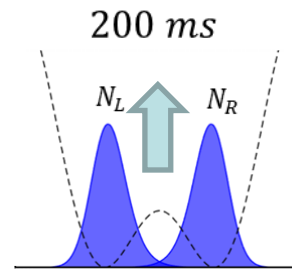


Spin ECHO

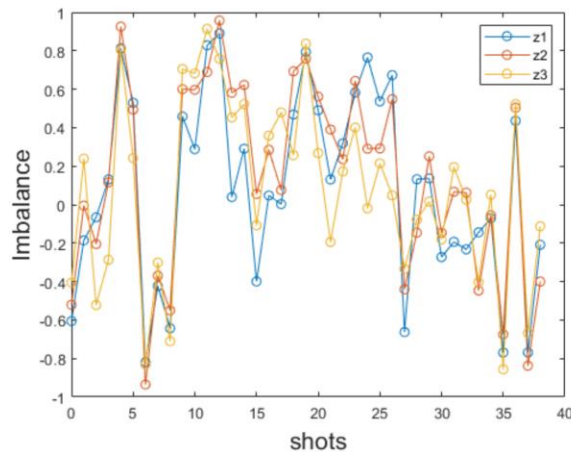


Towards number squeezing with repulsive interactions

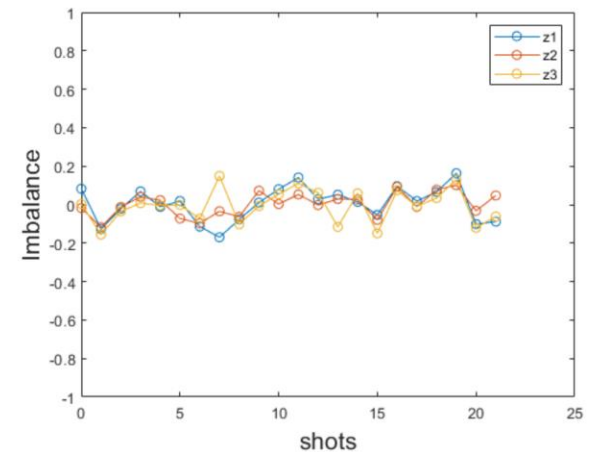
We introduce repulsive interactions during the raising of the barrier



$$a = 0a_0$$



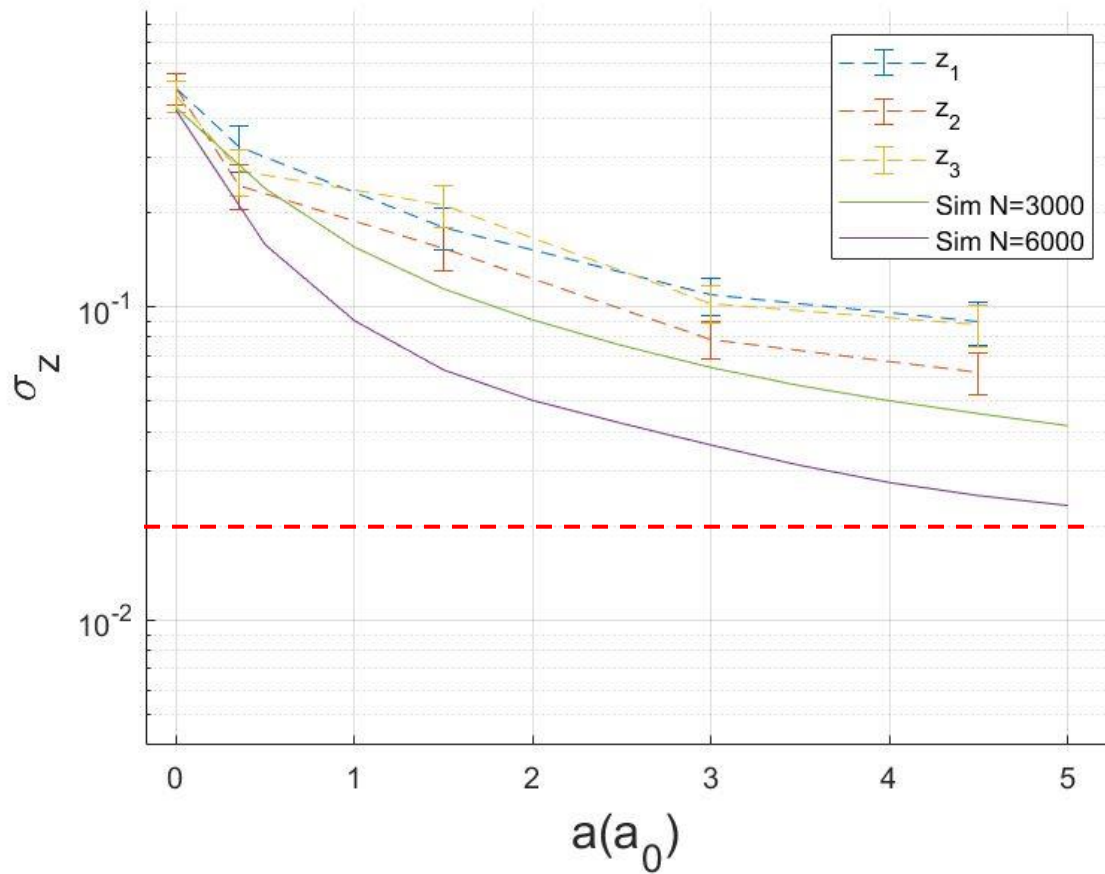
$$a = 4,5a_0$$



J. Esteve, et al. Nature **455** 1216 (2008)

C. Gross, et al. **464** 1165 (2010)
M. F. Riedel, et al. **464** 1170 (2010)

Towards number squeezing with repulsive interactions

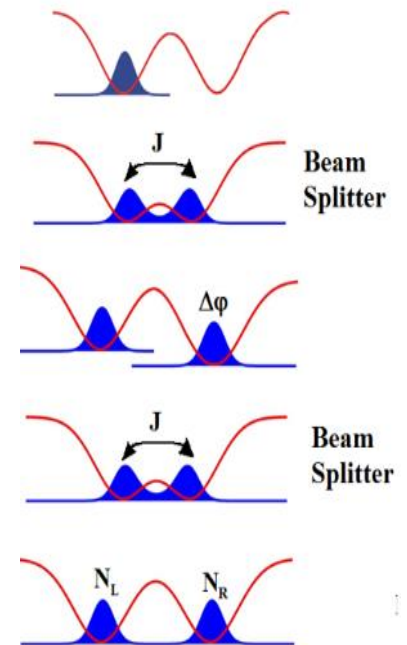
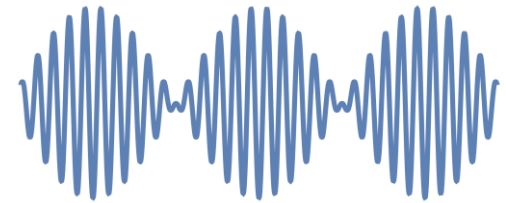
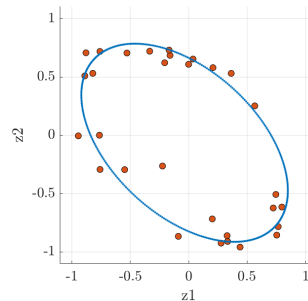
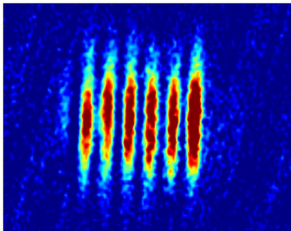


Atomic imbalance
measurements
Vs
Theoretical predictions

Shot noise limit

Summary

- We have proposed a new kind of lattice potential, i.e. Beat Note Superlattice
- We have demonstrated a trapped atom linear Mach-Zehnder with BECs with tunable interactions
- Simultaneous operation of several interferometers
-> trapped atom gradiometer



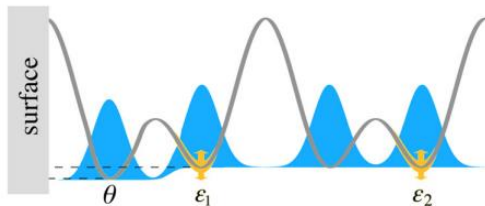
- Characterization of interaction induced decoherence
- Mandatory: identify sources of uncorrelated noise.

Outlook

- Next order interaction effects using decoherence: three body elastic scattering, light induced dipolar interactions.
- Gradiometry with quantum entangled states

.....in the long term

- Force measurements with high spatial resolution



- Compact, transportable device

The group



A. Santoni C. Mazzinghi

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M. Fattori



Collaborations



G. Modugno



M. Inguscio



D. Trypogeorgos



F.S. Cataliotti

Theoretical support



L. Pezzè



A. Smerzi



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